

CONSTRUCTION OF FISCHER'S SPORADIC GROUP Fi'_{24} INSIDE $\text{GL}_{8671}(13)$

HYUN KYU KIM AND GERHARD O. MICHLER

ABSTRACT. In this article we construct an irreducible simple subgroup $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{r}, \mathfrak{t}, \mathfrak{w} \rangle$ of $\text{GL}_{8671}(13)$ from an irreducible subgroup T of $\text{GL}_{11}(2)$ isomorphic to Mathieu's simple group \mathcal{M}_{24} by means of Algorithm 2.5 of [13]. We also use the first author's similar construction of Fischer's sporadic simple group $G_1 = \text{Fi}_{23}$ described in [11]. He starts from an irreducible subgroup T_1 of $\text{GL}_{11}(2)$ contained in T which is isomorphic to \mathcal{M}_{23} . In [7] J. Hall and L. S. Soicher published a nice presentation of Fischer's original 3-transposition group Fi_{24} [6]. It is used here to show that \mathfrak{G} is isomorphic to the simple commutator subgroup Fi'_{24} of Fi_{24} . We also determine a faithful permutation representation of \mathfrak{G} of degree 306936 with stabilizer $\mathfrak{G}_1 = \langle \mathfrak{q}, \mathfrak{r}, \mathfrak{w} \rangle \cong \text{Fi}_{23}$. It enabled MAGMA to calculate the character table of \mathfrak{G} automatically.

Furthermore, we prove that \mathfrak{G} has two conjugacy classes of involutions \mathfrak{z} and \mathfrak{u} such that $C_{\mathfrak{G}}(\mathfrak{u}) = \langle \mathfrak{q}, \mathfrak{r}, \mathfrak{t} \rangle \cong 2\text{Aut}(\text{Fi}_{22})$. Moreover, we determine a presentation of $\mathfrak{H} = C_{\mathfrak{G}}(\mathfrak{z})$ and a faithful permutation representation of degree 258048 for which we document a stabilizer.

1. INTRODUCTION

In 1971 B. Fischer [6] discovered 3 sporadic simple groups by characterizing all finite groups G which can be generated by a conjugacy class $D = z^G$ of 3-transpositions. This means that the product of 2 elements of D in G has order 1, 2 or 3. The largest of these 3 sporadic groups turned out to be the commutator subgroup Fi'_{24} of the 3-transposition group Fi_{24} . In [5] Fischer constructed for each of the 3 groups Fi_k a graph \mathcal{G}_k and showed that Fi_k is isomorphic to the automorphism group $\text{Aut}(\mathcal{G}_k)$, $k \in \{22, 23, 24\}$. Using the structure of \mathcal{G}_{24} J. Hall and L. Soicher determined a nice presentation of Fi_{24} , see [7] and [15], p. 124.

The results of this article are part of our joint research project *Simultaneous construction of the sporadic simple groups of Conway, Fischer and Janko*. Its goal is to provide uniform existence proofs for the sporadic simple groups discovered by Conway, Fischer and Janko by means of Algorithm 2.5 of [13] constructing finite simple groups from irreducible subgroups T of $\text{GL}_n(2)$. In [10] we constructed Conway's and Fischer's sporadic groups Co_2 and Fi_{22} simultaneously from the irreducible subgroup \mathcal{M}_{22} in $\text{GL}_{10}(2)$. In [11] the first author applied the same methods to the irreducible subgroup \mathcal{M}_{23} of $\text{GL}_{11}(2)$ and realized Fi_{23} as an irreducible subgroup of $\text{GL}_{782}(17)$.

In Lemma 2.1 of Section 2 we construct a presentation of the unique non split extension E of \mathcal{M}_{24} by the natural $GF(2)$ -vector space V of dimension 11 by means of Holt's Algorithm [8] implemented in MAGMA. By Fischer's work [6] it is known that this extension group has a Sylow 2-subgroup S which is isomorphic to the ones of his sporadic simple group Fi'_{24} . Lemma 2.1 also states that E has a unique conjugacy class of 2-central involutions z . Furthermore, it has a subgroup N_1 of index 24 and a non 2-central involution u such that $N_1/V \cong \mathcal{M}_{23}$ and $E = \langle N_1, C_E(u) \rangle$.

In our first attempt we applied Algorithm 2.5 of [13] to E and constructed a finitely presented group H containing a Sylow 2-subgroup S_1 having a maximal elementary abelian normal subgroup A such that $N_H(A) \cong D = C_E(z)$ and $|H : N_H(A)|$ is odd. Furthermore, we calculated the character tables of E , H and D and applied Algorithm 7.4.8 of [12] to show that the free product $Q = H *_D E$ with amalgamated subgroup D has 939,080,024,064 irreducible representations of minimal dimension 8671 over $GF(13)$. In view of our time constraints we decided not to start Thompson's amalgamation process described in Theorem 7.2.2 of [12] in order to find an irreducible representation of Q of degree 8671 which has a Sylow 2-subgroup isomorphic to $S_1 \cong S$. Instead we now construct such a matrix representation \mathfrak{G} of Q using the first author's work [11].

There he applied Algorithm 2.5 of [13] to N_1 and obtained a simple subgroup \mathfrak{G}_1 of $GL_{782}(17)$ which he showed to be isomorphic to Fischer's sporadic group Fi_{23} . He also determined its character table, a faithful permutation representation PG_1 of degree 31671 and a presentation of the centralizer $\mathfrak{H}_1 = C_{\mathfrak{G}_1}(z_1)$ of a 2-central involution u_1 of \mathfrak{G}_1 . Since $\mathfrak{H}_1 \cong 2Fi_{22}$ and $C_{N_1}(u)$ have isomorphic Sylow 2-subgroups by [11], we determine in Lemma 3.1 of Section 2 a presentation for $A_1 = 2Aut(Fi_{22})$, its character table, a system of representatives of its conjugacy classes and a faithful permutation representation PA_1 . Thus we can show that A_1 and $C_E(u)$ have isomorphic Sylow 2-subgroups where u is the non 2-central involution of E mentioned above.

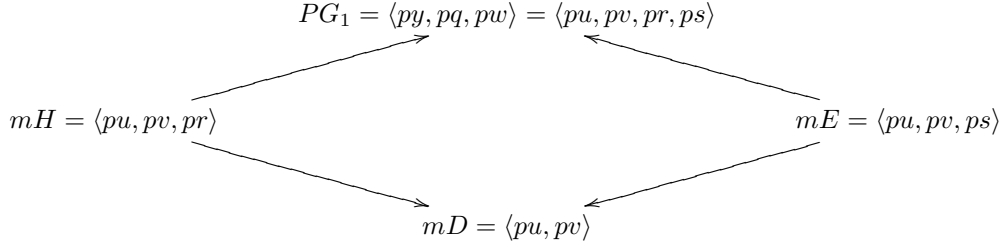
Lemma 3.2 of Section 3 asserts that the amalgam $\mathfrak{G}_1 \leftarrow \mathfrak{H}_1 \rightarrow A_1$ has 8 distinct compatible pairs of semi-simple characters of \mathfrak{G}_1 and A_1 of degree 8671. All these compatible pairs have the same semi-simple character $\tau = \tau_3 + \tau_4$ of \mathfrak{G}_1 . Its two irreducible constituents τ_3 and τ_4 have respective degrees 3588 and 5083. Each compatible pair (τ, χ_i) determines a subgroup \mathfrak{K}_i of $GL_{8671}(13)$. In the following two sections we construct these 8 matrix groups.

Since \mathfrak{G}_1 does not have any suitable subgroups whose permutation characters of \mathfrak{G}_1 contain τ_3 or τ_4 as irreducible constituents we first construct a pair mH and mE of subgroups of the permutation representation PG_1 of \mathfrak{G}_1 such that

$$PG_1 = \langle mH, mE \rangle \quad \text{and} \quad mD = mH \cap mE \cong G_2(3) \times Sym(3).$$

These subgroups enable us to construct the irreducible representations of PG_1 in $GL_{3588}(13)$ and $GL_{5083}(13)$ corresponding to τ_3 or τ_4 , respectively, by means of Thompson's methods described in Theorem 7.2.2 of [12]. The character tables of mH and mE are Tables B.2 and B.3 of the Appendix, respectively. They were automatically computed by MAGMA using PG_1 .

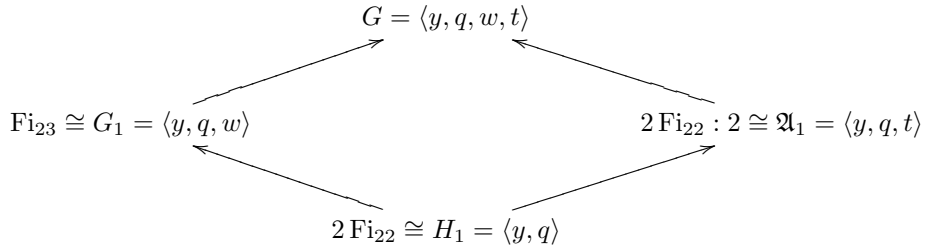
By the main result of [11] we have 3 permutations py , pq and pw and a 2-central involution pz_1 of PG_1 such that $PG_1 = \langle py, pq, pw \rangle \cong \mathfrak{G}_1$ and $C_{PG_1}(pz_1) = \langle py, pq \rangle \cong \mathfrak{H}_1$. We found these two subgroups mH and mE of PG_1 by means of the MAGMA command `LowIndexSubgroups(PG_1: n)` for $n = 137632$ and $n = 148642560$. An application of the first author's program `GetShortGens(PG_1, U)` provides short words pu , p_v , pr and ps in terms of the generators py , pq and pw of PG_1 such that $mD = \langle pu, p_v \rangle$, $mH = \langle mD, pr \rangle$ and $mE = \langle mD, ps \rangle$. For the construction of the target matrix group G it is necessary to get short words of the old generators py , pq and pw in terms of the new generators of PG_1 . This is also done in Lemma 4.1 of Section 4. Thus we obtain the following diagram of permutation groups.



The restrictions of the characters τ_3 and τ_4 of PG_1 to the two subgroups mH and mE are determined in Lemma 4.1 of Section 4. Each of their irreducible constituents is an irreducible constituent of a permutation character of mH or mE . In Propositions 4.2 and 4.3 we apply the first author's efficient implementation of Algorithm 5.7.1 of [12] and get the various matrix representations corresponding to the irreducible constituents of the permutation characters of mH and mE determined in Lemma 4.1. They enable us to construct the correct blocked matrices u, v, r and s of the generators pu, pv, pr and ps of PG_1 in $\text{GL}_{8671}(13)$ corresponding to the restrictions of the irreducible characters τ_3 and τ_4 of PG_1 to mH, mE and mD . Let y, q and w be the matrices of $\text{GL}_{8671}(13)$ obtained by inserting the matrices u, v, r and s into the words of py, pq and pw in terms of pu, pv, pr and ps . Then $G_1 = \langle y, q, w \rangle \cong PG_1$, and $H_1 = \langle y, q \rangle \cong PH_1$.

Since the restriction of $\tau_3 + \tau_4$ to mD is not multiplicity-free we have to solve a difficult amalgamation problem in order to get the corresponding irreducible representations of PG_1 . By Lemma 4.1 we know that the generator pr of mH is an involution which commutes in the known group PG_1 with the involution $pf = (pu^3pvpu^3psv)^9$. Using this information and the structure of the blocked matrices r, s, u and v we are able to solve the amalgamation problems in Propositions 4.2 and 4.3 by calculating the solutions of well determined systems of algebraic equations with coefficients in $GF(13)$.

As H_1 is isomorphic to a normal subgroup of index 2 in A_1 we use Clifford's Theorem to determine exactly 8 semi-simple representations of A_1 corresponding to the 8 compatible pairs having the same restriction to PH_1 as $\tau_3 \oplus \tau_4$, see Proposition 5.1. In Remark 5.3 we show that for exactly one compatible pair of Lemma 3.2(e) we can construct a matrix $t \in \text{GL}_{8671}(13)$ such that the matrix subgroup $G = \langle y, q, w, t \rangle$ may have a Sylow 2-subgroup isomorphic to the ones of E . For this matrix t we have the following amalgam of matrix groups:



In Section 6 we show that this matrix group G is isomorphic to the commutator subgroup P' of finitely presented group P of Hall and Soicher [7], see Lemma 6.2. Thus $G \cong \text{Fi}'_{24}$. In particular, the subgroup $G = \langle q, y, w, t \rangle$ of $\text{GL}_{8671}(13)$ is a simple

group of order $2^{21} \cdot 3^{16} \cdot 5^2 \cdot 7^3 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29$ and it has a faithful permutation representation of degree 306936 with stabilizer $G_1 = \langle q, y, w \rangle$, see Theorem 6.3.

In Section 7 we determine generators and a presentation of $H = C_G(z)$ of a 2-central involution z of G , see Proposition 7.1. Furthermore, we construct a faithful permutation representation of degree 258048 of H with a documented stabilizer. It has been used to calculate a system of representatives of its 167 conjugacy classes and its character table, see Tables A.3 and B.4., respectively.

In Section 8 we show that G has 2 conjugacy classes of involutions. They are represented by $u = [(y(y^5t)^7)^{14}]$ and $z = (xyw)^8$. Their centralizers are $C_G(u) = \langle q, y, t \rangle = A_1$ and $C_G(z) = H$. Using their character tables we calculate the group order of G using Thompson's group order formula and Theorem 6.1.4 of [14], see Proposition 8.1.

In the appendix we collect all the systems of representatives of conjugacy classes in terms of the given generators of the local subgroups of G which have been used to construct this matrix group $G \cong \text{Fi}'_{24}$. We also state the character tables of these subgroups. The four generating matrices of the simple subgroup $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{r}, \mathfrak{t}, \mathfrak{w} \rangle$ of $\text{GL}_{8671}(13)$ can be downloaded from the first author's website <http://www.math.yale.edu/~hk47/Fi24/index.html>.

Concerning our notation and terminology we refer to the books [3] and [12]. The computer algebra system MAGMA is described in Cannon-Playoust [1].

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2. AN EXTENSION OF MATHIEU'S GROUP \mathcal{M}_{24}

In [6] B. Fischer stated that his simple group Fi'_{24} has a subgroup E which is isomorphic to a non split extension of the Mathieu group \mathcal{M}_{24} by a well determined 11-dimensional irreducible \mathcal{M}_{24} -module V over $GF(2)$ such that E contains a Sylow 2-subgroup S of his simple group Fi'_{24} . In this section we construct such an extension by means of Holt's Algorithm [8] implemented in MAGMA. It follows that E is uniquely determined by \mathcal{M}_{24} up to isomorphism.

A faithful permutation representation of degree 24 of \mathcal{M}_{24} is stated in Lemma 8.2.2 of [12]. The irreducible 2-modular representations of the Mathieu group \mathcal{M}_{24} were determined by G. James [9]. Here only the 2 non isomorphic simple modules V_i , $i = 1, 2$, of dimension 11 over $F = GF(2)$ will be used. Todd's permutation representations of the Mathieu groups are stated in Lemma 8.2.2 of [12]. Therefore all conditions of Holt's Algorithm [8] implemented in MAGMA are satisfied. It constructs all split and non split extensions of \mathcal{M}_{24} by V_1 and V_2 up to isomorphism. Here only the presentation of the non split extension of \mathcal{M}_{24} by V_2 is stated.

Lemma 2.1. *Let $\mathcal{M}_{24} = \langle a, b, c, d, t, g, h, i, j, k \rangle$ be the finitely presented group of Definition 8.2.1 of [12]. Then the following statements hold:*

- (a) *The first irreducible representation V_1 of \mathcal{M}_{24} is described by the following matrices:*

$$a_1 = \begin{pmatrix} 10000000000 \\ 01000000100 \\ 00100000100 \\ 00010000100 \\ 00000100000 \\ 00001000000 \\ 00000000010 \\ 00000000101 \\ 00000000100 \\ 00000010000 \\ 00000001100 \end{pmatrix}, \quad b_1 = \begin{pmatrix} 10000001101 \\ 01000001101 \\ 00100001101 \\ 00010000100 \\ 00000011000 \\ 00000000111 \\ 00001000001 \\ 00000000001 \\ 00000000100 \\ 00000101100 \\ 00000001000 \end{pmatrix},$$

$$c_1 = \begin{pmatrix} 10000010010 \\ 01000010010 \\ 00100010110 \\ 00010000100 \\ 00000011100 \\ 00000000011 \\ 00000000110 \\ 00001000010 \\ 00000000100 \\ 00000010100 \\ 00000110100 \end{pmatrix}, \quad d_1 = \begin{pmatrix} 10000010010 \\ 010001110001 \\ 00100111000 \\ 00010111000 \\ 00000001110 \\ 00000010001 \\ 00000001011 \\ 00000100010 \\ 00001111011 \\ 00000011001 \\ 00000101011 \end{pmatrix},$$

$$t_1 = \begin{pmatrix} 10000010010 \\ 01000100110 \\ 00100001010 \\ 00010000100 \\ 00001011110 \\ 00000101100 \\ 00000110100 \\ 00000011100 \\ 00000111100 \\ 00000101011 \\ 00000110010 \end{pmatrix}, \quad g_1 = \begin{pmatrix} 10000000011 \\ 01000000100 \\ 00100000001 \\ 00001000111 \\ 00010000111 \\ 00000100111 \\ 00000010110 \\ 00000001101 \\ 00000000100 \\ 00000000010 \\ 00000000001 \end{pmatrix},$$

$$\begin{aligned}
h_1 &= \begin{pmatrix} 10000010010 \\ 01000000010 \\ 00010000010 \\ 00100000010 \\ 00001010000 \\ 00000100010 \\ 00000010000 \\ 00000010000 \\ 00000010001 \\ 00000010110 \\ 00000000010 \\ 00000011000 \end{pmatrix}, \quad i_1 = \begin{pmatrix} 10000000000 \\ 00100011000 \\ 01000110100 \\ 00010111000 \\ 00001101100 \\ 00000001100 \\ 00000101000 \\ 00000011100 \\ 00000111100 \\ 00000100101 \\ 00000110010 \end{pmatrix}, \\
j_1 &= \begin{pmatrix} 01000100000 \\ 10000100000 \\ 00100100100 \\ 00010000100 \\ 00001000100 \\ 00000100000 \\ 00000110100 \\ 00000101000 \\ 00000000100 \\ 00000000101 \\ 00000000110 \end{pmatrix} \quad \text{and} \quad k_1 = \begin{pmatrix} 11111110111 \\ 01000101000 \\ 00100111000 \\ 00010111000 \\ 00001010000 \\ 00000001000 \\ 00000010000 \\ 00000100000 \\ 00000111100 \\ 00000011001 \\ 00000110010 \end{pmatrix}.
\end{aligned}$$

- (b) The second irreducible representation V_2 of \mathcal{M}_{24} is described by the transpose inverse matrices of the generating matrices of \mathcal{M}_{24} defining V_1 :

$$a_2 = [a_1^{-1}]^T, \quad b_2 = [b_1^{-1}]^T, \quad c_2 = [c_1^{-1}]^T, \quad d_2 = [d_1^{-1}]^T, \quad t_2 = [t_1^{-1}]^T, \quad g_2 = [g_1^{-1}]^T, \quad h_2 = [h_1^{-1}]^T, \quad i_2 = [i_1^{-1}]^T, \quad j_2 = [j_1^{-1}]^T, \quad \text{and} \quad k_2 = [k_1^{-1}]^T.$$

- (c) $\dim_F[H^2(\mathcal{M}_{24}, V_1)] = 0$ and $\dim_F[H^2(\mathcal{M}_{24}, V_2)] = 1$.

In particular, there is a uniquely determined non split extension E of \mathcal{M}_{24} by V_2 .

- (d) The non split extension

$$E = E(Fi'_{24}) = \langle a, b, c, d, t, g, h, i, j, k, v_1, v_2, v_3, v_4, v_5, v_6, v_8, v_8, v_9, v_{10}, v_{11} \rangle$$

of \mathcal{M}_{24} by V_2 has a set $\mathcal{R}(E)$ of defining relations consisting of $\mathcal{R}_1(V_2 \rtimes \mathcal{M}_{24})$ and the following relations:

$$\begin{aligned}
v_i^2 &= 1 \quad \text{and} \quad v_k v_j = v_j v_k \quad \text{for all} \quad 1 \leq i, j, k \leq 11. \\
a^{-1} v_1 a v_1^{-1} v_2^{-1} v_5^{-1} v_6^{-1} v_7^{-1} v_8^{-1} v_{10}^{-1} &= a^{-1} v_2 a v_{10}^{-1} = a^{-1} v_3 a v_4^{-1} = 1, \\
a^{-1} v_4 a v_3^{-1} &= a^{-1} v_5 a v_6^{-1} = a^{-1} v_6 a v_5^{-1} = a^{-1} v_7 a v_8^{-1} = a^{-1} v_8 a v_7^{-1} = 1, \\
a^{-1} v_9 a v_9^{-1} &= a^{-1} v_{10} a v_2^{-1} = a^{-1} v_{11} a v_{11}^{-1} = b^{-1} v_1 b v_6^{-1} = b^{-1} v_2 b v_8^{-1} = 1, \\
b^{-1} v_3 b v_1^{-1} v_2^{-1} v_4^{-1} v_6^{-1} v_{10}^{-1} v_{11}^{-1} &= b^{-1} v_4 b v_1^{-1} v_3^{-1} v_6^{-1} v_7^{-1} v_8^{-1} v_{11}^{-1} = 1, \\
b^{-1} v_5 b v_1^{-1} v_2^{-1} v_5^{-1} v_6^{-1} v_7^{-1} v_8^{-1} v_{10}^{-1} &= b^{-1} v_6 b v_1^{-1} = b^{-1} v_7 b v_{10}^{-1} = 1, \\
b^{-1} v_8 b v_2^{-1} &= b^{-1} v_9 b v_9^{-1} = b^{-1} v_{10} b v_7^{-1} = b^{-1} v_{11} b v_{11}^{-1} = c^{-1} v_1 c v_{10}^{-1} = 1, \\
c^{-1} v_2 c v_1^{-1} v_2^{-1} v_5^{-1} v_6^{-1} v_7^{-1} v_8^{-1} v_{10}^{-1} &= c^{-1} v_3 c v_2^{-1} v_4^{-1} v_5^{-1} v_8^{-1} v_9^{-1} v_{10}^{-1} = 1, \\
c^{-1} v_4 c v_2^{-1} v_3^{-1} v_6^{-1} v_7^{-1} v_9^{-1} v_{10}^{-1} &= c^{-1} v_5 c v_8^{-1} = c^{-1} v_6 c v_7^{-1} = c^{-1} v_7 c v_6^{-1} = 1, \\
c^{-1} v_8 c v_5^{-1} &= c^{-1} v_9 c v_9^{-1} = c^{-1} v_{10} c v_1^{-1} = c^{-1} v_{11} c v_{11}^{-1} = 1, \\
d^{-1} v_1 d v_3^{-1} v_5^{-1} v_7^{-1} v_8^{-1} v_9^{-1} v_{10}^{-1} v_{11}^{-1} &= d^{-1} v_2 d v_1^{-1} v_3^{-1} v_6^{-1} v_7^{-1} v_8^{-1} v_{11}^{-1} = 1, \\
d^{-1} v_3 d v_7^{-1} &= d^{-1} v_4 d v_8^{-1} = d^{-1} v_5 d v_2^{-1} v_3^{-1} v_6^{-1} v_7^{-1} v_9^{-1} v_{10}^{-1} = 1, \\
d^{-1} v_6 d v_2^{-1} v_4^{-1} v_5^{-1} v_8^{-1} v_9^{-1} v_{10}^{-1} &= d^{-1} v_7 d v_3^{-1} = d^{-1} v_8 d v_4^{-1} = 1,
\end{aligned}$$

$$\begin{aligned}
d^{-1}v_9dv_9^{-1} &= d^{-1}v_{10}dv_1^{-1}v_2^{-1}v_4^{-1}v_6^{-1}v_{10}^{-1}v_{11}^{-1} = d^{-1}v_{11}dv_{11}^{-1} = 1, \\
t^{-1}v_1tv_6^{-1}v_{11}^{-1} &= t^{-1}v_2tv_{10}^{-1}v_{11}^{-1} = t^{-1}v_3tv_7^{-1}v_{11}^{-1} = 1, \\
t^{-1}v_4tv_2^{-1}v_4^{-1}v_5^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} &= t^{-1}v_5tv_8^{-1}v_{11}^{-1} = 1, \\
t^{-1}v_6tv_2^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} &= 1, \\
t^{-1}v_7tv_1^{-1}v_2^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1}v_{11}^{-1} &= 1, \\
t^{-1}v_8tv_1^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_8^{-1} &= t^{-1}v_9tv_{11}^{-1} = 1, \\
t^{-1}v_{10}tv_3^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1} &= t^{-1}v_{11}tv_9^{-1}v_{11}^{-1} = g^{-1}v_1gv_4^{-1}v_6^{-1} = 1, \\
g^{-1}v_2gv_4^{-1}v_{10}^{-1} &= g^{-1}v_3gv_3^{-1}v_4^{-1} = g^{-1}v_4gv_4^{-1} = g^{-1}v_5gv_4^{-1}v_8^{-1} = 1, \\
g^{-1}v_6gv_1^{-1}v_4^{-1} &= g^{-1}v_7gv_1^{-1}v_2^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} = 1, \\
g^{-1}v_8gv_4^{-1}v_5^{-1} &= g^{-1}v_9gv_2^{-1}v_5^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1} = g^{-1}v_{10}gv_2^{-1}v_4^{-1} = 1, \\
g^{-1}v_{11}gv_1^{-1}v_2^{-1}v_6^{-1}v_{10}^{-1}v_{11}^{-1} &= h^{-1}v_1hv_1^{-1}v_9^{-1} = h^{-1}v_2hv_9^{-1}v_{10}^{-1} = 1, \\
h^{-1}v_3hv_7^{-1}v_9^{-1} &= h^{-1}v_4hv_8^{-1}v_9^{-1} = h^{-1}v_5hv_2^{-1}v_4^{-1}v_5^{-1}v_8^{-1}v_{10}^{-1} = 1, \\
h^{-1}v_6hv_2^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1} &= h^{-1}v_7hv_3^{-1}v_9^{-1} = 1, \\
h^{-1}v_8hv_4^{-1}v_9^{-1} &= h^{-1}v_9hv_9^{-1} = h^{-1}v_{10}hv_2^{-1}v_9^{-1} = 1, \\
h^{-1}v_{11}hv_9^{-1}v_{11}^{-1} &= i^{-1}v_1iv_2^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1} = 1, \\
i^{-1}v_2iv_3^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1}v_{11}^{-1} &= i^{-1}v_3iv_5^{-1}v_9^{-1} = 1, \\
i^{-1}v_4iv_4^{-1}v_9^{-1} &= i^{-1}v_5iv_3^{-1}v_9^{-1} = i^{-1}v_6iv_6^{-1}v_9^{-1} = 1, \\
i^{-1}v_7iv_1^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} &= i^{-1}v_8iv_1^{-1}v_2^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1} = 1, \\
i^{-1}v_9iv_9^{-1} &= i^{-1}v_{10}iv_9^{-1}v_{10}^{-1} = i^{-1}v_{11}iv_9^{-1}v_{11}^{-1} = j^{-1}v_1jv_6^{-1} = 1, \\
j^{-1}v_2jv_1^{-1}v_2^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} &= 1, \\
j^{-1}v_3jv_3^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} &= 1, \\
j^{-1}v_4jv_4^{-1} &= j^{-1}v_5jv_8^{-1} = j^{-1}v_6jv_1^{-1} = j^{-1}v_7jv_{10}^{-1} = 1, \\
j^{-1}v_8jv_5^{-1} &= j^{-1}v_9jv_{11}^{-1} = j^{-1}v_{10}jv_7^{-1} = j^{-1}v_{11}jv_9^{-1} = 1, \\
k^{-1}v_1kv_3^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1}v_{11}^{-1} &= 1, \\
k^{-1}v_2kv_2^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1} &= k^{-1}v_3kv_7^{-1}v_9^{-1} = 1, \\
k^{-1}v_4kv_4^{-1}v_9^{-1} &= k^{-1}v_5kv_1^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} = 1, \\
k^{-1}v_6kv_9^{-1}v_{10}^{-1} &= k^{-1}v_7kv_3^{-1}v_9^{-1} = k^{-1}v_8kv_8^{-1}v_9^{-1} = k^{-1}v_9kv_9^{-1} = 1, \\
k^{-1}v_{10}kv_6^{-1}v_9^{-1} &= k^{-1}v_{11}kv_9^{-1}v_{11}^{-1} = a^2v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1} = 1, \\
b^2v_1^{-1}v_3^{-1}v_4^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{11}^{-1} &= c^2v_1^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} = 1, \\
d^2v_1^{-1}v_2^{-1}v_3^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{11}^{-1} &= b^{-1}aba^{-1}v_1^{-1}v_3^{-1}v_4^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1} = 1, \\
c^{-1}aca^{-1}v_2^{-1}v_{10}^{-1} &= d^{-1}ada^{-1}v_5^{-1}v_6^{-1}v_9^{-1} = 1, \\
c^{-1}bcb^{-1}v_1^{-1}v_3^{-1}v_4^{-1}v_8^{-1}v_9^{-1} &= d^{-1}bdb^{-1}v_2^{-1}v_4^{-1} = 1, \\
d^{-1}cdc^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_{10}^{-1}v_{11}^{-1} &= t^3v_1^{-1}v_3^{-1}v_8^{-1} = 1, \\
t^{-1}atd^{-1}c^{-1}v_2^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} &= 1, \\
t^{-1}btd^{-1}a^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1}v_{11}^{-1} &= 1, \\
t^{-1}ctd^{-1}b^{-1}v_1^{-1}v_2^{-1}v_4^{-1}v_5^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} &= 1, \\
t^{-1}dtd^{-1}b^{-1}a^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} &= 1, \\
g^2v_5^{-1}v_8^{-1} &= gagagav_2^{-1}v_3^{-1}v_4^{-1}v_7^{-1}v_8^{-1}v_{11}^{-1} = 1, \\
gbgbgbv_1^{-1}v_3^{-1}v_4^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1} &= gcgcgcgv_1^{-1}v_2^{-1}v_5^{-1}v_6^{-1}v_9^{-1} = 1,
\end{aligned}$$

$$\begin{aligned}
gtgtv_4^{-1}v_{11}^{-1} &= h^2v_4^{-1}v_8^{-1}v_9^{-1} = h^{-1}aha^{-1}v_2^{-1}v_3^{-1}v_4^{-1}v_9^{-1}v_{10}^{-1} = 1, \\
h^{-1}bhd^{-1}b^{-1}a^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} &= 1, \\
h^{-1}chc^{-1}a^{-1}v_1^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_{10}^{-1}v_{11}^{-1} &= 1, \\
h^{-1}dhd^{-1}v_2^{-1}v_4^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1} &= 1, \\
h^{-1}thtv_1^{-1}v_2^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1}v_{11}^{-1} &= 1, \\
ghghghv_1^{-1}v_3^{-1}v_4^{-1}v_6^{-1}v_8^{-1}v_9^{-1} &= 1, \\
i^2v_1^{-1}v_5^{-1}v_6^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} &= i^{-1}aid^{-1}c^{-1}v_3^{-1}v_5^{-1}v_7^{-1}v_9^{-1}v_{10}^{-1} = 1, \\
i^{-1}bid^{-1}a^{-1}v_2^{-1}v_3^{-1}v_4^{-1}v_6^{-1}v_9^{-1}v_{11}^{-1} &= 1, \\
i^{-1}cid^{-1}c^{-1}b^{-1}a^{-1}v_3^{-1}v_5^{-1}v_7^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} &= 1, \\
i^{-1}did^{-1}c^{-1}b^{-1}v_1^{-1}v_3^{-1}v_4^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} &= 1, \\
i^{-1}titv_1^{-1}v_5^{-1}v_6^{-1}v_8^{-1} &= i^{-1}gig^{-1}t^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_8^{-1}v_{10}^{-1}v_{11}^{-1} = 1, \\
hihihiv_1^{-1}v_2^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1}v_{11}^{-1} &= j^2v_9^{-1}v_{11}^{-1} = 1, \\
j^{-1}ajc^{-1}b^{-1}a^{-1}v_1^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_9^{-1} &= 1, \\
j^{-1}bjb^{-1}v_9^{-1}v_{11}^{-1} &= j^{-1}cjc^{-1}v_1^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} = 1, \\
j^{-1}dj d^{-1}c^{-1}v_2^{-1}v_5^{-1}v_8^{-1} &= j^{-1}tjtv_3^{-1}v_5^{-1}v_8^{-1}v_{10}^{-1} = 1, \\
j^{-1}ggj^{-1}v_2^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} &= j^{-1}hjh^{-1}t^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_8^{-1}v_{11}^{-1} = 1, \\
ijijijv_1^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_8^{-1} &= k^2v_1^{-1}v_5^{-1}v_6^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} = 1, \\
k^{-1}akd^{-1}a^{-1}v_4^{-1}v_6^{-1}v_8^{-1}v_9^{-1} &= k^{-1}bk d^{-1}c^{-1}v_1^{-1}v_2^{-1}v_7^{-1}v_9^{-1} = 1, \\
k^{-1}ckd^{-1}b^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} &= 1, \\
k^{-1}dkd^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_7^{-1} &= k^{-1}tktv_2^{-1}v_5^{-1}v_6^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} = 1, \\
k^{-1}gkg^{-1}t^{-1}v_2^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_9^{-1}v_{10}^{-1} &= k^{-1}hkh^{-1}v_2^{-1}v_4^{-1}v_8^{-1}v_{10}^{-1} = 1, \\
k^{-1}iki^{-1}v_2^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1}v_{11}^{-1} &= jkjkjv_2^{-1}v_5^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1} = 1.
\end{aligned}$$

- (e) E has a faithful permutation representation PE of degree 1518 with stabilizer $T = \langle g, h, i, (dg)^5, (dhjk)^3, (ijkj)^2, (dhjdg)^3 \rangle$.
- (f) $z = a^2$ is a 2-central involution of E with centralizer $D = C_E(z) = \langle x, y \rangle$ of order $2^{21} \cdot 3^3 \cdot 5$ where $x = a(agik)^3$ and $y = d(cgihj)^4$ have orders 4 and 6, respectively and $z = (xy^3)^{12}$. Furthermore, $E = \langle x, y, e \rangle$, where $e = g$ has order 4.
- (g) $E = \langle x, y, e \rangle$ has 73 conjugacy classes. A system of their representatives is given in Table A.1.
- (h) Table B.1 is the character table of E .
- (i) V_2 is the unique maximal elementary abelian normal subgroup of each Sylow 2-subgroup S of the extension group E .
- (j) $C_E(V_2) = V_2$.
- (k) $N_1 = \langle a, b, c, d, t, g, h, i, j, V_2 \rangle$ is a non split extension of M_{23} by V_2 .
- (l) $u = (agt)^5$ is an involution of N_1 generating the center $Z(R_2)$ of $C_{R_1}(u)$.
- (m) $E = \langle N_1, C_E(u) \rangle$.

Proof. (a) The 2 irreducible $F\mathcal{M}_{24}$ -modules V_i , $i = 1, 2$, occur as composition factors with multiplicity 1 in the permutation module $(1_{\mathcal{M}_{23}})^{\mathcal{M}_{24}}$ and can easily be constructed using the faithful permutation representation of \mathcal{M}_{24} stated in (a) and the Meataxe algorithm implemented in MAGMA. The corresponding matrices of the generators of \mathcal{M}_{24} with respect to the first irreducible representation of \mathcal{M}_{24} are stated in (a). The second irreducible representation V_2 of \mathcal{M}_{24} is dual to V_1 and so it is defined by the equations given in (b).

(c) The cohomological dimensions $d_i = \dim_F[H^2(\mathcal{M}_{24}, V_i)]$, $i = 1, 2$, have been calculated by means of MAGMA using Holt's Algorithm 7.4.5 of [12], the presentation of \mathcal{M}_{24} of Definition 8.2.1 of [12] and all the data stated in (a) and (b). It follows that $d_1 = 0$ and $d_2 = 1$.

(d) The presentation of E has been obtained by means of step 3 of Holt's Algorithm 7.4.5 of [12] and MAGMA.

(e) Using a stand-alone program due Paul Young we found a faithful permutation representation pE of E of degree 24288 with stabilizer $\langle (i^{-1}j^{-1}(bg)^2, (i^{-1}bth^{-1})^4) \rangle$. Applying the MAGMA command `DegreeReduction(pE)` we obtained the faithful permutation representation PE of degree 1518. The given generators of its stabilizer T were obtained by means of the first author's program `GetShortGens(PE, BasicStabilizer(PE, 2))`.

(f) Using MAGMA and the faithful permutation representation PE of E the reader easily verifies that the centralizer $C_E(z)$ of $z = a^2$ has order $2^{21} \cdot 3^3 \cdot 5$. Hence z is a 2-central involution of E by (d). The words of the generators x, y of D were calculated by means of PE , MAGMA and the first author's program `GetShortGens(PE, PD)`. Another check with MAGMA and PE verifies that $E = \langle D, g \rangle$.

(g) Using Kratzer's Algorithm 5.3.18 of [12], the faithful permutation representation PE and MAGMA we observed that E has 73 conjugacy classes. Their representatives are given in Table A.1.

(h) The character table of E was automatically computed by MAGMA using PE .

The remaining 4 statements can be checked with MAGMA and the faithful permutation representation PE . \square

3. THE 2-FOLD COVER OF THE AUTOMORPHISM GROUP $\text{Aut}(\text{Fi}_{22})$

Applying Algorithm 2.5 of [13] to an extension group isomorphic to the subgroup N_1 of $E = \langle x, y, e \rangle$ described in Lemma 2.1(k) H. Kim realized Fischer's second sporadic simple group Fi_{23} as an irreducible subgroup G_1 of $\text{GL}_{782}(17)$ in his senior thesis [11]. He showed that the centralizer $H_1 = C_{G_1}(u)$ of a 2-central involution u of G_1 is isomorphic to the 2-fold cover 2Fi_{22} of Fischer's smallest sporadic simple group Fi_{22} . Furthermore, he constructed a faithful permutation representation PG_1 of G_1 of degree 31671. In this section we use these results to construct the 2-fold cover A_1 of the automorphism group $\text{Aut}(H_1)$ and show that H_1 is its commutator subgroup. Thus we obtain an amalgam $A_1 \leftarrow H_1 \rightarrow G_1$ such that A_1 and $D_1 = C_E(z_1)$ have isomorphic Sylow 2-subgroups where z_1 is the involution e^2 of E . Using the character tables of the 3 groups of the amalgam we also show that it has 8 compatible pairs of semi-simple characters of degree 8671.

Lemma 3.1. *Let A_1 be the 2-fold cover of the automorphism group $\text{Aut}(\text{Fi}_{22})$ of Fischer's simple group Fi_{22} and let $H_1 = A'_1$ be its derived subgroup. Let $E = \langle x, y, e \rangle$ be the non split extension of \mathcal{M}_{24} by its simple $\text{GF}(2)$ -module constructed in Lemma 2.1. Then the following assertions hold:*

- (a) $H_1 = \langle a, b, c, d, e, f, g, h, i, z \rangle$ has the following set $\mathcal{R}(H_1)$ of defining relations:

$$\begin{aligned}
a^2 &= b^2 = c^2 = d^2 = e^2 = f^2 = g^2 = h^2 = i^2 = 1, \\
(ab)^3 &= 1, (bc)^3 = z, (cd)^3 = (de)^3 = 1, (ef)^3 = (fg)^3 = z, \\
(ac)^2 &= (ad)^2 = (ae)^2 = (af)^2 = (ag)^2 = (ah)^2 = (ai)^2 = 1, \\
(bd)^2 &= (be)^2 = (bf)^2 = (bg)^2 = (bh)^2 = (bi)^2 = 1, \\
(ce)^2 &= (cf)^2 = (cg)^2 = (ch)^2 = (ci)^2 = 1, \\
(df)^2 &= (dg)^2 = (eg)^2 = (eh)^2 = (ei)^2 = 1, \\
(dh)^3 &= (hi)^3 = (di)^2 = (fh)^2 = (fi)^2 = (gh)^2 = (gi)^2 = 1, \\
(dcbdefdhi)^{10} &= (abcdefh)^9 = (bcdefgh)^9 = 1, \\
z^2 &= (z, a) = (z, b) = (z, c) = (z, d) = (z, e) = (z, f) = 1, \\
(z, g) &= (z, h) = (z, i) = 1.
\end{aligned}$$

- (b) $A_1 = \langle H_1, t \rangle$ has a set $\mathcal{R}(A_1)$ of defining relations consisting of $\mathcal{R}(H_1)$ and the following relations:

$$t^2 = 1, \quad (z, t) = 1, \quad a^t g = 1, \quad b^t f = c^t e = (dt)^2 = (ht)^2 = (it)^2 = z.$$

- (c) $A_1 = 2\text{Aut}(\text{Fi}_{22})$ has a faithful permutation representation PA_1 of degree 56320 with stabilizer $\langle bz, c, d, e, fz, g, h, i \rangle$.
(d) A system of representatives a_i of the 150 conjugacy classes of A_1 and the corresponding centralizers orders $|C_A(a_i)|$ are given in Table A.2.
(e) The character table of A_1 is given in Table B.5.
(f) The group A_1 and the centralizer $C_E(z_1)$ of the involution $z_1 = e^2$ of E have isomorphic Sylow 2-subgroups of order 2^{19} .

Proof. (a) The given presentation of H_1 is a restatement of Proposition 6.2.3 of [14] due to H. Kim, see [11].

(b) By that result we also know that H_1 has a faithful permutation representation PH_1 of degree 28160 with stabilizer $U = \langle bz, c, d, e, fz, g, h, i \rangle$. Using it and the MAGMA command `AutomorphismGroup(H_1)` we see that $|\text{Aut}(H_1)| = |H_1|$. As $H_1/\langle z \rangle \cong \text{Fi}_{22}$ has the same presentation as Fi_{22} given in [15], p. 110 we can quote the presentation of $\text{Aut}(\text{Fi}_{22})$ given in [15], p. 111, where z is replaced by 1. Now (b) follows from (a) and 2^6 MAGMA calculations with PH_1 checking whether 1 or z has to be on the right hand side of the six new relations stated in (b) different from $t^2 = 1$ and $[z, t] = 1$. It follows that there is exactly one solution.

(c) This statement has been verified by means of the MAGMA command `CosetAction(A_1, U)`.

(d) The system of representatives of the conjugacy classes of A_1 has been calculated by means of the permutation representations PA_1 of $A_1 = 2\text{Aut}(H_1)$, MAGMA and Kratzer's Algorithm 5.3.18 of [12].

(e) The character table of A_1 has been calculated by means of PA_1 and MAGMA.

(f) Let PE be the faithful permutation representation of E constructed in Lemma 2.1. Let $C = C_E(u)$ for the involution $u = e^2$ of $E = \langle x, y, e \rangle$. Now (f) can be verified by using the permutation representations PA_1 and PE together with the Cannon-Holt isomorphism test implemented in MAGMA. \square

Lemma 3.2. *Keep the notation of Lemma 3.1. Let $G_1 = \langle x, y, q, w \rangle \cong \text{Fi}_{23}$ be the simple subgroup of $\text{GL}_{782}(17)$ of order $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$ with centralizer $H_1 = \langle x, y, q \rangle = C_{G_1}(z)$ of the 2-central involution $z = (xy^2)^7$ constructed in [11]. Let $A_1 = 2\text{Aut}(H_1)$. Then the following assertions hold:*

- (a) $H_1 = \langle a, b, c, d, e, f, g, h, i, z \rangle$ where

$$\begin{aligned} a &= (xyx)^7, & b &= [(qy)^2 qy^3 q^2 y^3 qy]^7, & c &= (y^2 xyxy^3)^5, \\ d &= (qyq^2 yqyqyqy^2 q^2)^{15}, & e &= (yxy^5 x)^5, & f &= (yqyq^2 yq^2 y^2 qy^4 q^2)^5, \\ g &= (xy^2 xy^3 x)^7, & h &= (y^5 xyx)^5, & i &= (q^2 y^2 qyq^2)^7. \end{aligned}$$
- (b) $H_1 = \langle a, b, c, d, e, f, g, h, i, z \rangle$ satisfies the set $\mathcal{R}(H_1)$ of defining relations stated in Lemma 3.1(a). Its character table is Table 6.5.2 of [14].
- (c) The character table of G_1 is stated in the Atlas [4], its pp. 178 - 179.
- (d) The amalgam $A_1 \leftarrow H_1 \rightarrow G_1$ has Goldschmidt index 1.
- (e) The amalgam $A_1 \leftarrow H_1 \rightarrow G_1$ has eight compatible pairs

$$(\chi, \tau) \in \text{mfchar}_{\mathbb{C}}(A_1) \times \text{mfchar}_{\mathbb{C}}(G_1)$$

of degree 8671. All have the same restriction

$$\delta_2 + \delta_6 + \delta_7 + \delta_8 + \delta_9 \in \text{mfchar}_{\mathbb{C}}(H_1).$$

They are:

- (1) $(\chi_3 + \chi_{11} + \chi_{13} + \chi_{17}, \quad \tau_3 + \tau_4),$
- (2) $(\chi_3 + \chi_{11} + \chi_{14} + \chi_{17}, \quad \tau_3 + \tau_4),$
- (3) $(\chi_3 + \chi_{12} + \chi_{13} + \chi_{17}, \quad \tau_3 + \tau_4),$
- (4) $(\chi_3 + \chi_{12} + \chi_{14} + \chi_{17}, \quad \tau_3 + \tau_4),$
- (5) $(\chi_4 + \chi_{11} + \chi_{13} + \chi_{17}, \quad \tau_3 + \tau_4),$
- (6) $(\chi_4 + \chi_{11} + \chi_{14} + \chi_{17}, \quad \tau_3 + \tau_4),$
- (7) $(\chi_4 + \chi_{12} + \chi_{13} + \chi_{17}, \quad \tau_3 + \tau_4),$
- (8) $(\chi_4 + \chi_{12} + \chi_{14} + \chi_{17}, \quad \tau_3 + \tau_4).$

Proof. By Kim's Theorem 6.3.1 of [14] the simple matrix subgroup $G_1 \cong \text{Fi}_{23}$ of $\text{GL}_{782}(17)$ has a faithful permutation representation PG_1 of degree 31671 with stabilizer H_1 . It is used throughout this proof.

(a) The words of the new generators a, b , etc. of H_1 in terms of the given generators x, y and q of H_1 are quoted from Kim's Proposition 6.2.3 of [14].

(b) Using the faithful permutation representation PG_1 and MAGMA it has been checked that the new generators a, b etc. of H_1 given in statement (a) satisfy all the relations of $\mathcal{R}(H_1)$ of Lemma 3.1(a).

(c) This assertion is a restatement of Theorem 6.3.1 of [14].

(d) Kratzer's Algorithm 7.1.10 of [12] could not be applied to calculate the Goldschmidt index. When trying to calculate $\text{Aut}(G_1)$ using the Cannon-Holt Algorithm of [2] MAGMA answered: "Sorry, the top factor of order 4089470473293004800 is not currently stored". However, using the faithful permutation representation PA_1 of A_1 stated in Lemma 3.1(c) MAGMA established that the outer automorphism groups $\text{Out}(H_1)$ and $\text{Out}(A_1)$ of H_1 and A_1 are both cyclic of order 2. Hence the Goldschmidt index of the amalgam $A_1 \leftarrow H_1 \rightarrow G_1$ is 1 by Step 3 of Algorithm 7.1.10 of [12].

(e) The eight compatible pairs of degree 8671 of the amalgam $A_1 \leftarrow H_1 \rightarrow G_1$ were determined by means of Kratzer's Algorithm 7.3.10 of [12] and MAGMA. \square

4. A SEMI-SIMPLE REPRESENTATION OF Fi_{23} OVER $GF(13)$

In this section H. Kim's results of his senior thesis [11] are used for the construction of two irreducible representations of degrees 3588 and 5083 of $G_1 \cong \text{Fi}_{23}$ over the prime field $GF(13)$. They correspond to the 2 irreducible characters τ_3 and τ_4 of 13-defect zero of G_1 occurring in the 8 compatible pairs constructed in Lemma 3.2(e). For the construction of these fairly large representations we first determine generators of two large subgroups mH and mE of G_1 and their intersection mD . We also calculate the character tables of these 3 subgroups of G_1 .

Lemma 4.1. *Keep the notation of Lemmas 3.1 and 3.2. Let $G_1 = \text{Fi}_{23} = \langle x, y, q, w \rangle$ be the simple subgroup of $\text{GL}_{782}(17)$ of order $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$ with faithful permutation representation PG_1 of degree 31671 and stabilizer $H_1 = \langle x, y, q \rangle$ constructed in Kim's Theorem 6.3.1 of [14]. Let $r = s_1 = (yqy^2q)^7$, $s_2 = (yqqyq)^7$, $s_3 = (yeyeq)^{21}$, $s_4 = (qqqey^2)^7$, $u = s_4^2(s_1s_2s_4)^3$, $s = qqeqy^2$ and $v = (s_1s_3s_1s_4)^2(s_1s_3s_1s_3s_1s_4)^3s_3s_4s_3s_4^2s_3$. Then the following assertions hold:*

- (a) $mH = \langle s_1, s_2, s_3, s_4 \rangle = \langle r, u, v \rangle \cong O_8^+(3) : S_3$.
- (b) The character table of mH is given in Table B.2.
- (c) $mE = \langle u, v, s \rangle \cong O_7(3) \times S_3$.
- (d) The character table of mE is Table B.3.
- (e) $mD = mH \cap mE = \langle u, v \rangle \cong G_2(3) \times S_3$.
- (f) $G_1 = \langle u, v, r, s \rangle$ and the original generators q, y, w and x of G_1 are equal to the following words in its generators u, v, r and s :

$$\begin{aligned}
q &= [(us^2vsr)^9[(svs^3rsr)^{12}(vrsrsvu)^{12}]^2]^{24}, \\
w &= [(w_2w_4w_2w_4w_3w_2w_4)^3(w_1w_2w_3w_1w_4w_3w_1w_4)^7]^3, \\
y &= (m_1m_2m_1m_2m_3)^7[(m_1m_2)^5(n_3n_1n_2n_3n_1n_3^2n_1)^5]^{14}, \\
x &= [(yq^2yqqyq^2)^{11}(q^2y^2qqyq)^{11}(qy^2qqyqqyq)^4]^{12}, \quad \text{where} \\
w_1 &= (vuvs^2)^9, \quad w_2 = (v^2uruvr)^{18}, \quad w_3 = (v^2s^2urv)^{11}, \\
w_4 &= (uvrv^4r)^{10}, \quad n_1 = m_1m_2m_3m_2^2m_1, \quad n_2 = m_2m_3m_1m_3m_2^3, \\
n_3 &= (m_3m_1m_2m_3m_2m_1m_2)^2, \quad m_1 = (t_1t_3t_1t_3t_1^2t_3t_1)^2, \\
m_2 &= (t_1t_3t_1t_3t_1^2t_2t_3t_2)^2, \quad m_3 = (t_2t_1t_3t_2t_3t_1t_2t_3t_2)^9, \\
t_1 &= (srsrsrs)^5, \quad t_2 = (s^2rs^4)^{11}, \quad \text{and} \quad t_3 = (rs^7)^3.
\end{aligned}$$

- (g) The restrictions of the irreducible character τ_3 of degree 3588 of G_1 to mH and mE are $\pi_8 + \pi_{16} \in \text{mfchar}_{\mathbb{C}}(mH)$ and $\psi_4 + \psi_{10} + \psi_{19} + \psi_{36} + \psi_{61} \in \text{mfchar}_{\mathbb{C}}(mE)$, respectively.
- (h) The restrictions of the irreducible character τ_4 of degree 5083 of G_1 to mH and mE are $\pi_{12} + \pi_{15} \in \text{mfchar}_{\mathbb{C}}(mH)$ and $\psi_{14} + \psi_{22} + \psi_{45} + \psi_{74} \in \text{mfchar}_{\mathbb{C}}(mE)$, respectively.
- (i) The irreducible characters π_9 , π_{11} and π_{15} of mH are constituents of the permutation characters $1_{mH_9}^H$, $1_{mH_{11}}^H$ and $1_{mH_{15}}^H$ of the subgroups

$$\begin{aligned}
mH_9 &= \langle (vru^2)^{13}, (rvur)^2, (uvrv^2)^4 \rangle, \\
mH_{11} &= \langle (uvurv)^4, (v^5ur)^9, (ururv^2u^2)^{13} \rangle \quad \text{and} \\
mH_{15} &= \langle (uvuru)^{13}, (uv^2uv)^2, (u^5v)^3 \rangle
\end{aligned}$$

of mH with indices 3240, 72800 and 2274480, respectively.

- (j) The linear character π_2 of mH has values -1 and 1 at v and u, r , respectively. Furthermore, $\pi_8 = \pi_2 \otimes \pi_9$, $\pi_{12} = \pi_2 \otimes \pi_{11}$, and $\pi_{16} = \pi_2 \otimes \pi_{15}$.
- (k) The irreducible characters ψ_5 and ψ_{10} of mE are constituents of the permutation character $1_{mE_1}^{mE}$ of the subgroup $mE_1 = \langle (s^2vs)^9, (svsv^2s^2)^2 \rangle$ of index 2106.
- (l) The irreducible characters $\psi_{14}, \psi_{22}, \psi_{36}, \psi_{45}, \psi_{61}$ and ψ_{74} of mE are constituents of the permutation characters $1_{mE_{14}}^{mE}, 1_{mE_{22}}^{mE}, 1_{mE_{36}}^{mE}, 1_{mE_{45}}^{mE}, 1_{mE_{61}}^{mE}$ and $1_{mE_{74}}^{mE}$ of the subgroups
- $$\begin{aligned} mE_{14} &= \langle (s^2us)^6, (su^2s^2)^4, (susu^2)^2, (us^3us)^7 \rangle, \\ mE_{22} &= \langle (us^4u^2)^6, usu^2susus \rangle, \\ mE_{36} &= \langle (s^2vs)^9, (v^2svsv^2)^5, (vsv^2svs)^2 \rangle, \\ mE_{45} &= \langle (s^2v^2)^7, (v^5s)^3, (vsv^2sv^3sv)^6 \rangle, \\ mE_{61} &= \langle (vs^2)^7, (v^4s^2v^2)^{12}, (v^2svsv^4)^{10}, (vs^2v^3s^3)^{30} \rangle \quad \text{and} \\ mE_{74} &= \langle (s^2v)^7, (v^2s^2)^{21}, (s^2v^2)^{21}, (svsv^2sv)^3 \rangle \end{aligned}$$
- of mE with indices 702, 2160, 19656, 7280, 85293 and 29484, respectively.
- (m) The linear character ψ_2 of mE has values -1 and 1 at v and u, s , respectively. Furthermore, $\psi_4 = \psi_2 \otimes \psi_5$, and $\psi_{19} = \psi_2 \otimes \psi_{22}$.
- (n) Both r and $f = (u^3vsv)^9$ are involutions of G_1 such that $(r, f) = 1$, $rf \notin mH$ and $rf \notin mE$.

Proof. (a) The subgroup mH of G_1 has been constructed by means of the faithful permutation representation PG_1 of G_1 of degree 31671 and the MAGMA command `LowIndexSubgroups(PG_1, 137632)`. The four generators s_i of mH , $1 \leq i \leq 4$, were calculated with Kim's program `GetShortGens(PG_1, mH)`. Another application of MAGMA determined the composition factors of mH .

(b) The character table of mH was calculated by MAGMA using PG_1 .

(c) and (e) By Table 6.5.4 of [14] $|C_{G_1}((qw)^4)| = 2^9 \cdot 3^{10} \cdot 5 \cdot 7 \cdot 13$. Let $mX = N_{G_1}(\langle (qw)^4 \rangle)$. Using PG_1 and MAGMA we searched for an element $x \in mX$ of order 3 such that $|N_{mH}(\langle x \rangle)| = 2^7 \cdot 3^7 \cdot 7 \cdot 13$. MAGMA found such an element and stated that $mD = N_{mH}(\langle x \rangle) = \langle u, v \rangle$. Furthermore, $mE = N_{G_1}(\langle x \rangle) = \langle mD, s \rangle$ where u, v and s are defined in the statement of this lemma. The composition factors of mD and mE have been determined by means of MAGMA.

(d) The character table of mE was calculated by means of PG_1 and MAGMA.

(f) Using the faithful permutation representation PG_1 of G_1 and MAGMA one verifies that $G_1 = \langle mH, mE \rangle$. Hence $G_1 = \langle u, v, r, s \rangle$ by (a) and (c). The words for q, y, w and x can easily be checked computationally.

(g) G_1 has a unique character τ_3 of degree 3588 by the character table of $G_1 \cong \text{Fi}_{23}$, see [4], p. 178. Its restrictions to mH and mE given in the statements have been determined by means of PG_1 , the character tables of the subgroups mH and mE of G_1 , and MAGMA.

(h) This assertion is proved as (g).

(i) Using the MAGMA command `LowIndexSubgroups(mH, k)` we searched for conjugacy classes of subgroups H_k of index $|mH : mH_k| = m_k$ such that π_k is an irreducible constituent of the permutation character $1_{mH_k}^{mH}$ for $k \in \{9, 11, 15\}$. Thus we found 3 subgroups mH_k of respective indices $m_9 = 3240$, $m_{11} = 72800$

and $m_{15} = 2274480$. Their given generators have been obtained by means of Kim's program `GetShortGens(mH, mH_k)`.

(j) π_2 is the unique non trivial linear character of mH by its character table. The character equations of the statement are easily verified by means of Table B.2.

The statements (k) and (l) are proved similarly as (i).

(m) mE has a unique non trivial linear character ψ_2 , see Table B.3. The character equations of the statement are easily verified by means of Table B.2.

(n) Using PG_1 and MAGMA we checked that r and f are commuting involutions such that $rf \notin mH$ and $rf \notin mE$. \square

Proposition 4.2. *Keep the notation of Lemmas 3.1, 3.2 and 4.1. Let PG_1 be the faithful permutation representation of the simple group $G_1 = \langle x, y, q, w \rangle = \langle u, v, r, s \rangle$ of degree 31671 with stabilizer $H_1 = \langle x, y, q \rangle$. Let $mH = \langle u, v, r \rangle$, $mD = \langle u, v \rangle$ and $mE = \langle u, v, s \rangle$. Let F^* be the multiplicative group of the prime field $F = \text{GF}(13)$. Let $Y = \text{GL}_{3588}(13)$.*

Let \mathfrak{V} and \mathfrak{W} be the up to isomorphism uniquely determined faithful semi-simple 3588-dimensional modules of mH and mE over F corresponding to the restrictions $\tau_{\mathfrak{3}|mH}$ and $\tau_{\mathfrak{3}|mE}$ of the irreducible character $\tau_{\mathfrak{3}}$ of G_1 , respectively.

Let $\kappa_{\mathfrak{V}} : mH \rightarrow \text{GL}_{3588}(13)$ and $\kappa_{\mathfrak{W}} : mE \rightarrow \text{GL}_{3588}(13)$ be the representations of mH and mE afforded by the modules \mathfrak{V} and \mathfrak{W} , respectively.

Let $\mathfrak{r} = \kappa_{\mathfrak{V}}(r)$, $\mathfrak{u} = \kappa_{\mathfrak{V}}(u)$, $\mathfrak{v} = \kappa_{\mathfrak{W}}(v)$ in $\kappa_{\mathfrak{V}}(mH) \leq \text{GL}_{3588}(13)$.

Then $\mathfrak{V}|_{mD} \cong \mathfrak{W}|_{mD}$, and there is a transformation matrix $\mathcal{T}_1 \in \text{GL}_{3588}(13)$ such that

$$\mathfrak{u} = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(u) \mathcal{T}_1, \mathfrak{v} = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(v) \mathcal{T}_1.$$

Let $\mathfrak{m}\mathfrak{D} = \langle \mathfrak{u}, \mathfrak{v} \rangle$, $\mathfrak{m}\mathfrak{H} = \langle \mathfrak{u}, \mathfrak{v}, \mathfrak{r} \rangle$. Let $\mathcal{D} = C_Y(\mathfrak{m}\mathfrak{D})$ and $\mathcal{H} = C_Y(\mathfrak{m}\mathfrak{H})$. Let $\mathfrak{s}_1 = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(s) \mathcal{T}_1$. Let $\mathfrak{m}\mathfrak{E} = \langle \mathfrak{m}\mathfrak{D}, \mathfrak{s}_1 \rangle$ and $\mathcal{E} = C_Y(\mathfrak{m}\mathfrak{E})$. Then the following statements hold:

(a) *There is an isomorphism*

$$\alpha : \mathcal{D} \rightarrow \mathcal{D}_1 = \text{GL}_2(13) \times \text{GL}_2(13) \times F^{*5} \leq \text{GL}_9(13).$$

(b) $\mathcal{H}_1 = \alpha(\mathcal{H})$ *is generated by the two blocked diagonal matrices*

$$a_1 = \text{diag}((\begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix}), (\begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix}), 2, 1, 2, 1, 1) \quad \text{and} \quad a_2 = \text{diag}((\begin{smallmatrix} 1 & 0 \\ 0 & 2 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 0 & 2 \end{smallmatrix}), 1, 2, 1, 2, 2),$$

(c) $\mathcal{E}_1 = \alpha(\mathcal{E})$ *is generated by the five blocked diagonal matrices*

$$\begin{aligned} b_1 &= \text{diag}((\begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}), 1, 1, 1, 1, 1), & b_2 &= \text{diag}((\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}), (\begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix}), 1, 1, 1, 1, 1), \\ b_3 &= \text{diag}((\begin{smallmatrix} 1 & 0 \\ 0 & 2 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}), 1, 2, 1, 1, 1), & b_4 &= \text{diag}((\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}), 2, 1, 1, 2, 1), \\ & & \text{and } b_5 &= \text{diag}((\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 0 & 2 \end{smallmatrix}), 1, 1, 2, 1, 2). \end{aligned}$$

(d) \mathcal{D} *has $2184^2 \times 12 = 57238272$ \mathcal{H} - \mathcal{E} double cosets.*

(e) *The free product $mH *_{mD} mE$ of mH and mE with amalgamated subgroup mD has an irreducible 3588-dimensional representation over F which induces an irreducible representation of G_1 . It corresponds to the \mathcal{H} - \mathcal{E} double coset representative*

$$\mathcal{F} = \text{diag}(w, z, 1^{182}, 1^{182}, 1^{364}, 1^{728}, 1^{1664}) \in \text{GL}_{3588}(13), \quad \text{where}$$

$$w = \begin{pmatrix} 1^{78} & 4^{78} \\ 12^{78} & 2^{78} \end{pmatrix}, \quad z = \begin{pmatrix} 1^{156} & 6^{156} \\ 5^{156} & 10^{156} \end{pmatrix},$$

and a^n denotes a diagonal $n \times n$ matrix with unique diagonal non zero entry $a \in GF(13)$.

Let $\mathfrak{s} = \mathcal{F}^{-1}\mathfrak{s}_1\mathcal{F}$ and $\mathfrak{G}_1 = \langle \mathfrak{u}, \mathfrak{v}, \mathfrak{r}, \mathfrak{s} \rangle$. Inserting these four generating matrices of \mathfrak{G}_1 into the formulas of Lemma 4.1(f) one obtains the matrices \mathfrak{x}_{3588} , \mathfrak{y}_{3588} , \mathfrak{q}_{3588} , and \mathfrak{w}_{3588} of the original generators x, y, q and w of $G_1 = \text{Fi}_{23}$ as words in the generators u, v, r and s . The matrices \mathfrak{x}_{3588} , \mathfrak{y}_{3588} , \mathfrak{q}_{3588} , and \mathfrak{w}_{3588} can be downloaded from the first author's website <http://www.math.yale.edu/~hk47/Fi24/index.html>.

Proof. Let \mathfrak{V} be the up to isomorphism uniquely determined faithful semi-simple 3588-dimensional module of mH over $F = GF(13)$ corresponding to the restriction $\tau_{\mathfrak{z}|mH}$. By Lemma 4.1(g) $\tau_{\mathfrak{z}|mH} = \pi_8 + \pi_{16}$. Lemma 4.1(j) states that $\pi_8 = \pi_2 \otimes \pi_9$.

The irreducible characters π_9 and π_{15} of mH are constituents of the permutation characters $1_{mH_9}^{mH}$ and $1_{mH_{15}}^{mH}$ of the respective subgroups mH_9 and mH_{15} of mH determined in Lemma 4.1(i). Using a stand-alone program of the first author which is based on Algorithm 5.7.1 of [12] we calculated the primitive idempotents of the endomorphism rings of these permutation modules. Thus we obtained the corresponding irreducible representations $M(\pi_9)$ and $M(\pi_{15})$ of the respective dimensions 780 and 2808 over F . The irreducible FmH -modules $M(\pi_8)$ and $M(\pi_{16})$ are the tensor products of $M(\pi_9)$ and $M(\pi_{15})$ with the linear character π_2 of mH over F . Thus $\mathfrak{V} = M(\pi_8) \oplus M(\pi_{16})$.

Let \mathfrak{W} be the up to isomorphism uniquely determined faithful semi-simple 3588-dimensional module of mE over F corresponding to the restriction $\tau_{\mathfrak{z}|mE}$. By Lemma 4.1(g) $\tau_{\mathfrak{z}|mE} = \psi_4 + \psi_{10} + \psi_{19} + \psi_{36} + \psi_{61}$. Lemma 4.1(m) states that $\psi_4 = \psi_2 \otimes \pi_5$ and $\psi_{19} = \psi_2 \otimes \pi_{22}$.

The irreducible characters ψ_5 and ψ_{10} of mE are constituents of the permutation character $1_{mE_1}^{mE}$ by 4.1(k). The irreducible characters ψ_{22} , ψ_{36} and ψ_{61} of mE are constituents of the permutation characters $1_{mE_{22}}^{mE}$, $1_{mE_{36}}^{mE}$ and $1_{mE_{61}}^{mE}$, respectively, see Lemma 4.1(m). Using a standalone program of the first author which is based on Algorithm 5.7.1 of [12] we calculated the primitive idempotents of the endomorphism rings of these four permutation modules. Thus we obtained the corresponding irreducible representations $N(\psi_5)$, $N(\psi_{10})$, $N(\psi_{22})$, $N(\psi_{36})$ and $N(\psi_{61})$ of the respective dimensions 78, 156, 260, 910 and 2184 over F . The irreducible FmE -modules $N(\psi_4)$ and $N(\psi_{19})$ are the tensor products of $N(\psi_5)$ and $N(\psi_{22})$ with the linear character ψ_2 of mE over F . Thus

$$\mathfrak{W} = N(\psi_4) \oplus N(\psi_{10}) \oplus N(\psi_{19}) \oplus N(\psi_{36}) \oplus N(\psi_{61}).$$

Fixing a basis in each irreducible constituent $M\pi_k$ of \mathfrak{V} we get a basis \mathcal{B}_V of \mathfrak{V} . It induces a representation $\kappa_{\mathfrak{V}} : mH \rightarrow \text{GL}_{3588}(13)$ of mH . Let $\mathfrak{r} = \kappa_{\mathfrak{V}}(r)$, $\mathfrak{u} = \kappa_{\mathfrak{V}}(u)$, $\mathfrak{v} = \kappa_{\mathfrak{V}}(v)$ in $\kappa_{\mathfrak{V}}(mH) \leq \text{GL}_{3588}(13)$.

Fixing a basis in each irreducible constituent $N\psi_j$ of \mathfrak{W} we get a basis \mathcal{B}_W of \mathfrak{W} . It induces a representation $\kappa_{\mathfrak{W}} : mE \rightarrow \text{GL}_{3588}(13)$ of mE . By Lemma 4.1(g) $\mathfrak{V}|_{mD} \cong \mathfrak{W}|_{mD}$. Let $Y = \text{GL}_{3588}(13)$. Applying now Parker's isomorphism test of Proposition 6.1.6 of [12] by means of the MAGMA command

`IsIsomorphic(GModule(sub<Y|V(u), V(v)>), GModule(sub<Y|W(u), W(v)>))`

one obtains the transformation matrix \mathcal{T}_1 satisfying $\mathfrak{u} = \kappa_{\mathfrak{W}}(u)^{\mathcal{T}_1}$ and $\mathfrak{v} = \kappa_{\mathfrak{W}}(v)^{\mathcal{T}_1}$.

(a) Let $\mathbf{m}\mathfrak{D} = \langle \mathbf{u}, \mathbf{v} \rangle$, $\mathbf{m}\mathfrak{H} = \langle \mathbf{u}, \mathbf{v}, \mathbf{r} \rangle$. Let $\mathcal{D} = C_Y(\mathbf{m}\mathfrak{D})$ and $\mathcal{H} = C_Y(\mathbf{m}\mathfrak{H})$. Let δ_{12a} , δ_{12b} and δ_{23a} , δ_{23b} be two distinct copies of the irreducible characters δ_{12} and δ_{23} of mD , respectively. Using PG_1 and MAGMA we checked that the irreducible characters π_8 and π_{16} of mH have the following restrictions to $mD = \langle u, v \rangle$:

$$\pi_{8|_{mD}} = \delta_{12a} + \delta_{23a} + \delta_{27} + \delta_{39}, \quad \pi_{16|_{mD}} = \delta_{12b} + \delta_{23b} + \delta_{29} + \delta_{54} + \delta_{69},$$

where the irreducible characters δ_{12} , δ_{23} , δ_{27} , δ_{29} , δ_{39} , δ_{54} , and δ_{69} of $mD \cong G_2(3) \times S_3$ have degrees 78, 156, 182, 182, 364, 728 and 1664, respectively.

(b) Furthermore, Schur's Lemma asserts that $\mathcal{H}_1 = \alpha(\mathcal{H})$ is generated by the two blocked diagonal matrices given in the statement because 2 is a primitive element of the multiplicative group F^* of $F = GF(13)$.

(c) Let $\mathfrak{s}_1 = \mathcal{T}_1^{-1} \kappa_{\mathfrak{W}}(s) \mathcal{T}_1$. Let $\mathbf{m}\mathfrak{E} = \langle \mathbf{m}\mathfrak{D}, \mathfrak{s}_1 \rangle$ and $\mathcal{E} = C_Y(\mathbf{m}\mathfrak{E})$. Let δ_{12a} , δ_{12b} and δ_{23a} , δ_{23b} are two distinct copies of the irreducible characters δ_{12} and δ_{23} of mD , respectively. Using PG_1 and MAGMA we checked that the irreducible characters ψ_4 , ψ_{10} , ψ_{19} , ψ_{36} and π_{61} of mE have the following restrictions to $mD = \langle u, v \rangle$:

$$\begin{aligned} \psi_{4|_{mD}} &= \delta_{12a}, & \psi_{10|_{mD}} &= \delta_{23a}, & \psi_{19|_{mD}} &= \delta_{12b} + \delta_{29}, \\ \psi_{36|_{mD}} &= \delta_{27} + \delta_{54}, & \psi_{61|_{mD}} &= \delta_{23b} + \delta_{39} + \delta_{69}. \end{aligned}$$

Now Schur's Lemma implies that $\mathcal{E}_1 = \alpha(\mathcal{E})$ is generated by the five blocked diagonal matrices b_j given in the statement.

(d) Every \mathcal{H} - \mathcal{E} double coset representative is of the form $\text{diag}(A, B, 1, 1, 1, 1, v)$ for some $A, B \in Y$ and $v \in F^*$. By multiplying from left and right, we observe that $\text{diag}(A, B, 1, 1, 1, 1, v)$ and $\text{diag}(A', B', 1, 1, 1, 1, v)$ represent the same double coset if and only if the first columns of A and B are each a scalar multiple of the first columns of A' and B' , respectively. So, we have 12 choices for v , and $|\text{GL}(2, 13)|/12 = 2184$ choices for A and B . Thus there are $2184^2 \cdot 12 = 57238272$ \mathcal{H} - \mathcal{E} double cosets.

(e) By Theorem 7.2.2 of [12] the irreducible representations of the free product $mH *_{mD} mE$ of the groups mH and mE with amalgamated subgroup mD are described by the \mathcal{H} - \mathcal{E} double coset representatives T of \mathcal{D} . The elements r and $f = (u^3 v s v)^9$ are two commuting involutions of $G_1 \cong \text{Fi}_{23}$ by Lemma 4.1(o). Let \mathbf{u} and \mathbf{f} be their matrices in $\mathbf{m}\mathfrak{H}$ and $\mathbf{m}\mathfrak{E}$, respectively. If $T = \text{diag}((\begin{smallmatrix} a & c \\ b & d \end{smallmatrix}), (\begin{smallmatrix} p & t \\ q & u \end{smallmatrix}), 1, 1, 1, 1, v)$ describes a 3588-dimensional representation of G_1 over F then $(*) (\mathbf{r}, \mathcal{T}^{-1} \mathbf{f} \mathcal{T}) = 1$ holds, where $\mathcal{T} \in \text{GL}_{3588}(13)$ corresponds to T .

Since $\mathfrak{V}_{|_{mD}} \cong \mathfrak{W}_{|_{mD}}$ is a direct sum of 9 irreducible FmD -modules both matrices \mathbf{r} and \mathbf{f} consist of 81 blocks $R_{i,j}$ and $F_{i,j}$, $1 \leq i, j \leq 9$, respectively, such that all diagonal blocks $R_{i,i}$ and $F_{i,i}$ are non zero. Furthermore a non diagonal block $R_{i,j}$ of \mathbf{r} is non zero if and only if the i -th irreducible and the j -th irreducible representations of mD appear in the restriction of an irreducible representation of mH to mD . A similar description holds for the blocks of \mathbf{f} . Hence the system of equations in the proofs of (a) and (c) imply that

$$\mathfrak{r} = \begin{pmatrix} R_{1,1} & \cdot & R_{1,3} & \cdot & R_{1,5} & \cdot & R_{1,7} & \cdot & \cdot \\ \cdot & R_{2,2} & \cdot & R_{2,4} & \cdot & R_{2,6} & \cdot & R_{2,8} & R_{2,9} \\ R_{3,1} & \cdot & R_{3,3} & \cdot & R_{3,5} & \cdot & R_{3,7} & \cdot & \cdot \\ \cdot & R_{4,2} & \cdot & R_{4,4} & \cdot & R_{4,6} & \cdot & R_{4,8} & R_{4,9} \\ R_{5,1} & \cdot & R_{5,3} & \cdot & R_{5,5} & \cdot & R_{5,7} & \cdot & \cdot \\ \cdot & R_{6,2} & \cdot & R_{6,4} & \cdot & R_{6,6} & \cdot & R_{6,8} & R_{6,9} \\ R_{7,1} & \cdot & R_{7,3} & \cdot & R_{7,5} & \cdot & R_{7,7} & \cdot & \cdot \\ \cdot & R_{8,2} & \cdot & R_{8,4} & \cdot & R_{8,6} & \cdot & R_{8,8} & R_{8,9} \\ \cdot & R_{9,2} & \cdot & R_{9,4} & \cdot & R_{9,6} & \cdot & R_{9,8} & R_{9,9} \end{pmatrix},$$

$$\mathfrak{f} = \begin{pmatrix} F_{1,1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & F_{2,2} & \cdot & \cdot & \cdot & F_{2,6} & \cdot & \cdot & \cdot \\ \cdot & \cdot & F_{3,3} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & F_{4,4} & \cdot & \cdot & F_{4,7} & \cdot & F_{4,9} \\ \cdot & \cdot & \cdot & \cdot & F_{5,5} & \cdot & \cdot & F_{5,8} & \cdot \\ \cdot & F_{6,2} & \cdot & \cdot & \cdot & F_{6,6} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & F_{7,4} & \cdot & \cdot & F_{7,7} & \cdot & F_{7,9} \\ \cdot & \cdot & \cdot & \cdot & F_{8,5} & \cdot & \cdot & F_{8,8} & \cdot \\ \cdot & \cdot & \cdot & F_{9,4} & \cdot & \cdot & F_{9,7} & \cdot & F_{9,9} \end{pmatrix}.$$

Let $e = (ad - bc)^{-1}$ and $g = (pu - tq)^{-1}$. Then $e \neq 0 \neq g$. For each integer k let I_k denote the $k \times k$ identity matrix over F . Then

$$\mathcal{T}^{-1} = \begin{pmatrix} ed(I_{78}) & -ec(I_{78}) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -eb(I_{78}) & ea(I_{78}) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & gu(I_{156}) & -gt(I_{156}) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -gq(I_{156}) & gp(I_{156}) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & I_{182} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & I_{182} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & I_{364} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & I_{728} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & v^{-1}(I_{1664}) & \cdot \end{pmatrix}.$$

Hence $\mathfrak{f}' = \mathcal{T}^{-1}\mathfrak{f}\mathcal{T}$ equals the matrix

$$\begin{pmatrix} G_{1,1} & G_{1,2} & \cdot & \cdot & \cdot & -ecF_{2,6} & \cdot & \cdot & \cdot \\ G_{2,1} & G_{2,2} & \cdot & \cdot & \cdot & eaF_{2,6} & \cdot & \cdot & \cdot \\ \cdot & \cdot & G_{3,3} & G_{3,4} & \cdot & \cdot & -gtF_{4,7} & \cdot & -vgtF_{4,9} \\ \cdot & \cdot & G_{4,3} & G_{4,4} & \cdot & \cdot & gpF_{4,7} & \cdot & vgpF_{4,9} \\ \cdot & \cdot & \cdot & \cdot & F_{5,5} & \cdot & \cdot & F_{5,8} & \cdot \\ G_{6,1} & G_{6,2} & \cdot & \cdot & \cdot & F_{6,6} & \cdot & \cdot & \cdot \\ \cdot & \cdot & G_{7,3} & G_{7,4} & \cdot & \cdot & F_{7,7} & \cdot & vF_{7,9} \\ \cdot & \cdot & \cdot & \cdot & F_{8,5} & \cdot & \cdot & F_{8,8} & \cdot \\ \cdot & \cdot & G_{9,3} & G_{9,4} & \cdot & \cdot & v^{-1}F_{9,7} & \cdot & F_{9,9} \end{pmatrix},$$

where

$$\begin{aligned}
G_{1,1} &= e(adF_{1,1} - bcF_{2,2}), & G_{1,2} &= ecd(F_{1,1} - F_{2,2}), \\
G_{2,1} &= -eab(F_{1,1} - F_{2,2}), & G_{2,2} &= -e(bcF_{1,1} - adF_{2,2}), \\
G_{6,1} &= bF_{6,2}, & G_{6,2} &= dF_{6,2}, \\
G_{3,3} &= g(puF_{3,3} - qtF_{4,4}), \\
G_{3,4} &= gut(F_{3,3} - F_{4,4}), \\
G_{4,3} &= -gpq(F_{3,3} - F_{4,4}), \\
G_{4,4} &= -g(tqF_{3,3} - puF_{4,4}), \\
G_{7,3} &= qF_{7,4}, & G_{7,4} &= uF_{7,4}, \\
G_{9,3} &= qv^{-1}F_{9,4}, & G_{9,4} &= uv^{-1}F_{9,4}.
\end{aligned}$$

Now (*) implies the following equations

$$\begin{aligned}
(1, 1) : G_{1,1}R_{1,1} &= R_{1,1}G_{1,1}, \\
(1, 2) : G_{1,2}R_{2,2} + (-ec)F_{2,6}R_{6,2} &= R_{1,1}G_{1,2}, \\
(6, 1) : G_{6,1}R_{1,1} &= R_{6,2}G_{2,1} + R_{6,6}G_{6,1}, \\
(8, 1) : F_{8,5}R_{5,1} &= R_{8,2}G_{2,1} + R_{8,6}G_{6,1}.
\end{aligned}$$

Inserting the previous equations into these 4 equations yields:

$$\begin{aligned}
(1, 1) : e(adF_{1,1} - bcF_{2,2})R_{1,1} &= eR_{1,1}(adF_{1,1} - bcF_{2,2}), \\
(1, 2) : ec(d(F_{1,1} - F_{2,2})R_{2,2} - F_{2,6}R_{6,2}) &= ecdR_{1,1}(F_{1,1} - F_{2,2}), \\
(6, 1) : bF_{6,2}R_{1,1} &= b(-eaR_{6,2}(F_{1,1} - F_{2,2}) + R_{6,6}F_{6,2}), \\
(8, 1) : F_{8,5}R_{5,1} &= b(-eaR_{8,2}(F_{1,1} - F_{2,2}) + R_{8,6}F_{6,2}).
\end{aligned}$$

Since $e^{-1} = ad - bc \neq 0$, at least one of ad or bc is nonzero. Suppose ad is zero, and bc is nonzero. Then $(1, 1)$ yields $F_{1,1}R_{1,1} = R_{1,1}F_{1,1}$. A MAGMA calculation disproves this equation. Hence $ad \neq 0$. If bc is zero and ad is nonzero, then $(1, 1)$ equation implies $F_{2,2}R_{1,1} = R_{1,1}F_{2,2}$ which is also wrong by MAGMA. Therefore all a, b, c, d are nonzero. We modify T so that $a = 1$ by multiplying some power of $\text{diag}((\begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}), 1, 1, 1, 1, 1)$ from the right.

Since ec is nonzero, it can be cancelled on both sides of equation $(1, 2)$. Hence

$$(1, 2) : d(F_{1,1} - F_{2,2})R_{2,2} - F_{2,6}R_{6,2} = dR_{1,1}(F_{1,1} - F_{2,2}).$$

Using MAGMA it can be verified that this equation holds only for $d = 2$. As $b \neq 0$ and $a = 1$ equation $(6, 1)$ implies

$$F_{6,2}R_{1,1} = -eR_{6,2}(F_{1,1} - F_{2,2}) + R_{6,6}F_{6,2}.$$

By MAGMA it has the solution $e = 11$. Now equation $(8, 1)$ implies that

$$F_{8,5}R_{5,1} = b(-11R_{8,2}(F_{1,1} - F_{2,2}) + R_{8,6}F_{6,2}).$$

This equation has the solution $b = 4$ by MAGMA. From the equation $ad - bc = e^{-1}$ we now deduce that $c = 12$.

In order to determine the remaining coefficients of the matrix T we use the following matrix equations derived from (*).

$$\begin{aligned} (9, 8) : G_{9,4}R_{4,8} + F_{9,9}R_{9,8} &= R_{9,8}F_{8,8}, \\ (9, 9) : G_{9,4}R_{4,9} + F_{9,9}R_{9,9} &= vgpR_{9,4}F_{4,9} + R_{9,9}F_{9,9}, \\ (4, 5) : G_{4,3}R_{3,5} + gpF_{4,7}R_{7,5} &= R_{4,8}F_{8,5}. \end{aligned}$$

Inserting the first set of equations yields:

$$\begin{aligned} (9, 8) : uv^{-1}F_{9,4}R_{4,8} + F_{9,9}R_{9,8} &= R_{9,8}F_{8,8}, \\ (9, 9) : uv^{-1}F_{9,4}R_{4,9} + F_{9,9}R_{9,9} &= vgpR_{9,4}F_{4,9} + R_{9,9}F_{9,9}, \\ (4, 5) : -gpq(F_{3,3} - F_{4,4})R_{3,5} + gpF_{4,7}R_{7,5} &= R_{4,8}F_{8,5}. \end{aligned}$$

By MAGMA the equation (9,8) has the solution $uv^{-1} = 10$. Now MAGMA asserts that the equation (9,9) has the solution $vgp = 11$. Hence $p \neq 0$. By multiplying some power of $\text{diag}((\begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}), 1, 1, 1, 1, 1)$ from the right we can modify T so that $p = 1$. Thus $vg = 11$ and the third equation (4,5) implies that

$$(4, 5) : g(-q(F_{3,3} - F_{4,4})R_{3,5} + F_{4,7}R_{7,5}) = R_{4,8}F_{8,5}.$$

This is non linear equation in the unknowns q and g has a unique solution $q = 6$ and $g = 11$ as has been checked by running with MAGMA through all 13^2 cases. From $6 = g^{-1} = pu - tq = 10 - 6t$ we now deduce that $t = 5$. This completes the determination of the coefficients of the two matrices w and z of statement (e). The remaining assertions are now clear. \square

Proposition 4.3. *Keep the notation of Lemmas 3.1, 3.2 and 4.1. Let PG_1 be the faithful permutation representation of the simple group $G_1 = \langle x, y, q, w \rangle = \langle u, v, r, s \rangle$ of degree 31671 with stabilizer $H_1 = \langle x, y, q \rangle$. Let $mH = \langle u, v, r \rangle$, $mD = \langle u, v, r \rangle$ and $mE = \langle u, v, s \rangle$. Let F^* be the multiplicative group of the prime field $F = \text{GF}(13)$. Let $Y = \text{GL}_{5083}(13)$.*

Let \mathfrak{V} and \mathfrak{W} be the up to isomorphism uniquely determined faithful semi-simple 5083-dimensional modules of mH and mE over F corresponding to the restrictions $\tau_{4|mH}$ and $\tau_{4|mE}$ of the irreducible character τ_4 of G_1 , respectively.

Let $\kappa_{\mathfrak{V}} : mH \rightarrow \text{GL}_{5083}(13)$ and $\kappa_{\mathfrak{W}} : mE \rightarrow \text{GL}_{5083}(13)$ be the representations of mH and mE afforded by the modules \mathfrak{V} and \mathfrak{W} , respectively.

Let $\mathfrak{r} = \kappa_{\mathfrak{V}}(r)$, $\mathfrak{u} = \kappa_{\mathfrak{V}}(u)$, $\mathfrak{v} = \kappa_{\mathfrak{W}}(v)$ in $\kappa_{\mathfrak{V}}(mH) \leq \text{GL}_{5083}(13)$.

Then $\mathfrak{V}_{|mD} \cong \mathfrak{W}_{|mD}$, and there is a transformation matrix $T \in \text{GL}_{5083}(13)$ such that

$$\mathfrak{u} = T^{-1}\kappa_{\mathfrak{W}}(u)T, \mathfrak{v} = T^{-1}\kappa_{\mathfrak{W}}(v)T.$$

Let $\mathfrak{mD} = \langle \mathfrak{u}, \mathfrak{v} \rangle$, $\mathfrak{mH} = \langle \mathfrak{u}, \mathfrak{v}, \mathfrak{r} \rangle$. Let $\mathcal{D} = C_Y(\mathfrak{mD})$ and $\mathcal{H} = C_Y(\mathfrak{mH})$. Let $\mathfrak{s}_1 = T^{-1}\kappa_{\mathfrak{W}}(s)T$. Let $\mathfrak{mE} = \langle \mathfrak{mD}, \mathfrak{s}_1 \rangle$ and $\mathcal{E} = C_Y(\mathfrak{mE})$. Then the following statements hold:

(a) *There is an isomorphism*

$$\alpha : \mathcal{D} \rightarrow \mathcal{D}_1 = \text{GL}_2(13) \times F^{*8} \leq \text{GL}_{10}(13).$$

(b) *$\mathcal{H}_1 = \alpha(\mathcal{H})$ is generated by the two blocked diagonal matrices*

$$a_1 = \text{diag}((\begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix}), 1, 2, 1, 2, 2, 1, 2, 1) \quad \text{and} \quad a_2 = \text{diag}((\begin{smallmatrix} 1 & 0 \\ 0 & 2 \end{smallmatrix}), 2, 1, 2, 1, 1, 2, 1, 2),$$

- (c) $\mathcal{E}_1 = \alpha(\mathcal{E})$ is generated by the four blocked diagonal matrices
- $$b_1 = \text{diag}((\begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix}), 1, 1, 1, 1, 1, 1, 1, 1), \quad b_2 = \text{diag}((\begin{smallmatrix} 1 & 0 \\ 0 & 2 \end{smallmatrix}), 2, 1, 1, 1, 1, 1, 1, 1),$$
- $$b_3 = \text{diag}((\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}), 1, 2, 1, 2, 2, 2, 1, 1), \quad b_4 = \text{diag}((\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}), 1, 1, 2, 1, 1, 1, 2, 2).$$
- (d) \mathcal{D} has $2184 \times 12^4 = 4587424$ $\mathcal{H}\text{-}\mathcal{E}$ double cosets.
- (e) The free product $mH *_{mD} mE$ of mH and mE with amalgamated subgroup mD has an irreducible 5083-dimensional representation over F which induces an irreducible representation of G_1 . It corresponds to the $\mathcal{H}\text{-}\mathcal{E}$ double coset representative

$$\mathcal{F} = \text{diag}(w, 1^{78}, 1^{91}, 11^{156}, 6^{273}, 11^{273}, 1^{728}, 1^{1456}, 1^{1664}) \in \text{GL}_{5083}(13), \quad \text{where}$$

$$w = \begin{pmatrix} 1^{182} & 9^{182} \\ 11^{182} & 9^{182} \end{pmatrix},$$

and a^n denotes a diagonal $n \times n$ matrix with unique diagonal non zero entry $a \in GF(13)$.

Let $\mathfrak{s} = \mathcal{F}^{-1}\mathfrak{s}_1\mathcal{F}$ and $\mathfrak{G}_1 = \langle \mathfrak{u}, \mathfrak{v}, \mathfrak{r}, \mathfrak{s} \rangle$. Inserting these four generating matrices of \mathfrak{G}_1 into the formulas of Lemma 4.1(f) one obtains the matrices \mathfrak{x}_{5083} , \mathfrak{y}_{5083} , \mathfrak{q}_{5083} , and \mathfrak{w}_{5083} of the original generators x , y , q and w of $G_1 = \text{Fi}_{23}$ as words in the generators u , v , r and s . The matrices \mathfrak{x}_{5083} , \mathfrak{y}_{5083} , \mathfrak{q}_{5083} , and \mathfrak{w}_{5083} can be downloaded from the first author's website <http://www.math.yale.edu/~hk47/Fi24/index.html>.

Proof. Let \mathfrak{V} be the up to isomorphism uniquely determined faithful semi-simple 5083-dimensional module of mH over $F = GF(13)$ corresponding to the restriction $\tau_{4|mH}$. By Lemma 4.1(g) $\tau_{4|mH} = \pi_{12} + \pi_{15}$.

The irreducible characters π_{11} and π_{15} of mH are constituents of the permutation characters $1_{mH_{11}}^{mH}$ and $1_{mH_{15}}^{mH}$ of the respective subgroups mH_{11} and mH_{15} of mH determined in Lemma 4.1(i). Using a stand-alone program of the first author which is based on Algorithm 5.7.1 of [12] we calculated the primitive idempotents of the endomorphism rings of these permutation modules. Thus we obtained the corresponding irreducible representations $M(\pi_{11})$ and $M(\pi_{15})$ of the respective dimensions 2275 and 2808 over $F = GF(13)$. Hence the irreducible FmH -module $M(\pi_{12})$ is the tensor product of $M(\pi_{11})$ with the linear character π_2 of mH over F . Thus $\mathfrak{V} = M(\pi_{12}) \oplus M(\pi_{15})$.

Let \mathfrak{W} be the up to isomorphism uniquely determined faithful semi-simple 5083-dimensional module of mE over F corresponding to the restriction $\tau_{4|mE} = \psi_{14} + \psi_{22} + \psi_{45} + \psi_{74}$, see Lemma 4.1(i).

The irreducible characters ψ_{14} , ψ_{22} , ψ_{45} and ψ_{74} of mE are constituents of the permutation characters $1_{mE_{14}}^{mE}$, $1_{mE_{22}}^{mE}$, $1_{mE_{45}}^{mE}$ and $1_{mE_{74}}^{mE}$, respectively, by Lemma 4.1(l). Using a stand alone program of the first author which is based on Algorithm 5.7.1 of [12] we calculated the primitive idempotents of the endomorphism rings of these four permutation modules. Thus we obtained the corresponding irreducible representations $N(\psi_{14})$, $N(\psi_{22})$, $N(\psi_{45})$ and $N(\psi_{74})$ of the respective dimensions 182, 260, 1365 and 3276 over F . Hence

$$\mathfrak{W} = N(\psi_{14}) \oplus N(\psi_{22}) \oplus N(\psi_{45}) \oplus N(\psi_{74}).$$

Fixing a basis in each irreducible constituent $M\pi_k$ of \mathfrak{V} we get a basis $\mathcal{B}_{\mathfrak{V}}$ of \mathfrak{V} . It induces a representation $\kappa_{\mathfrak{V}} : mH \rightarrow \text{GL}_{5083}(13)$ of mH . Let $\mathfrak{r} = \kappa_{\mathfrak{V}}(r)$, $\mathfrak{u} = \kappa_{\mathfrak{V}}(u)$, $\mathfrak{v} = \kappa_{\mathfrak{V}}(v)$ in $\kappa_{\mathfrak{V}}(mH) \leq \text{GL}_{5083}(13)$.

Fixing a basis in each irreducible constituent $N\psi_j$ of \mathfrak{W} we get a basis \mathcal{B}_W of \mathfrak{W} . It induces a representation $\kappa_{\mathfrak{W}} : mE \rightarrow \text{GL}_{5083}(13)$ of mE . By Lemma 4.1(h) $\mathfrak{V}_{|mD} \cong \mathfrak{W}_{|mD}$. Let $Y = \text{GL}_{5083}(13)$. Applying now Parker's isomorphism test of Proposition 6.1.6 of [12] by means of the MAGMA command

`IsIsomorphic(GModule(sub<Y|V(u), V(v)>), GModule(sub<Y|W(u), W(v)>))`

one obtains the transformation matrix T_1 satisfying $\mathbf{u} = \kappa_{\mathfrak{W}}(u)^{T_1}$ and $\mathbf{v} = \kappa_{\mathfrak{W}}(v)^{T_1}$.

(a) Let $\mathbf{m}\mathfrak{D} = \langle \mathbf{u}, \mathbf{v} \rangle$, $\mathbf{m}\mathfrak{H} = \langle \mathbf{u}, \mathbf{v}, \mathbf{r} \rangle$. Let $\mathcal{D} = C_Y(\mathbf{m}\mathfrak{D})$ and $\mathcal{H} = C_Y(\mathbf{m}\mathfrak{H})$. Let δ_{32a} and δ_{32b} be two distinct copies of the irreducible character δ_{32} of mD . Using PG_1 and MAGMA it can be checked that the irreducible characters π_{12} and π_{15} of mH have the following restrictions to $mD = \langle u, v \rangle$:

$$\begin{aligned}\pi_{12}|_{mD} &= \delta_{16} + \delta_{32a} + \delta_{34} + \delta_{37} + \delta_{66}, \\ \pi_{15}|_{mD} &= \delta_{11} + \delta_{23} + \delta_{32b} + \delta_{53} + \delta_{69},\end{aligned}$$

where the irreducible characters δ_{11} , δ_{16} , δ_{23} , δ_{32} , δ_{34} , δ_{37} , δ_{53} , δ_{66} , and δ_{69} of $mD \cong G_2(3) \times S_3$ have degrees 78, 91, 156, 182, 273, 273, 728, 1456 and 1664, respectively.

Since $\mathfrak{V}_{|mD}$ is a semi-simple FmD -module the Theorem 2.1.27 of [12] implies that there is an isomorphism

$$\alpha : \mathcal{D} \rightarrow \mathcal{D}_1 = \text{GL}_2(13) \times F^{*8} \leq \text{GL}_{10}(13).$$

(b) Furthermore, Schur's Lemma asserts that $\mathcal{H}_1 = \alpha(\mathcal{H})$ is generated by the two blocked diagonal matrices a_1 and a_2 given in the statement because 2 is a primitive element in the multiplicative group F^* of $F = GF(13)$.

(c) Let $\mathfrak{s}_1 = T_1^{-1} \kappa_{\mathfrak{W}}(s) T_1$. Let $\mathbf{m}\mathfrak{E} = \langle \mathbf{m}\mathfrak{D}, \mathfrak{s}_1 \rangle$ and $\mathcal{E} = C_Y(\mathbf{m}\mathfrak{E})$. Let δ_{32a} and δ_{32b} be two distinct copies of the irreducible character δ_{32} of mD . Using PG_1 and MAGMA it can be checked that the irreducible characters ψ_{14} , ψ_{22} , ψ_{45} and ψ_{74} of mE have the following restrictions to $mD = \langle u, v \rangle$:

$$\begin{aligned}\psi_{14}|_{mD} &= \delta_{32a}, & \psi_{22}|_{mD} &= \delta_{11} + \delta_{32b}, \\ \psi_{45}|_{mD} &= \delta_{16} + \delta_{34} + \delta_{37} + \delta_{53}, & \psi_{74}|_{mD} &= \delta_{23} + \delta_{66} + \delta_{69}.\end{aligned}$$

Now Schur's Lemma implies that $\mathcal{E}_1 = \alpha(\mathcal{E})$ is generated by the four blocked diagonal matrices b_j given in the statement.

(d) Every \mathcal{H} - \mathcal{E} double coset representative is of the form

$$\text{diag}(A, 1, v_2, v_3, v_4, v_5, 1, 1, 1)$$

for some $A \in Y$ and $v \in F^*$. By multiplying from left and right, we can find out that $\text{diag}(A, 1, v_2, v_3, v_4, v_5, 1, 1, 1)$ and $\text{diag}(A', 1, v_2, v_3, v_4, v_5, 1, 1, 1)$ represent the same double coset if and only if the first column of A is a scalar multiple of that of A' . So, we have 12^4 choices for v_i , and $|\text{GL}(2, 13)|/12 = 2184$ choices for A . Thus there are $2184 \cdot 12^4 = 4587424$ \mathcal{H} - \mathcal{E} double cosets.

(e) By Theorem 7.2.2 of [12] the irreducible representations of the free product $mH *_{mD} mE$ of the groups mH and mE with amalgamated subgroup mD are described by the \mathcal{H} - \mathcal{E} double coset representatives T of \mathcal{D} . The elements r and $f = (u^3 v s v)^9$ are two commuting involutions of $G_1 \cong \text{Fi}_{23}$ by Lemma 4.1(o). Let \mathbf{r} and \mathbf{f} be their matrices in $\mathbf{m}\mathfrak{H}$ and $\mathbf{m}\mathfrak{E}$, respectively. If $T_1 =$

$\text{diag}((\begin{smallmatrix} a & c \\ b & d \end{smallmatrix}), 1, v_2, v_3, v_4, v_5, 1, 1, 1)$ describes a 5083-dimensional irreducible representation of G_1 over F then $(**)$ $(\mathfrak{r}, \mathcal{T}_1^{-1}\mathfrak{f}\mathcal{T}_1) = 1$ holds.

Since $\mathfrak{V}_{|mD} \cong \mathfrak{W}_{|mD}$ is a direct sum of 10 irreducible FmD -modules both matrices \mathfrak{r} and \mathfrak{f} consist of 100 blocks $R_{i,j}$ and $F_{i,j}$, $1 \leq i, j \leq 10$ such that all diagonal blocks $R_{i,i}$ and $F_{i,i}$ are nontrivial. Furthermore, a non diagonal block $R_{i,j}$ of \mathfrak{r} is non zero if and only if the i -th irreducible and the j -th irreducible representations of mD appear in the restriction of an irreducible representation of mH to mD . A similar description holds for the blocks of \mathfrak{f} . Hence the system of equations in the proofs of (a) and (c) imply that

$$\mathfrak{r} = \begin{pmatrix} R_{1,1} & \cdot & \cdot & R_{1,4} & \cdot & R_{1,6} & R_{1,7} & \cdot & R_{1,9} & \cdot \\ \cdot & R_{2,2} & R_{2,3} & \cdot & R_{2,5} & \cdot & \cdot & R_{2,8} & \cdot & R_{2,10} \\ \cdot & R_{3,2} & R_{3,3} & \cdot & R_{3,5} & \cdot & \cdot & R_{3,8} & \cdot & R_{3,10} \\ R_{4,1} & \cdot & \cdot & R_{4,4} & \cdot & R_{4,6} & R_{4,7} & \cdot & R_{4,9} & \cdot \\ \cdot & R_{5,2} & R_{5,3} & \cdot & R_{5,5} & \cdot & \cdot & R_{5,8} & \cdot & R_{5,10} \\ R_{6,1} & \cdot & \cdot & R_{6,4} & \cdot & R_{6,6} & R_{6,7} & \cdot & R_{6,9} & \cdot \\ R_{7,1} & \cdot & \cdot & R_{7,4} & \cdot & R_{7,6} & R_{7,7} & \cdot & R_{7,9} & \cdot \\ \cdot & R_{8,2} & R_{8,3} & \cdot & R_{8,5} & \cdot & \cdot & R_{8,8} & \cdot & R_{8,10} \\ R_{9,1} & \cdot & \cdot & R_{9,4} & \cdot & R_{9,6} & R_{9,7} & \cdot & R_{9,9} & \cdot \\ \cdot & R_{10,2} & R_{10,3} & \cdot & R_{10,5} & \cdot & \cdot & R_{10,8} & \cdot & R_{10,10} \end{pmatrix},$$

$$\mathfrak{f} = \begin{pmatrix} F_{1,1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & F_{2,2} & F_{2,3} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & F_{3,2} & F_{3,3} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & F_{4,4} & \cdot & F_{4,6} & F_{4,7} & F_{4,8} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & F_{5,5} & \cdot & \cdot & \cdot & F_{5,9} & F_{5,10} \\ \cdot & \cdot & \cdot & F_{6,4} & \cdot & F_{6,6} & F_{6,7} & F_{6,8} & \cdot & \cdot \\ \cdot & \cdot & \cdot & F_{7,4} & \cdot & F_{7,6} & F_{7,7} & F_{7,8} & \cdot & \cdot \\ \cdot & \cdot & \cdot & F_{8,4} & \cdot & F_{8,6} & F_{8,7} & F_{8,8} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & F_{9,5} & \cdot & \cdot & \cdot & F_{9,9} & F_{9,10} \\ \cdot & \cdot & \cdot & \cdot & F_{10,5} & \cdot & \cdot & \cdot & F_{10,9} & F_{10,10} \end{pmatrix}.$$

Let $e = (ad - bc)^{-1}$ and $g = (pu - tq)^{-1}$. Then $e \neq 0 \neq g$. For each integer k let I_k denote the $k \times k$ identity matrix over F . Then

$$\mathcal{T}^{-1} = \begin{pmatrix} ed(I_{182}) & -ec(I_{182}) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -eb(I_{182}) & ea(I_{182}) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & I_{78} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & v_2^{-1}(I_{91}) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & v_3^{-1}(I_{156}) & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & v_4^{-1}(I_{273}) & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & v_5^{-1}(I_{273}) & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & I_{728} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & I_{1456} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & I_{1664} \end{pmatrix}.$$

Hence $\mathfrak{f}' = \mathcal{T}^{-1}\mathfrak{f}\mathcal{T}$ equals the matrix

$$\begin{pmatrix} G_{1,1} & G_{1,2} & G_{1,3} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ G_{2,1} & G_{2,2} & G_{2,3} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ bF_{3,2} & dF_{3,2} & F_{3,3} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & F_{4,4} & \cdot & G_{4,6} & G_{4,7} & v_2^{-1}F_{4,8} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & F_{5,5} & \cdot & \cdot & \cdot & v_3^{-1}F_{5,9} & v_3^{-1}F_{5,10} \\ \cdot & \cdot & \cdot & G_{6,4} & \cdot & F_{6,6} & G_{6,7} & v_4^{-1}F_{6,8} & \cdot & \cdot \\ \cdot & \cdot & \cdot & G_{7,4} & \cdot & G_{7,6} & F_{7,7} & v_5^{-1}F_{7,8} & \cdot & \cdot \\ \cdot & \cdot & \cdot & v_2F_{8,4} & \cdot & v_4F_{8,6} & v_5F_{8,7} & F_{8,8} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & v_3F_{9,5} & \cdot & \cdot & \cdot & F_{9,9} & F_{9,10} \\ \cdot & \cdot & \cdot & \cdot & v_3F_{10,5} & \cdot & \cdot & \cdot & F_{10,9} & F_{10,10} \end{pmatrix},$$

where

$$\begin{aligned} G_{1,1} &= e(adF_{1,1} - bcF_{2,2}), & G_{1,2} &= ecd(F_{1,1} - F_{2,2}), & G_{1,3} &= -ecF_{2,3}, \\ G_{2,1} &= -eab(F_{1,1} - F_{2,2}), & G_{2,2} &= -e(bcF_{1,1} - adF_{2,2}), & G_{2,3} &= eaF_{2,3}, \\ G_{4,6} &= v_2^{-1}v_4F_{4,6}, & G_{4,7} &= v_2^{-1}v_5F_{4,7}, & G_{6,4} &= v_4^{-1}v_2F_{6,4}, \\ G_{6,7} &= v_4^{-1}v_5F_{6,7}, & G_{7,4} &= v_5^{-1}v_2F_{7,4}, & G_{7,6} &= v_5^{-1}v_4F_{7,6}. \end{aligned}$$

Now (**) implies the following equations

$$\begin{aligned} (10, 1) : F_{10,9}R_{9,1} &= R_{10,2}G_{2,1} + R_{10,3}(bF_{3,2}), \\ (9, 1) : F_{9,9}R_{9,1} &= R_{9,1}G_{1,1}, \\ (9, 2) : v_3F_{9,5}R_{5,2} + F_{9,10}R_{10,2} &= R_{9,1}G_{1,2}, \\ (8, 1) : v_2F_{8,4}R_{4,1} + v_4F_{8,6}R_{6,1} + v_5F_{8,7}R_{7,1} &= R_{8,2}G_{2,1} + R_{8,3}(bF_{3,2}). \end{aligned}$$

Inserting the first set of equations yields:

$$\begin{aligned} (10, 1) : F_{10,9}R_{9,1} &= b[-eaR_{10,2}(F_{1,1} - F_{2,2}) + R_{10,3}F_{3,2}], \\ (9, 1) : F_{9,9}R_{9,1} &= eR_{9,1}(adF_{1,1} - bcF_{2,2}), \\ (9, 2) : v_3F_{9,5}R_{5,2} + F_{9,10}R_{10,2} &= ecdR_{9,1}(F_{1,1} - F_{2,2}). \\ (8, 1) : v_2F_{8,4}R_{4,1} + v_4F_{8,6}R_{6,1} + v_5F_{8,7}R_{7,1} &= b[-eaR_{8,2}(F_{1,1} - F_{2,2}) + R_{8,3}F_{3,2}]. \end{aligned}$$

Equation (10, 1) has only one solution $b = 9$, $ea = 1$ in $F = GF(13)$ as has been checked by means of MAGMA. Hence $a \neq 0$. By multiplying some power of $\text{diag}((\begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix}), 1, 1, 1, 1, 1, 1, 1, 1)$ from the right we can modify the matrix T so that $a = 1$. Thus also $e = 1$. Now equation (9, 1) becomes

$$(9, 1) : F_{9,9}R_{9,1} = R_{9,1}(dF_{1,1} - 9cF_{2,2}).$$

Another MAGMA calculation running through 13^2 pairs (c, d) of elements of F shows that (9, 1) has the unique solution $c = 11$ and $d = 9$.

Inserting the known values for c , d and e into equation (9, 2) yields that $v_3 = 11$ as has been checked by means of MAGMA. Finally, inserting the known values for a , b and e into equation (8, 1) yields

$$(8, 1) : v_2F_{8,4}R_{4,1} + v_4F_{8,6}R_{6,1} + v_5F_{8,7}R_{7,1} = 9[-R_{8,2}(F_{1,1} - F_{2,2}) + R_{8,3}F_{3,2}].$$

A MAGMA calculation shows that this equation has the unique solution $v_2 = 1$, $v_4 = 6$, $v_5 = 11$. This completes the proof because all remaining statements of (e) are straightforward. \square

5. CONSTRUCTION OF THE IRREDUCIBLE SUBGROUP \mathfrak{G} OF $\mathrm{GL}_{8671}(13)$

In this section we construct the 8 semi-simple representations of the 2-fold cover A_1 of the automorphism group $\mathrm{Aut}(\mathrm{Fi}_{22})$ of Fi_{22} corresponding to the 8 compatible pairs of Lemma 3.2(e). Since $H_1 = \langle q, y \rangle = C_{G_1}(u) \cong A'_1$ for some involution u of $G_1 = \langle q, y, w \rangle$ and A'_1 has index 2 in A_1 it is not difficult to construct the irreducible constituents of these representations of A_1 from the 3588-dimensional irreducible representation of $G_1 \cong \mathrm{Fi}_{23} = \langle q, y, w \rangle$ by means of Clifford's Theorem. In fact, we construct 8 new matrices t_i of order 2 such that $K_i = \langle G_1, t_i \rangle$ is an irreducible subgroup of $\mathrm{GL}_{8671}(13)$. It turns out that only K_3 corresponding to the compatible pair (3) of Lemma 3.2(e) may have a Sylow 2-subgroup which is isomorphic to a Sylow 2-subgroup of the extension group E of Lemma 2.1.

Proposition 5.1. *Let $\mathfrak{G}_1 = \langle \mathfrak{y}_{3588}, \mathfrak{q}_{3588}, \mathfrak{w}_{3588} \rangle$ be the simple subgroup of $Y = \mathrm{GL}_{3588}(13)$ of order $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$ constructed in Proposition 4.2. Let $y_1 = \mathfrak{y}_{3588}$, $q_1 = \mathfrak{q}_{3588}$. Let $\mathfrak{H}_1 = \langle y_1, q_1 \rangle$.*

Let $A_1 = 2\mathrm{Aut}(\mathrm{Fi}_{22}) = \langle a, b, c, d, e, f, g, h, i, z, t \rangle$ be the finitely presented group defined in Lemma 3.1. Then the following assertions hold:

- (a) *There is an isomorphism ϕ from the subgroup $H_1 = \langle a, b, c, d, e, f, g, h, i, z \rangle$ of A_1 to \mathfrak{H}_1 .*
- (b) *$A_1 = \langle y, q, t \rangle$, where $y = \phi^{-1}(y_1)$ and $q = \phi^{-1}(q_1)$.*
- (c) *There is a transformation matrix $T \in Y$ such that*

$$\begin{aligned} T^{-1}\mathfrak{y}T &= \mathrm{diag}(\mathfrak{y}_{78}, \mathfrak{y}_{1430}, \mathfrak{y}_{2080}) \in Y, \\ T^{-1}\mathfrak{q}T &= \mathrm{diag}(\mathfrak{q}_{78}, \mathfrak{q}_{1430}, \mathfrak{q}_{2080}) \in Y, \end{aligned}$$

where $y_k, q_k \in \mathrm{GL}_k(13)$ for $k \in \{78, 1430, 2080\}$.

- (d) *A_1 has a faithful irreducible representation $\lambda : A_1 \rightarrow \mathrm{GL}_{4160}(13)$ such that*

$$\begin{aligned} \lambda(y) &= \mathrm{diag}(\mathfrak{y}_{2080}, \phi(y^t)_{2080}) \in \mathrm{GL}_{4160}(13), \\ \lambda(q) &= \mathrm{diag}(\mathfrak{h}_{2080}, \phi(q^t)_{2080}) \in \mathrm{GL}_{4160}(13), \\ \lambda(t) &= \begin{pmatrix} 0 & I_{2080} \\ I_{2080} & 0 \end{pmatrix}. \end{aligned}$$

where I_{2080} denotes the identity matrix of $\mathrm{GL}_{2080}(13)$.

- (e) *The irreducible characters χ_3 , χ_4 , χ_{11} and χ_{12} of Table B.5 of respective degrees 78, 78, 1430 and 1430 are constituents of the permutation character $1_U^{A_1}$ of the subgroup*

$$U = \langle (q_1^2 y_1^3 q_1 y_1^3)^4, (y_1^2 q_1 y_1^3 q_1^2 y_1 q_1)^6, (y_1^4 q_1 y_1 q_1 y_1 q_1 y_1^2)^2 \rangle$$

of A_1 of index 2358720.

- (f) *The tensor product $\chi_3 \otimes \chi_3$ contains a copy of the irreducible character χ_{13} . Furthermore, $\chi_{14} = \chi_{13} \otimes \chi_2$, where χ_2 is the non trivial linear character of A_1 .*

Proof. (a) In the simple subgroup \mathfrak{G}_1 of $Y = \text{GL}_{3588}(13)$ let

$$\begin{aligned} x_1 &= [(y_1 q_1^2 y_1 q_1 y_1 q_1^2)^{11} (q_1^2 y_1^2 q_1 y_1 q_1 y_1)^{11} (q_1 y_1^2 q_1 y_1 q_1 y_1 q_1 y_1 q_1)^4]^{12}, \\ a_1 &= (x_1 y_1 x_1)^7, \quad b_1 = [(q_1 y_1)^2 q_1 y_1^3 q_1^2 y_1^3 q_1 y_1]^7, \quad c_1 = (y_1^2 x_1 y_1 x_1 y_1^3)^5, \\ d_1 &= (q_1 y_1 q_1^2 y_1 q_1 y_1 q_1 y_1 q_1 y_1^2 q_1^2)^{15}, \quad e_1 = (y_1 x_1 y_1^5 x_1)^5, \\ f_1 &= (y_1 q_1 y_1 q_1^2 y_1 q_1^2 y_1^2 q_1 y_1^4 q_1^2)^5, \quad g_1 = (x_1 y_1^2 x_1 y_1^3 x_1)^7, \\ h_1 &= (y_1^5 x_1 y_1 x_1)^5, \quad i_1 = (q_1^2 y_1^2 q_1 y_1 q_1^2)^7. \end{aligned}$$

By Lemma 3.2 and Proposition 4.2 the matrix subgroup $G_1 = \langle x, y, q, w \rangle$ of $\text{GL}_{782}(17)$ and the matrix subgroup $\mathfrak{G}_1 = \langle \mathfrak{h}_{3588}, \mathfrak{q}_{3588}, \mathfrak{w}_{3588} \rangle$ are isomorphic under the map θ sending x, y, q and w to x_1, y_1, q_1 and w_{3588} , respectively. Thus Lemma 3.2 (b) implies that the normal subgroup $H_1 = \langle a, b, c, d, e, f, g, h, i, z \rangle$ of A_1 is isomorphic to $\mathfrak{H}_1 = \langle a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, z_1 \rangle$, where $z_1 = (x_1 y_1^2)^7$. In particular, the map $\phi : H_1 \rightarrow \mathfrak{H}_1$ sending a, b, \dots, z to a_1, b_1, \dots, z_1 is an isomorphism such that $x = \phi^{-1}(x_1)$, $y = \phi^{-1}(y_1)$, $q = \phi^{-1}(q_1)$.

Statement (b) is an immediate consequence of (a) and Lemma 3.1(b).

(c) The natural F -vector space $V = F^{3588}$ is an $F\mathfrak{H}_1$ -module because $\mathfrak{H}_1 = \langle y_1, q_1 \rangle$. Applying the Meataxe algorithm it follows that V has three composition factors V_{78} , V_{1430} and V_{2080} of dimensions 78, 1430 and 2080, respectively. Since all three dimensions are divisible by 13 and 13 divides $|\mathfrak{H}_1| = |2\text{Fi}_{22}|$ to the first power only all three simple composition factors of V are projective $F\mathfrak{H}$ -modules by Theorems 3.12.2 and 3.12.4 of [12]. Hence V is isomorphic to their direct sum. Thus (c) holds.

(d) The group A_1 of Lemma 3.1 has a unique irreducible character χ_{17} of degree 4160 Table B.5. Clifford's Theorem 2.6.15 of [12] asserts that its restriction to H_1 is a sum of two inequivalent irreducible characters ν and ν^t of degree 2080. In particular, the induced FA_1 -module $V_{2080}^{A_1}$ is the reduction modulo 13 of a lattice which affords the irreducible character χ_{17} of A_1 . It corresponds to the irreducible representation $\lambda : A_1 \rightarrow \text{GL}_{4160}(13)$ of 13-defect zero defined in statement (d). It is well defined because Lemma 3.1(b) implies that $a^t = g^{-1}$, $b^t = f^{-1}$, $c^t = e^{-1}$, $d^t = d^{-1}$, $h^t = h^{-1}$ and $i^t = i^{-1}$. Therefore $\phi(y)_{2080}$ and $\phi(q)_{2080}$ are well defined by (a) and (c).

(e) Using the MAGMA command `LowIndexSubgroups(A_1, m)` we searched for conjugacy classes of subgroups U of index $|A_1 : U| = m$ such that χ_k is an irreducible constituent of the permutation character $1_U^{A_1}$ for $k \in \{3, 4, 11, 12\}$. Thus we found a subgroup U of index $m = 2358720$ such that its permutation character contains all four irreducible characters χ_k . Its generators have been obtained by means of Kim's program `GetShortGens(A_1, U)`.

Statement (f) can be verified by means of the character table of A_1 . \square

Proposition 5.2. *Keep the notation of Lemma 3.1 and Propositions 4.2, 4.3. Let $A_1 \leftarrow H_1 \rightarrow G_1$ be the amalgam constructed in Lemma 3.2, where $G_1 \cong \text{Fi}_{23}$. Let $\sigma : H_1 \rightarrow A_1$ denote its monomorphism of H_1 into G_1 . Let $Y = \text{GL}_{8671}(13)$. Let $\sigma(y) = \text{diag}(\mathfrak{h}_{3588}, \mathfrak{h}_{5083})$, $\sigma(q) = \text{diag}(\mathfrak{q}_{3588}, \mathfrak{q}_{5083})$, $\mathfrak{w}_1 = \text{diag}(\mathfrak{w}_{3588}, \mathfrak{w}_{5083})$ in Y . Let $\mathfrak{H}_1 = \langle \sigma(y), \sigma(q) \rangle$ and $\mathfrak{G}_1 = \langle \mathfrak{H}_1, \mathfrak{w}_1 \rangle$. Keep the notation of Table B.5, Table 6.6.3 of [14] and of the character table of Fi_{23} , see [4], its p. 178 - 179.*

Then the following statements hold:

(a) *There is a compatible pair of characters*

$(\chi, \tau) = (\chi_3 + \chi_{12} + \chi_{13} + \chi_{17}, \tau_3 + \tau_4) \in \text{mfchar}_{\mathbb{C}}(A_1) \times \text{mfchar}_{\mathbb{C}}(G_1)$
of degree 8671 of the groups $A_1 = \langle H_1, t \rangle$ and $G_1 = \langle H_1, e_1 \rangle$ with common restriction

$$\tau|_{H_1} = \chi|_{H_1} = \delta_2 + \delta_6 + \delta_7 + \delta_8 + \delta_9 \in \text{mfchar}_{\mathbb{C}}(H_1),$$

where irreducible characters with bold face indices denote faithful irreducible characters.

(b) *Let \mathfrak{V} and \mathfrak{W} be the up to isomorphism uniquely determined faithful semi-simple multiplicity-free 8671-dimensional modules of A_1 and G_1 over $F = \text{GF}(13)$ corresponding to the compatible pair χ, τ , respectively.*

Let $\kappa_{\mathfrak{V}} : H \rightarrow \text{GL}_{8671}(13)$ and $\kappa_{\mathfrak{W}} : E \rightarrow \text{GL}_{8671}(13)$ be the representations of A_1 and G_1 afforded by the modules \mathfrak{V} and \mathfrak{W} , respectively.

Let $\mathfrak{q} = \kappa_{\mathfrak{V}}(q)$, $\mathfrak{y} = \kappa_{\mathfrak{V}}(y)$, $\mathfrak{t} = \kappa_{\mathfrak{V}}(t)$ in $\kappa_{\mathfrak{V}}(A_1) \leq \text{GL}_{8671}(13)$. Then the following assertions hold:

(1) $\mathfrak{V}|_{\mathfrak{H}_1} \cong \mathfrak{W}|_{\mathfrak{H}_1}$, and there is a transformation matrix $T \in \text{GL}_{8671}(13)$ such that

$$\mathfrak{q} = T^{-1} \kappa_{\mathfrak{W}}(\sigma(q)) T, \mathfrak{y} = T^{-1} \kappa_{\mathfrak{W}}(\sigma(y)) T.$$

Let $\mathfrak{w} = T^{-1} \kappa_{\mathfrak{W}}(w_1) T \in \text{GL}_{8671}(13)$.

(2) *The subgroup $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{y}, \mathfrak{t}, \mathfrak{w} \rangle$ of Y is the uniquely determined irreducible representation of the free product $P = G_1 *_{H_1} A_1$ of G_1 and A_1 with amalgamated subgroup H_1 corresponding to the compatible pair (3) of Lemma 3.2(e). Its four generating matrices can be downloaded from the first author's website*

<http://www.math.yale.edu/~hk47/Fi24/index.html>.

Proof. (a) By Lemma 3.2(e) the amalgam $A_1 \leftarrow H_1 \rightarrow G_1$ has 8 compatible pairs of degree 8671. We constructed the corresponding semi-simple representations of $P = G_1 *_{H_1} A_1$ for each of them. But we give a proof only for that pair (3) of Lemma 3.2(e). It belongs to a group of a suitable order.

(b) By Propositions 4.2 and 4.3 the semi-simple FG_1 -module $\mathfrak{W} = \mathfrak{W}_3 \oplus \mathfrak{W}_4$ of dimension 8671 corresponding to the multiplicity free character $\tau_3 + \tau_4$ is described by the three matrices $\mathfrak{y} = \sigma(y)$, $\mathfrak{q} = \sigma(q)$, \mathfrak{w} of Y . The semi-simple FA_1 -module \mathfrak{V} of the same dimension corresponding to the multiplicity free character $\chi_3 + \chi_{12} + \chi_{14} + \chi_{17}$ of A_1 is defined by three blocked diagonal matrices $\mathfrak{q} = \text{diag}(\mathfrak{q}_3, \mathfrak{q}_{12}, \mathfrak{q}_{13}, \mathfrak{q}_{17})$, $\mathfrak{y} = \text{diag}(\mathfrak{y}_3, \mathfrak{y}_{12}, \mathfrak{y}_{13}, \mathfrak{y}_{17})$ and $\mathfrak{t} = \text{diag}(\mathfrak{t}_3, \mathfrak{t}_{12}, \mathfrak{t}_{13}, \mathfrak{t}_{17})$ whose entries can be calculated by means of Proposition 5.1 as follows.

Assertion (d) of Proposition 5.1 states that $\mathfrak{q}_{17} = \lambda(q)$, $\mathfrak{y}_{17} = \lambda(y)$, $\mathfrak{t}_{17} = \lambda(t)$ in $\text{GL}_{4160}(13)$. By Proposition 5.1(e) the irreducible characters χ_3 and χ_{12} are constituents of the permutation character $1_U^{A_1}$ of a well determined subgroup U of A_1 of index 2358720. Let P_U be the corresponding permutation module over $F = \text{GF}(13)$. Using a stand alone program of the first author which is based on Algorithm 5.7.1 of [12] we calculated the primitive idempotents E_{χ_3} and $E_{\chi_{12}}$ of the endomorphism ring $\text{End}_{FA_1}(P_U)$ of P_U . Since the irreducible characters χ_3 and χ_{12} are of 13-defect zero the FA_1 -modules $\mathfrak{V}_3 = E_{\chi_3} P_U$ and $\mathfrak{V}_{12} = E_{\chi_{12}} P_U$ are irreducible by Theorems 3.12.2 and 3.12.4 of [12]. After fixing a basis in each of them the actions of the generators q , y and t of A_1 on P_U induce the matrices \mathfrak{q}_3 , \mathfrak{q}_{12} , \mathfrak{y}_3 , \mathfrak{y}_{12} and \mathfrak{t}_3 , \mathfrak{t}_{12} , respectively.

The tensor product $\chi_3 \otimes \chi_3$ contains a copy of the irreducible character χ_{13} by Proposition 5.1(f). Since \mathfrak{V}_3 has dimension 78 the Meataxe algorithm implemented

in MAGMA can be applied to the tensor product $\mathfrak{V}_3 \otimes \mathfrak{V}_3$. This application provides the three 3003×3003 matrices \mathfrak{q}_{13} , \mathfrak{h}_{13} and \mathfrak{t}_{13} corresponding to the irreducible FA_1 -module \mathfrak{V}_{13} . Hence $\mathfrak{V} = \mathfrak{V}_3 \oplus \mathfrak{V}_{12} \oplus \mathfrak{V}_{13} \oplus \mathfrak{V}_{17}$.

By (a) the restrictions $\mathfrak{V}_{3|\mathfrak{H}_1}$, $\mathfrak{V}_{12|\mathfrak{H}_1}$ and $\mathfrak{V}_{13|\mathfrak{H}_1}$ to \mathfrak{H}_1 are irreducible. By the proof of Proposition 5.1(d) we know that

$$\begin{aligned}\mathfrak{V}_{17|\mathfrak{H}_1} &= \mathfrak{V}_{2080} \oplus \mathfrak{V}_{2080} \otimes t, \\ \mathfrak{V}_{3|\mathfrak{H}_1} \oplus \mathfrak{V}_{12|\mathfrak{H}_1} \oplus \mathfrak{V}_{2080} &\cong \mathfrak{W}_{3|\mathfrak{H}_1}, \\ \mathfrak{V}_{13|\mathfrak{H}_1} \oplus \mathfrak{V}_{2080} \otimes t &\cong \mathfrak{W}_{4|\mathfrak{H}_1}.\end{aligned}$$

Let $X_3 = \text{GL}_{3588}(13)$, $X_4 = \text{GL}_{5083}(13)$, $V_3 = \mathfrak{V}_{3|\mathfrak{H}_1} \oplus \mathfrak{V}_{12|\mathfrak{H}_1}$, $V_4 = \mathfrak{V}_{13|\mathfrak{H}_1} \oplus \mathfrak{V}_{2080} \otimes t$, $W_3 = \mathfrak{W}_{3|\mathfrak{H}_1}$ and $W_4 = \mathfrak{W}_{4|\mathfrak{H}_1}$. By the proof of Proposition 5.1(d) $V_3 \cong W_3$ and $V_4 \cong W_4$ as $F\mathfrak{H}_1$ -modules of respective dimensions 3588 and 5083. Applying now Parker's isomorphism test of Proposition 6.1.6 of [12] by means of the MAGMA command

`IsIsomorphic(GModule(sub<X_i|V_i(h),V_i(y)>),GModule(sub<X_i|W_i(h),W_i(y)>)),`

$i \in \{3, 4\}$, one obtains the transformation matrices $\mathcal{T}_3 \in X_3$ and $\mathcal{T}_4 \in X_4$ such that $\mathcal{T} = \text{diag}(\mathcal{T}_3, \mathcal{T}_4) \in Y$ satisfies $\mathfrak{q} = \kappa_{\mathfrak{W}}(\sigma(q))^{\mathcal{T}}$ and $\mathfrak{h} = \kappa_{\mathfrak{W}}(\sigma(y))^{\mathcal{T}}$.

Let $\mathfrak{w} = \mathcal{T}^{-1} \kappa_{\mathfrak{W}}(w_1) \mathcal{T} \in \text{GL}_{8671}(13)$. Corollary 7.2.4 of [12] asserts that the matrix group $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{h}, \mathfrak{t}, \mathfrak{w} \rangle$ is uniquely determined by the compatible given in (a). \square

Remark 5.3. Using Proposition 5.1 we constructed for each of the eight compatible pairs (k) of Lemma 3.2(e) a matrix \mathfrak{t}_k for the new generator t of $A_1 = \langle q, y, w, t \rangle$ using the methods of the proof of Proposition 5.2. Thus we obtained 8 subgroups $\mathfrak{G}_k = \langle \mathfrak{q}, \mathfrak{h}, \mathfrak{t}_k, \mathfrak{w} \rangle$ of $\text{GL}_{8671}(13)$. In each of them we tried to calculate the orders of the following products of the generators:

Group Name	$\mathfrak{w}\mathfrak{t}_k$	$\mathfrak{h}\mathfrak{w}\mathfrak{t}_k$	$\mathfrak{q}\mathfrak{w}\mathfrak{t}_k$	$\mathfrak{h}\mathfrak{w}\mathfrak{t}_k\mathfrak{q}$	$\mathfrak{h}\mathfrak{t}_k\mathfrak{w}\mathfrak{q}\mathfrak{t}_k$
\mathfrak{G}_1	12	fail	—	—	—
\mathfrak{G}_2	24	fail	—	—	—
\mathfrak{G}_3	4	24	24	21	33
\mathfrak{G}_4	8	24	fail	—	—
\mathfrak{G}_5	8	24	fail	—	—
\mathfrak{G}_6	4	24	24	42	66
\mathfrak{G}_7	24	fail	—	—	—
\mathfrak{G}_8	12	fail	—	—	—

where “fail” means the the product has an order which is greater than 100. The group \mathfrak{G} of Proposition 5.2 is \mathfrak{G}_3 . Looking at the orders of many random elements we saw that all such orders were bounded by 60. In particular, $\mathfrak{p} = \mathfrak{h}^2 \cdot \mathfrak{t}_3 \cdot \mathfrak{w} \cdot \mathfrak{q}$ has order 29.

Therefore we prove in the remainder of the article that \mathfrak{G} is isomorphic to Fischer's simple group Fi'_{24} . Most likely, \mathfrak{G}_6 is isomorphic to Fischer's non simple group Fi_{24} . In the other cases we were not able to calculate the orders of non trivial words of the generators in reasonable time.

6. ISOMORPHISM BETWEEN \mathfrak{G} AND FISCHER'S GROUP Fi'_{24}

In this section we construct an isomorphism between the matrix group $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{h}, \mathfrak{t}, \mathfrak{w} \rangle$ of Proposition 5.2 and the commutator subgroup of the finitely presented group G of Hall and Soicher, see [15], p.111. Hence \mathfrak{G} is isomorphic to Fischer's simple group Fi'_{24} .

Proposition 6.1. *Let $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{h}, \mathfrak{t}, \mathfrak{w} \rangle$ be the subgroup of $\text{GL}_{8671}(13)$ constructed in Proposition 5.2. Let $\mathfrak{H}_1 = \langle \mathfrak{h}, \mathfrak{q} \rangle$, $\mathfrak{A}_1 = \langle \mathfrak{H}_1, \mathfrak{t} \rangle$ and $\mathfrak{G}_1 = \langle \mathfrak{H}_1, \mathfrak{w} \rangle$.*

Let $E = \langle a, b, c, d, t, g, h, i, j, k, v_i \mid 1 \leq i \leq 11 \rangle$ be the non-split extension of the Mathieu group \mathcal{M}_{24} by its simple $GF(2)$ -module V_2 constructed in Lemma 2.1, and let $E_{23} = \langle a, b, c, d, t, g, h, i, j \rangle$.

Let $\mathfrak{x} = [(\mathfrak{h}\mathfrak{q}^2\mathfrak{h}\mathfrak{q}\mathfrak{h}\mathfrak{q}^2)^{11}(\mathfrak{q}^2\mathfrak{h}^2\mathfrak{q}\mathfrak{h}\mathfrak{q}\mathfrak{h})^{11}(\mathfrak{q}\mathfrak{h}^2\mathfrak{q}\mathfrak{h}\mathfrak{q}\mathfrak{h}\mathfrak{q}\mathfrak{h}\mathfrak{q})^4]^{12}$, $\mathfrak{u}_1 = (\mathfrak{r}\mathfrak{h}\mathfrak{r})^7$, $\mathfrak{u}_2 = (\mathfrak{r}\mathfrak{h}\mathfrak{r}\mathfrak{h}\mathfrak{r}\mathfrak{h}\mathfrak{r})^4$, $\mathfrak{u}_3 = (\mathfrak{r}\mathfrak{h}\mathfrak{r}\mathfrak{h}^2\mathfrak{r}\mathfrak{h}^2\mathfrak{r}\mathfrak{h}\mathfrak{r})^2$, $\mathfrak{u}_4 = (\mathfrak{r}\mathfrak{h}\mathfrak{r}\mathfrak{h}^5\mathfrak{r}\mathfrak{h}^4)^2$, and $\mathfrak{u}_5 = [\mathfrak{h}(\mathfrak{s}\mathfrak{w})^2\mathfrak{h}\mathfrak{w}]^7$.

Then the following assertions hold:

- (a) *The subgroup $\mathfrak{T}_1 = \langle \mathfrak{u}_i \mid 1 \leq i \leq 4 \rangle$ of $\mathfrak{D} = \langle \mathfrak{x}, \mathfrak{h} \rangle$ is a Sylow 2-subgroup of $\mathfrak{G}_1 = \langle \mathfrak{q}, \mathfrak{h}, \mathfrak{w} \rangle$ of order 2^{18} .*
- (b) *\mathfrak{T}_1 has a unique maximal elementary abelian normal subgroup \mathfrak{B} of order 2^{11} . It is generated by the 11 involutions:*

$$\begin{aligned} &\mathfrak{u}_1, \quad \mathfrak{u}_2^2, \quad (\mathfrak{u}_1\mathfrak{u}_2)^2, \quad (\mathfrak{u}_1\mathfrak{u}_3)^2, \quad (\mathfrak{u}_1\mathfrak{u}_4)^2, \quad (\mathfrak{u}_2\mathfrak{u}_4)^4, \\ &(\mathfrak{u}_1\mathfrak{u}_2\mathfrak{u}_3)^4, \quad (\mathfrak{u}_1\mathfrak{u}_2\mathfrak{u}_4)^4, \quad (\mathfrak{u}_1\mathfrak{u}_3\mathfrak{u}_4)^4, \quad (\mathfrak{u}_2^2\mathfrak{u}_3)^2, \quad (\mathfrak{u}_1\mathfrak{u}_2\mathfrak{u}_4\mathfrak{u}_2)^2. \end{aligned}$$

- (c) *$\mathfrak{s} = (\mathfrak{h}^5\mathfrak{t})^7$ is an involution of \mathfrak{A}_1 such that $\mathfrak{A}_1 = \langle \mathfrak{H}_1, \mathfrak{s} \rangle$, $\mathfrak{T}_1^{\mathfrak{s}} = \mathfrak{T}_1$, $\mathfrak{B}^{\mathfrak{s}} = \mathfrak{B}$, and $\mathfrak{G} = \langle \mathfrak{T}_1, \mathfrak{s} \rangle$ is a Sylow 2-subgroup of \mathfrak{A}_1 of order 2^{19} .*
- (d) *$\mathfrak{N}_1 = N_{\mathfrak{G}_1}(\mathfrak{B}) = \langle \mathfrak{x}, \mathfrak{h}, \mathfrak{w} \rangle$ is isomorphic to a non-split extension of \mathcal{M}_{23} by V_2 and $\mathfrak{D}_1 = N_{\mathfrak{A}_1}(\mathfrak{B}) = \langle \mathfrak{x}, \mathfrak{h}, \mathfrak{s} \rangle$*
- (e) *There is an isomorphism ρ between \mathfrak{N}_1 and the subgroup E_{23} of E such that $\rho(\mathfrak{h}) = (y_2^5 y_3 y_2 y_3)^3 (w_3 w_1 w_2 w_1 w_2 w_1 w_3 w_1^2 w_3 w_2 w_3 w_2 w_1 w_3)^{20}$, $\rho(\mathfrak{x}) = (x_1 x_2 x_4 x_5 x_4 x_2 x_5)^3$, $\rho(\mathfrak{w}) = (e_2 e_3 e_2 e_3^2)^7$, where*

$$\begin{aligned} &x_1 = (ij)^3, \quad x_2 = (gahigai)^2, \quad x_3 = (aghiagah)^4, \\ &x_4 = (jhighajai)^4, \quad x_5 = (aighjigai)^4, \\ &y_1 = i, \quad y_2 = ag, \quad y_3 = (ahj)^3, \\ &w_1 = (y_2 y_3^2)^2, \quad w_2 = (y_1 y_2 y_1 y_2 y_3)^3, \quad w_3 = (y_1 y_2 y_3 y_2^2)^3, \\ &e_1 = (agijih)^4, \quad e_2 = (ag^3 ihj)^7, \quad e_3 = ghghiai. \end{aligned}$$

- (f) *There is an isomorphism μ between $\mathfrak{D}_1 = N_{\mathfrak{A}_1}(\mathfrak{B})$ and the centralizer $C_E(u)$ of the involution $u = (\rho(\mathfrak{x})\rho(\mathfrak{h})^2)^7$ of E such that $\mu(\mathfrak{x}) = \rho(\mathfrak{x})$, $\mu(\mathfrak{h}) = \rho(\mathfrak{h})$ and $\mu(\mathfrak{s}) = (m_1^4 m_2 m_1 m_2)^2$, where $m_1 = agahj$, $m_2 = (ijhkj)^2$, $m_3 = (ahjagk)^5$.*
- (g) *The subgroup $\mathfrak{E} = \langle \mathfrak{x}, \mathfrak{h}, \mathfrak{w}, \mathfrak{s} \rangle$ of \mathfrak{G} has a faithful permutation representation $P\mathfrak{E}$ of degree 1518 with stabilizer $\langle (\mathfrak{h}\mathfrak{s})^7, (\mathfrak{w}\mathfrak{h}\mathfrak{s})^3, (\mathfrak{s}\mathfrak{h}^3)^2, (\mathfrak{h}^2\mathfrak{w}\mathfrak{h}^2)^3 \rangle$.*
- (h) *The groups \mathfrak{E} and E are isomorphic.*
- (i) *$\mathfrak{z} = (\mathfrak{r}\mathfrak{h}\mathfrak{w})^8$ is a 2-central involution of \mathfrak{E} with centralizer $C_{\mathfrak{E}}(\mathfrak{z})$ of order $2^{21} \cdot 3^3 \cdot 5$ generated by the elements $\mathfrak{r}_1 = (\mathfrak{s}\mathfrak{h}^3)^3$, $\mathfrak{r}_2 = (\mathfrak{h}^2\mathfrak{w}\mathfrak{h}\mathfrak{s})^6$, $\mathfrak{r}_3 = (\mathfrak{s}\mathfrak{h}\mathfrak{w}\mathfrak{h}\mathfrak{s})^2$, and $\mathfrak{r}_4 = (\mathfrak{s}\mathfrak{w}\mathfrak{h}\mathfrak{s}\mathfrak{w})^6$ with respective orders 2, 4, 4, and 2.*
- (j) *$C_{\mathfrak{G}_1}(\mathfrak{z})$ has order $2^{18} \cdot 3^5 \cdot 5$. It is generated by $\mathfrak{f}_1 = \mathfrak{r}\mathfrak{h}\mathfrak{w}$, $\mathfrak{f}_2 = (\mathfrak{r}\mathfrak{h}\mathfrak{r}\mathfrak{w})^7$, $\mathfrak{f}_3 = (\mathfrak{r}\mathfrak{h}\mathfrak{w}\mathfrak{h}\mathfrak{w}^2)^7$, and $\mathfrak{v} = (\mathfrak{w}\mathfrak{q}\mathfrak{w}\mathfrak{h}\mathfrak{q}\mathfrak{h})^7$.*

- (k) The subgroup $\langle \mathbf{vf}_2\mathbf{v}, (\mathbf{f}_1\mathbf{f}_2\mathbf{f}_1\mathbf{vf}_1)^4, (\mathbf{f}_2\mathbf{f}_1\mathbf{vf}_1\mathbf{v})^4 \rangle$ of $C_{\mathfrak{E}_1}(\mathfrak{z})$ has index 512. Furthermore, it does not contain \mathfrak{z} .
- (l) \mathfrak{B} is the Fitting subgroup of \mathfrak{E} . It is also the unique maximal elementary abelian normal subgroup of the Sylow 2-subgroup $\mathfrak{S} = \langle \mathbf{u}_i \mid 1 \leq i \leq 5 \rangle$ of \mathfrak{E} contained in $C_{\mathfrak{E}}(\mathfrak{z})$.

Proof. In order to simplify the notation of the proof we replace the German letters by Roman letters. In particular, we let $ME = \langle x, y, w, s \rangle$ be the subgroup \mathfrak{E} .

(a) Let PA_1 be the faithful permutation representation of A_1 of degree 56320 constructed in Lemma 3.1(c). By Lemma 3.2 and Lemma 4.1(f) we know that $H_1 = \langle y, q \rangle = \langle x, y, q \rangle$. Now [11] asserts that $D = \langle x, y \rangle$ has odd index in H_1 and therefore in G_1 . Thus D contains a Sylow 2-subgroup of G_1 . The given Sylow 2-subgroup T_1 of G_1 and its generators t_i have been found by using MAGMA, the permutation representation PA_1 , and the program `GetShortGens(H_1, T_1)`.

(b) Applying the MAGMA command

```
Subgroups(T_1: A1:=Normal, IsElementaryAbelian := true)
```

we observed that T_1 has 44 elementary abelian normal subgroups. Exactly one of them is maximal and has order 2^{11} . It is denoted by B . Its given generators have been calculated by means of the first author's program `GetShortGens(T_1, B)`.

(c) Since $|A_1 : H_1| = 2$ a Sylow 2-subgroup of A_1 has order 2^{19} . Let $W_1 = N_{A_1}(T_1)$. Applying PA_1 and the MAGMA command

```
exists(r){x: x \in A_1 | T_1^x = T_1 and x^2 eq 1 and x \notin H_1}
```

we found the involution $s \in A_1$ of the statement satisfying $s \notin H_1$. It satisfies the equation $T_1^s = T_1$. Hence $B^s = B$ holds trivially by (b).

(d) By another application of PA_1 and MAGMA we verified that $N_{A_1}(B) = \langle x, y, s \rangle$. Using the faithful permutation representation PG_1 of degree 31671 with stabilizer H_1 of Kim's Theorem 6.3.1 of [14] one establishes that $N_1 = N_{G_1}(B) = \langle x, y, w \rangle$. Hence N_1 is a non split extension of \mathcal{M}_{23} by B , see Lemma 6.1.2 and Theorem 6.3.1 of [14].

(e) By Lemma 2.1(e) $E = \langle a, b, c, d, t, g, h, i, j, k \rangle$ has a the faithful permutation representation PE with stabilizer $U_3 = \langle g, h, i, (dg)^5, (dhjk)^3, (ijkj)^2, (dhjidg)^3 \rangle$. Lemma 8.2.2 of [12] states that its subgroup $E_{23} = \langle a, b, c, d, t, g, h, i, j \rangle$ has index $|E : E_{23}| = 24$. Applying the command `IsIsomorphic(N_1, E_{23})` MAGMA establishes an isomorphism $\rho : N_1 \rightarrow E_{23}$. The words of the images $\rho(x)$, $\rho(y)$ and $\rho(w)$ of the generators x , y and w of N_1 are constructed as follows. Let $C = C_{E_{23}}(\rho(x))$. Using PE and MAGMA one sees that $|C| = 2^{14}$. The six generators $x_1 = (ij)^3$, $x_2 = (gahigai)^2$, $x_3 = (aghijagh)^4$, $x_4 = (jhighajji)^4$, $x_5 = (aighjigai)^4$ of C were obtained computationally by means of the program `GetShortGens(E_{23}, C)`. Using the program `LookupWord(C, \rho(x))` MAGMA returned $\rho(x) = (x_1x_2x_4x_5x_4x_2x_5)^3$. The expressions for $\rho(y)$ and $\rho(w)$ are obtained similarly.

(f) By Kim's Theorem 6.3.1 of [14] we know that $z_1 = (xy^2)^7$ is a 2-central involution of G_1 . Clearly $u = (\rho(x)\rho(y)^2)^7$ is an involution of E_{23} . Applying MAGMA and PE the reader can verify that $C_u = C_{E_{23}}(u)$ has order $2^{19} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$. Similarly one observes that $D_1 = N_{A_1}(B) = \langle x, y, s \rangle$ has the same order. Using PA_1 , PE and the command `IsIsomorphic(D_1, C_u)` MAGMA establishes an isomorphism $\mu :$

$D_1 \rightarrow C_u$. Applying the MAGMA command `IsConjugate(C_u, \mu(y), \rho(y))` we found an element $c_1 \in C_u$ such that $\mu(y)^{c_1} = \rho(y)$. Using the command

`exists(c){q: q in C_{C_u}(\rho(y)) | (\mu(x)^{c_1})^q = \rho(x)}`

one gets an element $c_2 \in C_u$ such that $\mu(x)^{c_1 c_2} = \rho(x)$. Hence $\mu' : D_1 \rightarrow C_u$ defined by $\mu'(d) = \mu(d)^{c_1 c_2}$, $d \in D_1$, is an isomorphism between D_1 and C_u such that $\mu'(x) = \rho(x)$ and $\mu'(y) = \rho(y)$. It has been checked that $C_E(\langle \rho(y), \rho(x) \rangle) = \langle u \rangle$. Furthermore, $\mu'(s)$ has a centralizer $C_{M_{23}}(\mu'(s))$ of order 2^{17} which is generated by the three elements m_1 , m_2 and m_3 of M_{23} given in the statement. Another application of the Lookup command yields the word $\mu'(s) = (m_1^4 m_2 m_1 m_2)^2$. Hence the map $\mu' : D_1 \rightarrow C_E(u)$ satisfies all conditions of (f).

(g) Using (e), (f), PE and MAGMA it has been verified that $E = \langle \rho(x), \rho(y), \rho(w), \mu(s) \rangle$. As $U_3 = \langle g, h, i, (dg)^5, (dhjk)^3, (ijkj)^2, (dhjidg)^3 \rangle$ is a stabilizer of PE we apply the program `GetShortGens(E, U_3)` w.r.t. the given generators of E . MAGMA returns $U_3 = \langle (\rho(y)\mu(s))^7, (\rho(w)\rho(y)\mu(s)\rho(y))^3, (\mu(s)\rho(y)^3)^2, (\rho(y)^2\rho(w)\rho(y)^2)^3 \rangle$.

Thus $MU = \langle (ys)^7, (wysy)^3, (sy^3)^2, (y^2wy^2)^3 \rangle$ is a subgroup of $ME = \langle x, y, w, s \rangle$ which is isomorphic to U_3 . Let V be the 8671-dimensional vector space over $F = GF(13)$. Using the Meataxe Algorithm implemented in MAGMA we see that the restriction V_{MU} of V to the subgroup MU has a 7-dimensional FMU -submodule W which has a complement of dimension 8664. Applying now the algorithm described in Theorem 6.2.1 of [12] we obtain a faithful permutation representation PME of the matrix group ME of degree 1518 with stabilizer MU .

(h) Using PE , PME and the isomorphism test `IsIsomorphic(PE, PME)` MAGMA established that $ME \cong E$.

(i) By (d) and Table 6.5.1 of [14] we know that $z = (xyw)^8$ is an involution of $N_1 = N_{G_1}(B) \cong E_{23}$ with centralizer $C_{N_1}(z)$ of order $2^{18} \cdot 3^2 \cdot 5$. Therefore we calculate $C_E(z)$ by means of PE and MAGMA. It follows that $|C_E(z)| = 2^{21} \cdot 3^3 \cdot 5$. Hence E has a Sylow 2-subgroup S_3 of order 2^{21} with center $Z(S_3) = \langle z \rangle$ by Table A.1. The given generators r_i of $C_z = C_E(z)$ have been determined by means of MAGMA and the program `GetShortGens(E, C_z)`.

(j) Table 6.5.6 of [14] implies that $|C_{G_1}(z)| = 2^{18} \cdot 3^5 \cdot 5$ because $z = (xyw)^8 \in G_1 = \langle x, y, w, q \rangle$. Using MAGMA and the faithful permutation representation PG_1 of G_1 we found the involution $v = (wqwyqq)^7$ such that $(z, v) = 1$, and $C_{G_1}(z) = \langle f_1, f_2, f_3, v \rangle$ for the elements $f_i \in G_1$ given in the statement.

(k) All assertions of the statement are easily checked by means of MAGMA and the faithful permutation representation PG_1 of G_1 .

(l) By (b) and (c) the elementary abelian subgroup B is normal in ME . Hence it is the Fitting subgroup ME by (h) and Lemma 2.1. Using the faithful permutation representation of ME given in (g) the remaining assertions can be verified by means of MAGMA. \square

The following presentation of the 3-transposition Fischer group $P = \text{Fi}_{24}$ is taken from [15], its p. 124. It is due to J. Hall and L. S. Soicher [7].

Lemma 6.2. *Let $\mathfrak{G} = \langle \mathfrak{h}, \mathfrak{q}, \mathfrak{t}, \mathfrak{w} \rangle$ be the subgroup of $\text{GL}_{8671}(13)$ constructed in Proposition 5.2. Let $\mathfrak{H}_1 = \langle \mathfrak{h}, \mathfrak{q} \rangle$, $\mathfrak{G}_1 = \langle \mathfrak{H}_1, \mathfrak{w} \rangle$ and $\mathfrak{A}_1 = \langle \mathfrak{H}_1, \mathfrak{t} \rangle$. Let $\mathfrak{s} = (\mathfrak{h}^5 \mathfrak{t})^7$ and $\mathfrak{x} = [(\mathfrak{h} \mathfrak{q}^2 \mathfrak{h} \mathfrak{q} \mathfrak{h} \mathfrak{q}^2)^{11} (\mathfrak{q}^2 \mathfrak{h}^2 \mathfrak{q} \mathfrak{h} \mathfrak{q} \mathfrak{h})^{11} (\mathfrak{q} \mathfrak{h}^2 \mathfrak{q} \mathfrak{h} \mathfrak{q} \mathfrak{h} \mathfrak{q} \mathfrak{h} \mathfrak{q} \mathfrak{h})^4]^{12}$. Let $\mathfrak{E} = \langle \mathfrak{x}, \mathfrak{h}, \mathfrak{w}, \mathfrak{s} \rangle$*

Let $P = \langle a, b, c, d, e, f, g, h, i, j, k, l \rangle$ be the finitely generated group with the following set $\mathcal{R}(P)$ of defining relations:

$$\begin{aligned} l^2 &= k^2 = a^2 = b^2 = c^2 = d^2 = e^2 = f^2 = g^2 = j^2 = h^2 = i^2 = 1, \\ (lk)^3 &= (ka)^3 = (ab)^3 = (bc)^3 = (cd)^3 = (de)^3 = (ef)^3 = (fg)^3 = (gj)^3 = 1, \\ (la)^2 &= (lb)^2 = (lc)^2 = (ld)^2 = (le)^2 = (lf)^2 = (lg)^2 = (lj)^2 = (lh)^2 = (li)^2 = 1, \\ (kb)^2 &= (kc)^2 = (kd)^2 = (ke)^2 = (kf)^2 = (kg)^2 = (kj)^2 = (kh)^2 = (ki)^2 = 1, \\ (ac)^2 &= (ad)^2 = (ae)^2 = (af)^2 = (ag)^2 = (aj)^2 = (ah)^2 = (ai)^2 = 1, \\ (bd)^2 &= (be)^2 = (bf)^2 = (bg)^2 = (bj)^2 = (bh)^2 = (bi)^2 = 1, \\ (ce)^2 &= (cf)^2 = (cg)^2 = (cj)^2 = (ch)^2 = (ci)^2 = (df)^2 = (dg)^2 = (dj)^2 = 1, \\ (eg)^2 &= (ej)^2 = (eh)^2 = (ei)^2 = (fj)^2 = (fh)^2 = (fi)^2 = (gh)^2 = (gi)^2 = 1, \\ (jh)^2 &= (ji)^2 = (dh)^3 = (hi)^3 = (di)^2 = 1, \\ l &= (abcdefh)^9, (dcbakldefgjdhi)^{17} = 1. \end{aligned}$$

Then the following statements hold:

- (a) P has a faithful permutation representation PP of degree 306936 with stabilizer $M = \langle a, b, c, d, e, f, g, h, i, j, l \rangle$.
- (b) The commutator subgroup $G = P'$ is a finite simple group of order $2^{21} \cdot 3^{16} \cdot 5^2 \cdot 7^3 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29$.
- (c) $G = \langle b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, j_1, k_1 \rangle$, where $b_1 = ab$, $c_1 = ac$, $d_1 = ad$, $e_1 = ae$, $f_1 = af$, $g_1 = ag$, $h_1 = ah$, $i_1 = ai$, $j_1 = aj$, and $k_1 = ak$.
Furthermore, G has a faithful permutation representation PG of degree 306936 with stabilizer $M_1 = \langle b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, j_1 \rangle$ and M_1 is a simple group of order $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$.
- (d) The centralizer $C_1 = C_G(c_1)$ of the involution c_1 of G has order $2^{19} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$, and $C_1 = \langle e_1, g_1, i_1, j_1, m_1, n_1 \rangle$, where $m_1 = (c_1 d_1 e_1 h_1)^3$ and $n_1 = (b_1 c_1 d_1 k_1 f_1)^3$ are involutions.

Furthermore, C_1 has a faithful permutation representation PC_1 of degree 56320 with stabilizer $\langle i_1, j_1, (e_1 g_1 n_1)^2, n_1 m_1 n_1 \rangle$.

- (e) There is an isomorphism ϕ between $\mathfrak{A}_1 = \langle \mathfrak{y}, \mathfrak{q}, \mathfrak{t} \rangle$ and the finitely presented group $A_1 = \langle a, b, c, d, e, f, g, h, i, z, t \rangle$ constructed in Lemma 3.1(b) such that

$$\begin{aligned} y &= \phi(\mathfrak{y}) = (S_1 S_2 S_4 S_1 S_4 S_2 S_4)^5 (T_1 T_3^2 T_1 T_3 T_1 T_2 T_1 T_3)^{20}, \\ q &= \phi(\mathfrak{q}) = [(p \cdot o \cdot j \cdot o \cdot k)^4 \cdot (j \cdot k \cdot o \cdot k^2 p)^4]^4, \\ t &= \phi(\mathfrak{t}), \quad \text{where} \\ S_1 &= (b \cdot a \cdot c \cdot b) \cdot (d \cdot e \cdot h \cdot d) \cdot (b \cdot a \cdot c \cdot b), \\ S_2 &= [(b \cdot a \cdot c \cdot b) \cdot (d \cdot e \cdot h \cdot d) \cdot (f \cdot e \cdot g \cdot f)]^2, \\ S_3 &= [(b \cdot a \cdot c \cdot b) \cdot (d \cdot e \cdot h \cdot d) \cdot (c \cdot d \cdot e \cdot h \cdot i)^4 \cdot (b \cdot a \cdot c \cdot b)]^2, \\ S_4 &= [(b \cdot a \cdot c \cdot b) \cdot (c \cdot d \cdot e \cdot h \cdot i)^4 \cdot (d \cdot e \cdot h \cdot d) \cdot (f \cdot e \cdot g \cdot f)]^4, \\ T_1 &= S_2 S_4 S_2^2, \quad T_2 = (S_4 S_2 S_1 S_3 S_4)^2, \quad T_3 = (S_4 S_2 S_4 S_2 S_1 S_3)^2, \\ j &= (c \cdot d \cdot e \cdot h)^3, \\ k &= (c \cdot d \cdot e \cdot f \cdot g)^2, \\ l &= (a \cdot b \cdot c \cdot d \cdot e \cdot h)^5, \\ o &= (l \cdot b \cdot k \cdot j \cdot b \cdot i)^6, \\ p &= (j \cdot k \cdot j \cdot l \cdot b \cdot i \cdot j \cdot i)^5. \end{aligned}$$

- (f) There is an isomorphism

$$\rho : A_1 = \langle a, b, c, d, e, f, g, h, i, z, t \rangle \rightarrow C_1 = \langle e_1, g_1, i_1, j_1, m_1, n_1 \rangle$$

such that $\rho(t) = (m_1 n_1)^3$ and:

$$\begin{aligned}
\rho(a) &= [(i_1 m_1 n_1)^6 \cdot (j_1 n_1 m_1 n_1) \cdot (g_1 n_1 e_1 g_1 n_1)^2 \cdot (i_1 m_1 n_1)^6 \\
&\quad \cdot (e_1 g_1 n_1 m_1 n_1 e_1)^2 \cdot (g_1 n_1 e_1 g_1 n_1)^2]^7, \\
\rho(b) &= [(i_1 n_1 g_1 j_1 m_1)^{12} \cdot (e_1 g_1 j_1 m_1 i_1 n_1)^4 \cdot (i_1 n_1 g_1 j_1 m_1)^{12} \cdot (e_1 g_1 j_1 m_1 i_1 n_1)^{12} \\
&\quad \cdot (i_1 n_1 g_1 j_1 m_1)^{12} \cdot (e_1 g_1 j_1 m_1 i_1 n_1)^8]^{11}, \\
\rho(c) &= [(j_1 n_1 m_1 n_1) \cdot (e_1 n_1 g_1 n_1 e_1) \cdot (e_1 g_1 i_1 j_1 m_1 n_1)^{18}]^{11}, \\
\rho(d) &= [(j_1 m_1) \cdot (n_1 g_1 n_1) \cdot (e_1 i_1 j_1 n_1 g_1 m_1)^{18} \cdot (j_1 m_1) \cdot (e_1 i_1 j_1 n_1 g_1 m_1)^{18} \\
&\quad \cdot (j_1 m_1) \cdot (e_1 i_1 j_1 n_1 g_1 m_1)^{18}]^{11}, \\
\rho(e) &= (bc)^3 \cdot c^t, \quad \rho(f) = (bc)^3 \cdot b^t, \quad \rho(g) = a^t, \\
\rho(h) &= [e_1 \cdot i_1 \cdot (m_1 i_1 n_1 m_1 n_1)^3 \cdot (e_1 g_1 n_1 g_1 j_1 m_1)^6 \cdot i_1 \cdot (e_1 g_1 n_1 g_1 j_1 m_1)^6]^{11}, \\
\rho(i) &= [(m_1 i_1 m_1) \cdot (e_1 g_1 j_1 n_1 m_1)^{12}]^9.
\end{aligned}$$

In particular, $\psi = \rho \circ \phi : \mathfrak{A}_1 \rightarrow C_1$ is an isomorphism such that

$y_1 = \psi(\mathfrak{y}) = \rho(y)$, $q_1 = \psi(\mathfrak{q}) = \rho(q)$, and $t_1 = \psi(\mathfrak{t}) = \rho(t)$ generate C_1 .

(g) In C_1 let $x_1 = \psi(\mathfrak{x})$ and $s_1 = \psi(\mathfrak{s}) = (y_1^5 t_1)^7$.

Then $S_1 = \langle u_1, u_2, u_3, u_4, s_1 \rangle$ is a Sylow 2-subgroup of C_1 , where

$$\begin{aligned}
u_1 &= (x_1 y_1 x_1)^7, \quad u_2 = (x_1 y_1 x_1 y_1 x_1 y_1 x_1)^4, \quad u_3 = (x_1 y_1 x_1 y_1^2 x_1 y_1^2 x_1 y_1 x_1 y_1)^2, \\
u_4 &= (x_1 y_1 x_1 y_1^5 x_1 y_1^4)^2.
\end{aligned}$$

Moreover, S_1 has a unique maximal elementary abelian normal subgroup B_1 . It is generated by the eleven involutions $u_1, u_2^2, (u_1 u_2)^2, (u_1 u_3)^2, (u_1 u_4)^2, (u_2 u_4)^4, (u_1 u_2 u_3)^4, (u_1 u_2 u_4)^4, (u_1 u_3 u_4)^4, (u_2^2 u_3)^2, (u_1 u_2 u_4 u_2)^2$.

(h) $N_1 = N_G(B_1) = \langle x_1, y_1, s_1, o_1 \rangle$, where $o_1 = [d_1 \cdot y_1 \cdot s_1 \cdot (y_1)^2 \cdot s_1]^{10}$ and $|N_1| = 2^{21} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 23$.

(i) There is an isomorphism $\mu : \mathfrak{E} \rightarrow N_1$ such that $\mu(\mathfrak{x}) = \psi(\mathfrak{x}) = x_1$, $\mu(\mathfrak{y}) = \psi(\mathfrak{y}) = y_1$, $\mu(\mathfrak{s}) = \psi(\mathfrak{t}) = s_1$ and

$$\mu(w) = w_1 = [(s_1 y_1 x_1 y_1)^2 \cdot (x_1 y_1 o_1 y_1 s_1 y_1^2)^5]^{10}.$$

(j) $G = \langle C_1, N_1 \rangle = \langle q_1, y_1, w_1, t_1 \rangle$.

(k) The map $\kappa : G \rightarrow \mathfrak{G}$ given by $\kappa(q_1) = \mathfrak{q}$, $\kappa(y_1) = \mathfrak{y}$, $\kappa(w_1) = \mathfrak{w}$ and $\kappa(s_1) = \mathfrak{s}$ is a group isomorphism.

In particular, \mathfrak{G} is a simple group of order $2^{21} \cdot 3^{16} \cdot 5^2 \cdot 7^3 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29$.

Proof. (a) This statement has been verified by running the Todd-Coxeter Algorithm `CosetAction(P,M)` built into MAGMA.

(b) Using (a) and MAGMA it has been checked that $G = P'$ is a simple group of the stated order.

(c) Using then the program `GetShortGens(P,G)` we found the given generators of G . Using the faithful permutation representation PP of (a) and MAGMA it has been checked that $M_1 = \langle b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, j_1 \rangle$ is a simple group of order $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$. It is the stabilizer of the faithful permutation representation PG of G of degree 306936 as has been checked by means of MAGMA and the command `CosetAction(G,M_1)`.

(d) The centralizer $C_1 = C_G(c_1)$ of the involution c_1 and $|C_1|$ have been determined by means of the permutation representation PG of G and MAGMA. Its given generators have been found by MAGMA using the program `GetShortGens(G,C_1)`. Applying the MAGMA command `DegreeReduction` we get a faithful permutation representation PC_1 of C_1 having degree 56320. Its stabilizer U_1 has been found

by means of the MAGMA command `BasicStabilizer(~,2)`. Its given generators were gotten by another application of `GetShortGens(C_1,U_1)`.

(e) This statement follows immediately from Lemma 3.1(b), Proposition 6.2.3 of [14], and Proposition 5.2.

(f) By Lemma 3.1(c) the finitely presented group $A_1 = \langle a, b, c, d, e, f, g, h, i, z, t \rangle$ has a faithful permutation representation PA_1 of degree 56320. Let PC_1 be the faithful permutation representation of C_1 constructed in (d). A successful application of the MAGMA command `IsIsomorphic(PA_1,PC_1)` provides an isomorphism $\eta : \mathfrak{A}_1 \rightarrow C_1$. Now $C_{C_1}(\eta(t))$ and $C_{C_1}((m_1 n_1)^3)$ have the same order. Using the MAGMA command `exists(w){x : x in C_1 | \eta(t)^x eq (m_1 n_1)^3}` we found an element $a \in C_1$ such that $\eta(t)^a = (m_1 n_1)^3$. Let α be the inner automorphism of C_1 induced by a . Then the map $\rho = \alpha \circ \eta$ is an isomorphism from A_1 onto C_1 such that $\rho(t) = (m_1 n_1)^3$. The centralizers of the images of $\rho(a), \rho(b), \dots, \rho(i)$ in C_1 all have the same order $2^{17} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$. Their generators and their words in them are obtained computationally using the programs `GetShortGens` and `LookupWord`, respectively, as follows:

- (1) $C_{C_1}(\rho(a))$ is generated by $a_1 = (i_1 m_1 n_1)^6$, $a_2 = j_1 n_1 m_1 n_1$, $a_3 = (g_1 n_1 e_1 g_1 n_1)^2$, $a_4 = (e_1 g_1 n_1 m_1 n_1 e_1)^2$, and $\rho(a) = (a_1 a_2 a_3 a_1 a_4 a_3)^7$.
- (2) $C_{C_1}(\rho(b))$ is generated by $b_1 = (i_1 n_1 g_1 j_1 m_1)^{12}$, $b_2 = (e_1 g_1 j_1 m_1 i_1 n_1)^4$, and $\rho(b) = (b_1 b_2 b_1 b_2^3 b_1 b_2^2)^{11}$.
- (3) $C_{C_1}(\rho(c))$ is generated by $c_1 = j_1 n_1 m_1 n_1$, $c_2 = e_1 n_1 g_1 n_1 e_1$, $c_3 = (e_1 g_1 i_1 j_1 m_1 n_1)^{18}$, and $\rho(c) = (c_1 c_2 c_3)^{11}$.
- (4) $C_{C_1}(\rho(d))$ is generated by $d_1 = j_1 m_1$, $d_2 = n_1 g_1 n_1$, $d_3 = (e_1 i_1 j_1 n_1 g_1 m_1)^{18}$, and $\rho(d) = (d_1 d_2 d_3 d_1 d_3 d_1 d_3)^{11}$.
- (5) $C_{C_1}(\rho(h))$ is generated by $h_1 = e_1$, $h_2 = i_1$, $h_3 = (m_1 i_1 n_1 m_1 n_1)^3$, $h_4 = (e_1 g_1 n_1 g_1 j_1 m_1)^6$, and $\rho(h) = (h_1 h_2 h_3 h_4 h_2 h_4)^{11}$.
- (6) $C_{C_1}(\rho(i))$ is generated by $i_1 = m_1 i_1 m_1$, $i_2 = j_1 n_1 m_1 n_1$, $i_3 = (e_1 g_1 j_1 n_1 m_1)^6$, and $\rho(i) = (i_1 i_2^3)^9$.

The given words for the images $\rho(e)$, $\rho(f)$ and $\rho(g)$ can now be calculated from these images and the relations of Lemma 3.1(b).

Statement (e) implies that the composition of the isomorphisms $\phi : \mathfrak{A}_1 \rightarrow A_1$ and $\rho : A_1 \rightarrow C_1$ is an isomorphism $\psi : \mathfrak{A}_1 \rightarrow C_1$. Furthermore, the given images $\psi(q)$, $\psi(q)$ and $\psi(q)$ in C_1 of the 3 generators of \mathfrak{A}_1 are well defined.

(g) Since $\mathfrak{x} \in \mathfrak{A}_1$ its image

$$\psi(\mathfrak{x}) = [(y_1 q_1^2 y_1 q_1 y_1 q_1^2)^{11} (q_1^2 y_1^2 q_1 y_1 q_1 y_1)^{11} (q_1 y_1^2 q_1 y_1 q_1 y_1 q_1 y_1 q_1)^4]^{12} = x_1$$

is a well defined element of C_1 .

Let u_i be the generators of the Sylow 2-subgroup \mathfrak{T}_1 of the subgroup $\mathfrak{G}_1 = \langle q, \eta, \mathfrak{w} \rangle$ of \mathfrak{G} given in Proposition 6.1(a). Hence \mathfrak{T}_1 is a subgroup of \mathfrak{A}_1 because its generators u_i are words in \mathfrak{x} and η by Proposition 6.1. Therefore their images $u_i = \psi(u_i)$, $1 \leq i \leq 4$, are well defined. So is $s_1 = \psi(\mathfrak{s}) = (y_1^5 t_1)^7$. Hence $S_1 = \psi(\mathfrak{S}) = \langle u_1, u_2, u_3, u_4, s_1 \rangle$ is a Sylow 2-subgroup of C_1 by Proposition 6.1(c) and (f).

Let $B_1 \psi(\mathfrak{B})$ where \mathfrak{B} is the unique maximal elementary abelian normal subgroup of \mathfrak{S} defined in Proposition 6.1(b and (d)). Then B_1 is generated by the 11 involutions given in the statement.

(h) Let $N_1 = N_G(B_1)$. Using the faithful permutation representation PG of G and MAGMA the reader can easily check that $|N_1| = 2^{21} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 23$ and that N_1 is generated by x_1, y_1, s_1 and the element $o_1 \in G$ given in the statement.

(i) By Proposition 6.1(g) the subgroup \mathfrak{E} of \mathfrak{G} has a faithful permutation representation $P\mathfrak{E}$ of degree 1518 with stabilizer $\langle (\eta\mathfrak{s})^7, (\mathfrak{w}\eta\mathfrak{s}\eta)^3, (\mathfrak{s}\eta^3)^2, (\eta^2\mathfrak{w}\eta^2)^3 \rangle$. Let PN_1 be the reduction of PG to $N_1 = N_G(B_1)$. A successful application of the MAGMA command `IsIsomorphic(PE, PN_1)` provides an isomorphism $\tau : \mathfrak{E} \rightarrow N_1$. As in the proof of (f) we find an element $a \in N_1$ such that $(\tau(\eta))^a = y_1 = \psi(\eta)$. Using MAGMA again we verified that $C_{N_1}(y_1)$ has order 56. Searching through its elements we find an element $b \in C_{N_1}(y_1)$ such that $(\tau(\mathfrak{x}))^{ab} = \psi(\mathfrak{x}) = x_1$ and $(\tau(\mathfrak{s}))^{ab} = \psi(\mathfrak{s}) = s_1$. Let β denote the inner automorphism of N_1 induced by conjugation with ab . Then the map $\mu = \beta \circ \nu$ is an isomorphism from \mathfrak{E} onto $N_1 = N_G(B_1)$ such that $\mu(\mathfrak{x}) = x_1$, $\mu(\eta) = y_1$ and $\mu(\mathfrak{s}) = s_1$. Let $w_1 = \mu(\mathfrak{w})$. Another application of MAGMA and PN_1 yields that $N_1 = \langle x_1, y_1, s_1, w_1 \rangle$.

The word for w_1 in the generators of G is obtained as follows. Let $C_2 = C_{N_1}(w_1)$. Using the generators of N_1 given in (h), MAGMA, and the program `GetShortGens(N_1, C_2)` we see that $C_2 = \langle v_i \mid 1 \leq i \leq 3 \rangle$, where $v_1 = (s_1 y_1 x_1 y_1)^2$, $v_2 = (y_1 o_1 x_1 s_1 o_1)^4$, $v_3 = (x_1 y_1 o_1 y_1 s_1 y_1^2)^5$. Applying then the command `LookupWord(C_2, w_1)` MAGMA provides the solution $w_1 = (v_1 v_3)^7$.

(j) Using the faithful permutation representation PG of G and MAGMA the reader easily verifies that $G = \langle C_1, N_1 \rangle$. Hence (f) and (i) imply that $G = \langle q_1, y_1, t_1, x_1, s_1, w_1 \rangle = \langle q_1, y_1, t_1, w_1 \rangle$.

(k) In \mathfrak{G} let $\mathfrak{E}_{23} = \langle \mathfrak{x}, \eta, \mathfrak{w} \rangle$ and $\mathfrak{H}_1 = \langle \mathfrak{q}, \eta \rangle$. Then Proposition 5.2 and Kim's Theorem 6.3.1 of [14] imply that $\mathfrak{G}_1 = \langle \mathfrak{H}_1, \mathfrak{E}_{23} \rangle$ is a simple subgroup of \mathfrak{G} such that $\mathfrak{D}_1 = \mathfrak{E}_{23} \cap \mathfrak{H}_1 = \langle \mathfrak{x}, \eta \rangle$ and $\mathfrak{H}_1 = C_{\mathfrak{G}_1}(\mathfrak{z}_1)$, where $\mathfrak{z}_1 = (\mathfrak{x}\eta^2)^7$ is a 2-central involution of \mathfrak{G}_1 .

Let $E_{23} = \mu(\mathfrak{E}_{23})$, $H_1 = \psi(\mathfrak{H}_1)$ and $G_1 = \langle E_{23}, H_1 \rangle$ in G . As μ and ψ agree on \mathfrak{H}_1 by (j) we have $D_1 = \langle x_1, y_1 \rangle = E_{23} \cap H_1$. Using PG and MAGMA it has been checked that G_1 is a simple group of order $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$. Furthermore, $C_{G_1}(z_1) = H_1$, where $z_1 = (x_1 y_1^2)^7 = \psi(\mathfrak{z}_1)$ is a 2-central involution of G_1 . Thus Kim's Theorem 6.3.1 of [14] implies that $G_1 \cong \text{Fi}_{23}$. In particular, the map $\kappa_1 : G_1 \rightarrow \mathfrak{G}_1$ defined by $\kappa_1(q_1) = \mathfrak{q}$, $\kappa_1(y_1) = \eta$, and $\kappa_1(w_1) = \mathfrak{w}$ is an isomorphism.

By Proposition 5.2 and Lemma 3.2(e) the amalgam $\mathfrak{A}_1 \leftarrow \mathfrak{H}_1 \rightarrow \mathfrak{G}_1$ has Goldschmidt index 1. Therefore by Corollary 7.1.9 of [12], (f) and (i) imply that the free products $\mathfrak{A}_1 *_{\mathfrak{H}_1} \mathfrak{G}_1$ and $C_1 *_{H_1} G_1$ with respective amalgamated subgroups \mathfrak{H}_1 and H_1 are isomorphic. Thus (j) and Proposition 5.2 imply that the map $\kappa : G \rightarrow \mathfrak{G}$ defined by $\kappa(q_1) = \mathfrak{q}$, $\kappa(y_1) = \eta$, $\kappa(w_1) = \mathfrak{w}$ and $\kappa(t_1) = \mathfrak{t}$ is an irreducible 8671-dimensional representation of the group G over $GF(13)$. Hence $\kappa : G \rightarrow \mathfrak{G}$ is an isomorphism because κ is surjective and G is simple by (b). This completes the proof. \square

Theorem 6.3. *Let $\mathfrak{G} = \langle \mathfrak{q}, \eta, \mathfrak{t}, \mathfrak{w} \rangle$ be the subgroup of $\text{GL}_{8671}(13)$ constructed in Proposition 5.2. Then the following statements hold:*

- (a) $\mathfrak{G} = \langle \mathfrak{q}, \eta, \mathfrak{t}, \mathfrak{w} \rangle$ has a faithful permutation representation of degree 306936 with stabilizer $\mathfrak{G}_1 = \langle \mathfrak{q}, \eta, \mathfrak{w} \rangle$.
- (b) \mathfrak{G} is a finite simple group of order

$$|\mathfrak{G}| = 2^{21} \cdot 3^{16} \cdot 5^2 \cdot 7^3 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29.$$

- (c) \mathfrak{G} is isomorphic to the commutator subgroup P' of the finitely presented group $P = \langle a, b, c, d, e, f, g, h, i, j, k, l \rangle$ with set of defining relations $\mathcal{R}(P)$ stated in Lemma 6.2.
- (d) The character table of \mathfrak{G} is equivalent to that of Fi'_{24} in the Atlas [4], its pp. 200–202.

Proof. The first three statements hold by Lemma 6.2(j) and (k).

(d) Let $G = P'$ be the commutator subgroup of the finitely presented group P . Then $\mathfrak{G} \cong G$ by (c). The character table of G has been calculated by means of MAGMA and the faithful permutation representation PG of G with stabilizer $M = \langle b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, j_1 \rangle$ given in Lemma 6.2(c). \square

7. PRESENTATION OF 2-CENTRAL INVOLUTION CENTRALIZER

In this section we determine generators and a presentation of the centralizer $\mathfrak{H} = C_{\mathfrak{G}}(\mathfrak{z})$ of a 2-central involution \mathfrak{z} of the simple subgroup $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{h}, \mathfrak{w}, \mathfrak{t} \rangle$ of $\text{GL}_{8671}(13)$. Thus we are able to show that \mathfrak{G} satisfies all conditions of Algorithm 2.5 of [13]. In particular, $\mathfrak{G} = \langle \mathfrak{H}, \mathfrak{E} \rangle$ and $\mathfrak{D} = N_{\mathfrak{H}}(\mathfrak{B}) = C_{\mathfrak{E}}(\mathfrak{z})$ where \mathfrak{B} is the unique maximal elementary abelian normal subgroup of a well defined Sylow 2-subgroup \mathfrak{S} of \mathfrak{G} and $\mathfrak{E} = N_{\mathfrak{G}}(\mathfrak{B})$. However, the free product $P = H *_D E$ of the groups $H = \mathfrak{H}$ and $E = \mathfrak{E}$ with amalgamated subgroup $D = \mathfrak{D}$ has 939, 080, 024, 064 irreducible representations of dimension 8671 over $GF(13)$. Therefore we have not tried to find one satisfying the Sylow 2-subgroup test of Step 5 c) of Algorithm 7.4.8 of [12].

Proposition 7.1. *Let $\mathfrak{G} = \langle \mathfrak{q}, \mathfrak{h}, \mathfrak{t}, \mathfrak{w} \rangle$ be the subgroup of $Y = \text{GL}_{8671}(13)$ constructed in Proposition 5.2. Let $\mathfrak{x} = [(\mathfrak{h}\mathfrak{q}^2\mathfrak{h}\mathfrak{q}\mathfrak{h}\mathfrak{q}^2)^{11}(\mathfrak{q}^2\mathfrak{h}^2\mathfrak{q}\mathfrak{h}\mathfrak{q}\mathfrak{h})^{11}(\mathfrak{q}\mathfrak{h}^2\mathfrak{q}\mathfrak{h}\mathfrak{q}\mathfrak{h}\mathfrak{q}\mathfrak{h})^4]^{12}$ and $\mathfrak{s} = (\mathfrak{h}\mathfrak{t})^7$. Let $\mathfrak{r}_1 = (\mathfrak{s}\mathfrak{h}^3)^3$, $\mathfrak{r}_2 = (\mathfrak{h}^2\mathfrak{w}\mathfrak{h}\mathfrak{s})^6$, $\mathfrak{r}_3 = (\mathfrak{s}\mathfrak{h}\mathfrak{w}\mathfrak{h}\mathfrak{s})^2$, $\mathfrak{r}_4 = (\mathfrak{s}\mathfrak{w}\mathfrak{h}\mathfrak{s}\mathfrak{w})^6$, and $\mathfrak{v} = (\mathfrak{w}\mathfrak{q}\mathfrak{w}\mathfrak{h}\mathfrak{q}\mathfrak{h})^7$. Let $\mathfrak{H} = \langle \mathfrak{r}_1, \mathfrak{r}_2, \mathfrak{r}_3, \mathfrak{r}_4, \mathfrak{v} \rangle$.*

Then the following statements hold:

- (a) *The element $\mathfrak{z} = (\mathfrak{r}\mathfrak{h}\mathfrak{w})^8$ is a 2-central involution $\mathfrak{z} = (\mathfrak{r}\mathfrak{h}\mathfrak{w})^8$ of \mathfrak{G} such that $C_{\mathfrak{G}}(\mathfrak{z}) = \mathfrak{H}$ has order $2^{21} \cdot 3^7 \cdot 5 \cdot 7$.*
- (b) *\mathfrak{H} has a faithful permutation representation $P\mathfrak{H}$ of degree 258048 with stabilizer $\mathfrak{U}_1 = \langle \mathfrak{v}\mathfrak{f}_2\mathfrak{v}, (\mathfrak{f}_1\mathfrak{f}_2\mathfrak{f}_1\mathfrak{v}\mathfrak{f}_1)^4, (\mathfrak{f}_2\mathfrak{f}_1\mathfrak{v}\mathfrak{f}_1\mathfrak{v})^4 \rangle$, where $\mathfrak{f}_1 = \mathfrak{r}\mathfrak{h}\mathfrak{w}$, $\mathfrak{f}_2 = (\mathfrak{r}\mathfrak{h}\mathfrak{r}\mathfrak{w})^7$, $\mathfrak{f}_3 = (\mathfrak{r}\mathfrak{w}\mathfrak{h}\mathfrak{w})^2$.*
- (c) *$\mathfrak{S} = \langle \mathfrak{s}, \mathfrak{u}_i \mid 1 \leq i \leq 5 \rangle$ is a Sylow 2-subgroup of \mathfrak{E} contained in \mathfrak{H} with center $Z(\mathfrak{H}) = \langle \mathfrak{z} \rangle$, where $\mathfrak{u}_1 = (\mathfrak{r}\mathfrak{h}\mathfrak{r})^7$, $\mathfrak{u}_2 = (\mathfrak{r}\mathfrak{h}\mathfrak{r}\mathfrak{h}\mathfrak{r}\mathfrak{h})^4$, $\mathfrak{u}_3 = (\mathfrak{r}\mathfrak{h}\mathfrak{r}\mathfrak{h}^2\mathfrak{r}\mathfrak{h}^2\mathfrak{r}\mathfrak{h}\mathfrak{r})^2$, $\mathfrak{u}_4 = (\mathfrak{r}\mathfrak{h}\mathfrak{r}\mathfrak{h}^5\mathfrak{r}\mathfrak{h}^4)^2$ and $\mathfrak{u}_5 = (\mathfrak{h}\mathfrak{s}\mathfrak{w}\mathfrak{s}\mathfrak{w}\mathfrak{h}\mathfrak{w})^7$.*
- (d) *$\mathfrak{B} = \langle \mathfrak{b}_i \mid 1 \leq i \leq 11 \rangle$ is the unique maximal elementary abelian normal subgroup of \mathfrak{S} where $\mathfrak{b}_1 = \mathfrak{u}_1$, $\mathfrak{b}_2 = \mathfrak{u}_2^2$, $\mathfrak{b}_3 = (\mathfrak{u}_1\mathfrak{u}_2)^2$, $\mathfrak{b}_4 = (\mathfrak{u}_1\mathfrak{u}_3)^2$, $\mathfrak{b}_5 = (\mathfrak{u}_1\mathfrak{u}_4)^2$, $\mathfrak{b}_6 = (\mathfrak{u}_2\mathfrak{u}_4)^4$, $\mathfrak{b}_7 = (\mathfrak{u}_1\mathfrak{u}_2\mathfrak{u}_3)^4$, $\mathfrak{b}_8 = (\mathfrak{u}_1\mathfrak{u}_2\mathfrak{u}_4)^4$, $\mathfrak{b}_9 = (\mathfrak{u}_1\mathfrak{u}_3\mathfrak{u}_4)^4$, $\mathfrak{b}_{10} = (\mathfrak{u}_2^2\mathfrak{u}_3)^2$, $\mathfrak{b}_{11} = (\mathfrak{u}_1\mathfrak{u}_2\mathfrak{u}_4\mathfrak{u}_2)^2$.*
- (e) *$N_{\mathfrak{G}}(\mathfrak{B}) = \langle \mathfrak{r}, \mathfrak{h}, \mathfrak{w}, \mathfrak{s} \rangle = \mathfrak{E}$.*
- (f) *$\mathfrak{D} = \langle \mathfrak{r}_i \mid 1 \leq i \leq 4 \rangle = N_{\mathfrak{H}}(\mathfrak{B}) = C_{\mathfrak{E}}(\mathfrak{z})$.*
- (g) *The Fitting subgroup \mathfrak{D} of \mathfrak{H} is extra-special of order 2^{13} and center $Z(\mathfrak{D}) = \langle \mathfrak{z} \rangle$. It is generated by the twelve involutions*

$$\begin{aligned} \mathfrak{p}_1 &= (\mathfrak{r}_2)^2, & \mathfrak{p}_2 &= (\mathfrak{r}_1\mathfrak{r}_2)^4, & \mathfrak{p}_3 &= (\mathfrak{r}_3\mathfrak{r}_4)^3, & \mathfrak{p}_4 &= (\mathfrak{r}_1\mathfrak{r}_2\mathfrak{r}_1)^2, \\ \mathfrak{p}_5 &= (\mathfrak{r}_1\mathfrak{r}_3\mathfrak{r}_4)^6, & \mathfrak{p}_6 &= (\mathfrak{r}_2^2\mathfrak{r}_4)^2, & \mathfrak{p}_7 &= (\mathfrak{r}_3\mathfrak{r}_4\mathfrak{r}_1)^6, & \mathfrak{p}_8 &= (\mathfrak{r}_4\mathfrak{r}_1\mathfrak{r}_3)^6, \\ \mathfrak{p}_9 &= (\mathfrak{r}_1\mathfrak{r}_2\mathfrak{r}_3^2)^4, & \mathfrak{p}_{10} &= (\mathfrak{r}_1\mathfrak{r}_2\mathfrak{r}_3\mathfrak{r}_4)^4, & \mathfrak{p}_{11} &= (\mathfrak{r}_1\mathfrak{r}_2\mathfrak{r}_1\mathfrak{r}_2\mathfrak{r}_4)^5, & \mathfrak{p}_{12} &= (\mathfrak{r}_1\mathfrak{r}_2\mathfrak{r}_3^2\mathfrak{r}_4)^4, \\ \text{and } \mathfrak{z} &= (\mathfrak{p}_1\mathfrak{p}_5)^2. \end{aligned}$$

- (h) $\mathfrak{D}/Z(\mathfrak{D})$ has a complement $\mathfrak{K} \cong 3U_4(3) : 2$ in $\mathfrak{H}/Z(\mathfrak{D})$.
- (i) $\mathfrak{H} = \langle \mathbf{a}, \mathbf{b}, \mathbf{p}_i \mid 1 \leq i \leq 12 \rangle = \langle \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{f}, \mathbf{z}, \mathbf{p}_i \mid 1 \leq i \leq 12 \rangle$, where
 $\mathbf{a} = \mathbf{r}_1 \mathbf{r}_3^3 \mathbf{v} \mathbf{r}_3$, $\mathbf{b} = [\mathbf{r}_1 \mathbf{r}_3 \mathbf{r}_1 \mathbf{r}_3^2 \mathbf{v}]^6 [\mathbf{r}_1 \mathbf{r}_3 \mathbf{r}_1 \mathbf{r}_3 \mathbf{v} \mathbf{r}_3 \mathbf{v}]^{12}$, $\mathbf{c} = (\mathbf{a} \mathbf{b})^2$, $\mathbf{d} = (\mathbf{b} \mathbf{c})^7$ and
 $\mathbf{f} = [(\mathbf{b} \mathbf{a}^3)^5 \cdot (\mathbf{a}^4 \mathbf{b}^2 \mathbf{a})^9 \cdot (\mathbf{a}^2 \mathbf{b}^3 \mathbf{a}^4 \mathbf{b})^3 \cdot (\mathbf{b} \mathbf{a}^3)^5]^3$.
- (j) \mathfrak{H} is isomorphic to the finitely presented group $H = \langle b, c, d, f, z, p_i \mid 1 \leq i \leq 12 \rangle$ having the following set $\mathcal{R}(H)$ of defining relations:

$$\begin{aligned}
& b^6 = c^9 = d^3 = f^2 = z^2 = 1, \quad p_i^2 = 1 \quad \text{for } 1 \leq i \leq 12, \\
& (z, b) = (z, c) = (z, d) = (z, f) = 1, \quad (z, p_i) = 1 \quad \text{for } 1 \leq i \leq 12, \\
& (b, d) = (c, d) = 1, \quad b^f = b^5, \quad c^f = d(b^3 c b^2 c^6 b c b c), \quad d^f = d^2, \\
& (c^{-1} b^{-1})^7 d = 1, \quad (c^{-1} b)^9 = z, \quad b^{-1} c^{-1} b^{-3} c^{-1} b^3 c^{-1} b^3 c^{-1} b^{-2} d = 1, \\
& (b c^{-2} b)^4 d^2 = (b c^{-3} b c^2 b)^2 d = b c^3 b^{-2} c b^3 c^{-1} b c^{-1} b c b^{-1} c b d^2 = 1, \\
& b^{-1} c^{-3} b^{-1} c^{-1} b c^{-1} b^2 c b^{-1} c b c^3 b^{-1} d = b^{-1} c^3 b c^{-1} b c^{-1} b^2 c b^{-1} c b^{-1} c^{-3} b^{-1} d = 1, \\
& c^{-2} b^{-2} c^{-1} b c^{-1} b^{-3} c^{-1} b c b^{-2} c b c^{-1} b d^2 = z, \\
& b^{-2} c^{-1} b c^{-1} b^{-2} c^{-1} b^{-2} c b^{-1} c b^{-1} c b^2 c^2 d^2 = c b^{-1} c^4 b c^{-1} b^{-2} c^{-1} b c b^{-1} c^2 b c b d^2 = 1, \\
& c^{-2} b^{-1} c^2 b^{-1} c^{-2} b^{-2} c^2 b c b^2 c^{-2} b^{-1} d = b^{-1} c^{-3} b c^2 b c^{-1} b^{-1} c b^{-1} c b^{-1} c^2 b^{-1} c^{-2} = z, \\
& c^2 b^{-1} c^{-1} b c^{-1} b^{-1} c^{-1} b^3 c^2 b^{-1} c^{-1} b c^{-1} b^2 c = z, \\
& b c^{-2} b c^{-2} b c^{-1} b^3 c^{-1} b^2 c^{-1} b^{-1} c b^2 c^{-1} = b^{-1} c^{-2} b^{-1} c^2 b c^{-1} b^3 c b^{-1} c^{-2} b c^2 b^{-2} = 1, \\
& b^{-2} c^2 b^{-2} c^{-2} b^3 c^{-1} b^{-1} c^{-2} b c^{-1} b^{-1} c^{-2} d = c b c b^{-1} c b^{-1} c^{-2} b^3 c^2 b^{-1} c b^{-1} c^2 b c d = 1, \\
& (p_1, p_2) = (p_1, p_3) = (p_1, p_4) = 1, \quad (p_1, p_5) = z, \\
& (p_1, p_6) = (p_1, p_7) = (p_1, p_8) = (p_1, p_9) = (p_1, p_{10}) = 1, \\
& (p_1, p_{11}) = (p_1, p_{12}) = (p_2, p_3) = z, \\
& (p_2, p_4) = (p_2, p_5) = 1, \quad (p_2, p_6) = z, \quad (p_2, p_7) = 1, \quad (p_2, p_8) = z, \\
& (p_2, p_9) = 1, \quad (p_2, p_{10}) = z, \quad (p_2, p_{11}) = 1, \quad (p_2, p_{12}) = z, \\
& (p_3, p_4) = 1, \quad (p_3, p_5) = z, \quad (p_3, p_6) = 1, \quad (p_3, p_7) = (p_3, p_8) = (p_3, p_9) = z, \\
& (p_3, p_{10}) = 1, \quad (p_3, p_{11}) = z, \quad (p_3, p_{12}) = 1, \\
& (p_4, p_5) = (p_4, p_6) = 1, \quad (p_4, p_7) = (p_4, p_8) = z, \\
& (p_4, p_9) = (p_4, p_{10}) = 1, \quad (p_4, p_{11}) = (p_4, p_{12}) = z, \\
& (p_5, p_6) = z, \quad (p_5, p_7) = (p_5, p_8) = 1, \\
& (p_5, p_9) = (p_5, p_{10}) = 1, \quad (p_5, p_{11}) = z, \quad (p_5, p_{12}) = 1, \\
& (p_6, p_7) = 1, \quad (p_6, p_8) = (p_6, p_9) = z, \quad (p_6, p_{10}) = (p_6, p_{11}) = (p_6, p_{12}) = 1, \\
& (p_7, p_8) = (p_7, p_9) = (p_7, p_{10}) = z, \quad (p_7, p_{11}) = (p_7, p_{12}) = 1, \\
& (p_8, p_9) = z, \quad (p_8, p_{10}) = 1, \quad (p_8, p_{11}) = (p_8, p_{12}) = (p_9, p_{10}) = (p_9, p_{11}) = z, \\
& (p_9, p_{12}) = 1, \quad (p_{10}, p_{11}) = (p_{10}, p_{12}) = (p_{11}, p_{12}) = z, \\
& p_1^b p_1 p_5 p_6 p_7 p_8 p_{12} = p_2^b p_5 p_8 p_9 p_{12} = p_3^b p_2 p_3 p_4 p_5 p_6 p_7 p_8 p_9 p_{10} p_{11} = 1, \\
& p_4^b p_1 p_2 p_4 p_5 p_8 p_9 p_{10} p_{11} p_{12} = p_5^b p_7 p_8 p_9 p_{12} = z, \quad p_6^b p_1 p_3 p_6 p_7 p_8 p_9 = 1, \\
& p_7^b p_2 p_5 p_6 p_9 p_{10} p_{11} = p_8^b p_1 p_2 p_5 p_7 p_{10} p_{11} p_{12} = 1, \\
& p_9^b p_3 p_5 p_7 p_{10} p_{11} p_{12} = z, \quad p_{10}^b p_1 p_4 p_5 p_7 p_8 p_9 p_{10} = 1, \\
& p_{11}^b p_2 p_4 p_6 p_7 p_8 p_9 p_{11} = z, \quad p_{12}^b p_1 p_2 p_4 p_5 p_9 p_{10} p_{11} = 1, \\
& p_1^c p_1 p_2 p_4 p_6 p_7 p_8 = 1, \quad p_2^c p_1 p_3 p_4 p_7 p_8 p_9 = z, \quad p_3^c p_1 p_3 p_6 p_7 p_8 p_9 p_{12} = 1, \\
& p_4^c p_3 p_4 p_5 p_6 p_8 p_{10} p_{11} = z, \quad p_5^c p_1 p_2 p_5 p_7 p_9 p_{12} = z, \quad p_6^c p_3 p_6 p_{11} p_{12} = 1,
\end{aligned}$$

$$\begin{aligned}
p_7^c p_1 p_2 p_3 p_6 p_7 p_8 p_9 p_{10} &= z, & p_8^c p_2 p_3 p_4 p_5 p_6 p_9 p_{12} &= 1, & p_9^c p_1 p_2 p_3 p_4 p_7 p_{11} &= z, \\
p_{10}^c p_2 p_3 p_5 p_7 p_9 p_{10} p_{12} &= z, & p_{11}^c p_1 p_4 p_7 p_8 p_{11} p_{12} &= z, & p_{12}^c p_1 p_2 p_8 p_9 &= 1, \\
p_1^d p_2 p_6 p_9 p_{10} &= p_2^d p_1 p_4 p_5 p_8 p_9 p_{12} = p_3^d p_1 p_2 p_3 p_4 p_6 p_9 = z, \\
p_4^d p_1 p_2 p_3 p_4 p_9 p_{10} &= p_5^d p_1 p_3 p_5 p_7 p_{10} p_{12} = z, & p_6^d p_2 p_3 p_6 p_9 &= z, \\
p_7^d p_3 p_4 p_5 p_7 p_8 p_{10} p_{11} &= z, & p_8^d p_2 p_3 p_4 p_5 p_6 p_8 p_{11} p_{12} &= z, & p_9^d p_2 p_3 p_5 p_8 p_{12} &= z, \\
p_{10}^d p_2 p_4 p_9 p_{10} &= p_{11}^d p_1 p_8 p_9 p_{10} p_{11} = 1, & p_{12}^d p_1 p_2 p_4 p_7 p_8 p_9 p_{10} p_{11} &= z, \\
p_1^f p_3 p_9 p_{11} p_{12} &= z, & p_2^f p_1 p_5 p_6 p_9 p_{12} &= z, & p_3^f p_1 p_2 p_3 p_7 p_8 p_9 &= 1, \\
p_4^f p_1 p_2 p_4 p_6 p_7 p_8 p_9 p_{10} &= p_5^f p_1 p_5 p_6 p_8 p_{10} p_{12} = 1, & p_6^f p_4 p_5 p_7 p_9 &= z, \\
p_7^f p_3 p_4 p_5 p_8 p_{10} p_{11} &= p_8^f p_3 p_4 p_8 p_{11} p_{12} = 1, & p_9^f p_2 p_3 p_6 p_7 p_8 p_9 p_{10} p_{11} p_{12} &= z, \\
p_{10}^f p_4 p_5 p_6 p_7 p_9 p_{10} &= 1, & p_{11}^f p_1 p_4 p_6 p_9 p_{10} p_{11} &= z, & p_{12}^f p_1 p_4 p_9 p_{12} &= 1.
\end{aligned}$$

- (k) $H = \langle b, c, d, f, z, p_i \mid 1 \leq i \leq 12 \rangle$ has a faithful permutation representation of degree 258048 with stabilizer $L_1 = \langle b, c^3, f, p_5 p_6 p_7 p_9 p_{10} p_{11} \rangle$.
- (l) In $H = \langle b, c, d, f, z, p_i \mid 1 \leq i \leq 12 \rangle$ let $r = [(fp_1)^2(cfb)^3(fp_1)^2(fc^4)]^6$ and $p = [(fp_1)^2(cfb)^3(fp_1)^2(b^2fc)]^5$. Then $H = \langle b, p, r \rangle$, and it has 167 conjugacy classes whose representatives are given in Table A.3.
- (m) Table B.4 is the character table of H .

Proof. (a) By Lemma 6.2 there is an isomorphism $\sigma : \mathfrak{G} \rightarrow G$ such that $\sigma(\mathfrak{q}) = q_1$, $\sigma(\mathfrak{h}) = y_1$, $\sigma(\mathfrak{w}) = w_1$, and $\sigma(\mathfrak{t}) = t_1$. In particular, $G = \langle q_1, y_1, w_1, t_1 \rangle$. Let x_1 be as in Lemma 6.2(i). Let $z_1 = \sigma(\mathfrak{z}) = (x_1 y_1 w_1)^8$. Using the faithful permutation representation PG of G given in Lemma 6.2(a) and MAGMA one sees that $C_G(z_1)$ has order $2^{21} \cdot 3^7 \cdot 5 \cdot 7$. Hence \mathfrak{z} is a 2-central involution of \mathfrak{G} by Lemma 6.2(k).

In G let $r_1 = s_1 y_1^3$, $r_2 = y_1^2 w_1 y_1 s_1$, $r_3 = (s_1 y_1 w_1 s_1)^2$, $r_4 = (s_1 w_1 y_1 s_1 w_1)^6$, and $v = (w_1 q_1 w_1 y_1 q_1 y_1)^7$. Let $H = \langle r_1, r_2, r_3, r_4, v \rangle = \sigma(\mathfrak{H})$. Then Proposition 6.1(i), (j) and Lemma 6.2(l) imply that $H \leq C_G(z_1)$. Another application of PG and MAGMA yields that $C_G(z_1) = H$ and that $Z(H) = \langle z_1 \rangle$.

(b) From Proposition 6.1(j) we deduce that $U = C_{G_1}(z_1) = \langle f_1, f_2, f_3, v \rangle$ has order $2^{18} \cdot 3^5 \cdot 5$, where $f_1 = x_1 y_1 w_1$, $f_2 = (x_1 y_1 x_1 w_1)^7$, $f_3 = (x_1 w_1 y_1 w_1 y_1^2)^7$. By Proposition 6.1(k) U has a subgroup U_1 of order $2^9 \cdot 3^5 \cdot 5$ which does not contain z generated by the elements of the statement. Hence $H_1 = C_G(z_1)$ has a faithful permutation representation PH_1 of degree 258048 by (a).

(c) By Proposition 6.1(l) the subgroup \mathfrak{S} is a Sylow 2-subgroup of $\mathfrak{E} = \langle \mathfrak{x}, \mathfrak{h}, \mathfrak{w}, \mathfrak{s} \rangle$. Lemmas 6.2(h) states that $\mathfrak{E} \cong E$ where E is the finitely presented group of Lemma 2.1. Thus $|\mathfrak{S}| = 2^{21}$ by Table B.1. Hence \mathfrak{S} is a Sylow 2-subgroup of \mathfrak{G} by (a). The equality $Z(\mathfrak{H}) = \langle \mathfrak{z} \rangle$ has been checked computationally in PG .

(d) This statement follows now immediately from Proposition 6.1(l).

(e) This assertion is true by (d) and Lemma 6.2(h), (i) and (k).

(f) By Proposition 6.1(i) and Lemma 6.2(l) we know that $\mathfrak{D} = C_{\mathfrak{E}}(\mathfrak{z}) = \langle \mathfrak{t}_i \mid 1 \leq i \leq 4 \rangle$, where $\mathfrak{z} = \kappa(z_1)$. Thus it suffices to check that $N_H(B_1) = \langle r_1, r_2, r_3, r_4 \rangle$. This has been done using the faithful permutation representation PG and MAGMA.

(g) We verified computationally that the Fitting subgroup O of H is extra-special of order 2^{13} and has center $Z(O) = \langle z \rangle$. The twelve involutions p_i generating the subgroup O have been calculated with MAGMA by means of PG and the program `GetShortGens(H, O)`. We also verified that $z = (p_1 p_5)^2$.

(h) Let $\alpha : H \rightarrow H/Z(H) = H_1$ be the canonical epimorphism of $H = C_G(z)$ with kernel $Z(H) = \langle z \rangle$. By (c) H has a faithful permutation representation PH with stabilizer $U_1 = \langle v f_2 v, (f_1 f_2 f_1 v f_1)^4, (f_2 f_1 v f_1 v)^4 \rangle$. As z does not belong to U_1 its cosets provide a faithful permutation representation PH of H having degree 258048 by (b). Using MAGMA we checked that the subgroup $\alpha(U_1)$ of H_1 is a stabilizer of a faithful permutation representation PH_1 of H_1 of degree 129024. Applying PH_1 and the MAGMA command `DegreeReduction(H_1)` MAGMA calculated a faithful permutation representation PH_{11} of $H_1 = \langle \alpha(r_i), \alpha(v) \rangle$ of degree 504 with stabilizer

$$U_{11} = \langle \alpha(r_1)\alpha(r_3)\alpha(r_1), [\alpha(r_1)\alpha(v)\alpha(r_3)\alpha(v)]^3, [\alpha(r_1)\alpha(v)\alpha(r_1)\alpha(r_3)\alpha(v)\alpha(r_3)]^3 \rangle.$$

Clearly, $V = O/Z(H)$ is an elementary abelian normal subgroup of H_1 of order 2^{12} . Using PH_{11} and the command `HasComplement(H_1, V)` MAGMA established a complement K_1 of V in H_1 . By means of the command `CompositionFactors(K_1)` we saw that $|K_1 : K'_1| = 2$, $|Z(K'_1)| = 3$, and $K'/Z(K') \cong U_4(3)$.

(i) Using PH_{11} a MAGMA calculation confirmed that H_1 is generated by $a_1 = \alpha(r_1)(\alpha(r_3)^3\alpha(v)\alpha(r_3))$ and $b_1 = [\alpha(r_1)\alpha(r_3)\alpha(r_1)\alpha(r_3)^2\alpha(v)]^6[\alpha(r_1)\alpha(r_3)\alpha(r_1)\alpha(r_3)\alpha(v)\alpha(r_3)\alpha(v)]^{12}$. Both generators have order 6. Furthermore, $K'_1 = \langle b_1, c_1 \rangle$, where $c_1 = (a_1 b_1)^2$. Using the command `GetShortGens(K_1', Z(K_1'))` we observed that the center $Z(K'_1)$ of K'_1 is generated by the element $d_1 = (b_1 c_1)^7$ of order 3.

Let $\beta : K'_1 \rightarrow K'_1/Z(K'_1) = K_2$ be the canonical epimorphism of K_1 with kernel $Z(K'_1) = \langle d_1 \rangle$. Let $U_{12} = U_{11} \cap K'_1$ and $U_{13} = \langle U_{12}, d_1 \rangle$. Then K_2 has a faithful permutation representation PK_2 of degree 126 with stabilizer $\beta(U_{13})$. Let $a_1 = \beta(b_1)$ and $a_2 = \beta(c_1)$. Then $K_2 = \langle a_1, a_2 \rangle$. Using the command `FPGROUP(K_2)` MAGMA calculates the following set $\mathcal{R}(K_2)$ of defining relations of K_2 :

$$\begin{aligned} a_1^6 &= 1, & a_2^9 &= 1, \\ (a_2^{-1}a_1^{-1})^7 &= 1, & (a_1a_2^{-2}a_1)^4 &= 1, & (a_2^{-1}a_1)^9 &= 1, \\ a_1^{-1}a_2^{-1}a_1^{-3}a_2^{-1}a_1^3a_2^{-1}a_1^{-2} &= a_1a_2^3a_1^{-2}a_2a_1^3a_2^{-1}a_1a_2^{-1}a_1a_2a_1^{-1}a_2a_1 = 1, \\ a_1^{-1}a_2^{-3}a_1^{-1}a_2^{-1}a_1a_2^{-1}a_1^2a_2a_1^{-1}a_2a_1a_2^3a_1^{-1} &= 1, \\ a_1^{-1}a_2^3a_1a_2^{-1}a_1a_2^{-1}a_1^2a_2a_1^{-1}a_2a_1^{-1}a_2^{-3}a_1^{-1} &= (a_1a_2^{-3}a_1a_2^2a_1)^2 = 1, \\ a_2^{-2}a_1^{-2}a_2^{-1}a_1a_2^{-1}a_1^{-3}a_2^{-1}a_1a_2a_1^{-2}a_2a_1a_2^{-1}a_1 &= 1, \\ a_1^{-2}a_2^{-1}a_1a_2^{-1}a_1^{-2}a_2^{-1}a_1^{-2}a_2a_1^{-1}a_2a_1^{-1}a_2a_1^2a_2^2 &= 1, \\ a_2a_1^{-1}a_2^4a_1a_2^{-1}a_1^{-2}a_2^{-1}a_1a_2a_1^{-1}a_2^2a_1a_2a_1 &= 1, \\ a_2^{-2}a_1^{-1}a_2^2a_1^{-1}a_2^{-2}a_1^{-2}a_2^2a_1a_2a_1^2a_2^{-2}a_1^{-1} &= 1, \\ a_1^{-1}a_2^{-3}a_1a_2^2a_1a_2^{-1}a_1^{-1}a_2a_1^{-1}a_2a_1^{-1}a_2^2a_1^{-1}a_2^{-2} &= 1, \\ a_2^2a_1^{-1}a_2^{-1}a_1a_2^{-1}a_1^{-1}a_2^{-1}a_1^3a_2^2a_1^{-1}a_2^{-1}a_1a_2^{-1}a_1^2a_2 &= 1, \\ a_1a_2^{-2}a_1a_2^{-2}a_1a_2^{-1}a_1^3a_2^{-1}a_1^2a_2^{-1}a_1^{-1}a_2a_1^2a_2^{-1} &= 1, \\ a_1^{-1}a_2^{-2}a_1^{-1}a_2^2a_1a_2^{-1}a_1^3a_2a_1^{-1}a_2^{-2}a_1a_2^2a_1^{-2} &= 1, \\ a_1^{-2}a_2^2a_1^{-2}a_2^{-2}a_1^3a_2^{-1}a_1^{-1}a_2^{-2}a_1a_2^{-1}a_1^{-1}a_2^{-2} &= 1, \\ a_2a_1a_2a_1^{-1}a_2a_1^{-1}a_2^{-2}a_1^3a_2^2a_1^{-1}a_2a_1^{-1}a_2^2a_1a_2 &= 1. \end{aligned}$$

Since $K'_1 = \langle b_1, c \rangle$, $Z(K'_1) = \langle d_1 \rangle$, $d_1 = (b_1 c_1)^7 \neq 1$, $d_1^3 = 1$ we lift the relations of $\mathcal{R}(K_2)$ to K'_1 by evaluating them in the permutation representation of $K'_1 =$

$\langle b_1, c_1, d_1 \rangle$. Thus we obtain the following set $\mathcal{R}(K'_1)$ of defining relations of K'_1 :

$$\begin{aligned}
b_1^6 &= c_1^9 = d_1^3 = 1, & (b_1, d_1) &= (c_1, d_1) = 1, \\
(c_1^{-1}b_1^{-1})^7 d_1 &= 1, & (c_1^{-1}b_1)^9 &= 1, & b_1^{-1}c_1^{-1}b_1^{-3}c_1^{-1}b_1^3c_1^{-1}b_1^3c_1^{-1}b_1^{-2}d_1 &= 1, \\
(b_1c_1^{-2}b_1)^4 d_1^2 &= (b_1c_1^{-3}b_1c_1^2b_1)^2 d_1 = b_1c_1^3b_1^{-2}c_1b_1^3c_1^{-1}b_1c_1^{-1}b_1c_1b_1^{-1}c_1b_1d_1^2 = 1, \\
b_1^{-1}c_1^{-3}b_1^{-1}c_1^{-1}b_1c_1^{-1}b_1^2c_1b_1^{-1}c_1b_1c_1^3b_1^{-1}d_1 &= 1, \\
b_1^{-1}c_1^3b_1c_1^{-1}b_1c_1^{-1}b_1^2c_1b_1^{-1}c_1b_1^{-1}c_1^{-3}b_1^{-1}d_1 &= 1, \\
c_1^{-2}b_1^{-2}c_1^{-1}b_1c_1^{-1}b_1^{-3}c_1^{-1}b_1c_1b_1^{-2}c_1b_1c_1^{-1}b_1d_1^2 &= 1, \\
b_1^{-2}c_1^{-1}b_1c_1^{-1}b_1^{-2}c_1^{-1}b_1^{-2}c_1b_1^{-1}c_1b_1^{-1}c_1b_1^2c_1^2d_1^2 &= 1, \\
c_1b_1^{-1}c_1^4b_1c_1^{-1}b_1^{-2}c_1^{-1}b_1c_1b_1^{-1}c_1^2b_1c_1b_1d_1^2 &= 1, \\
c_1^{-2}b_1^{-1}c_1^2b_1^{-1}c_1^{-2}b_1^{-2}c_1^2b_1c_1b_1^2c_1^{-2}b_1^{-1}d_1 &= 1, \\
b_1^{-1}c_1^{-3}b_1c_1^2b_1c_1^{-1}b_1^{-1}c_1b_1^{-1}c_1b_1^{-1}c_1^2b_1^{-1}c_1^{-2} &= 1, \\
c_1^2b_1^{-1}c_1^{-1}b_1c_1^{-1}b_1^{-1}c_1^{-1}b_1^3c_1^2b_1^{-1}c_1^{-1}b_1c_1^{-1}b_1^2c_1 &= 1, \\
b_1c_1^{-2}b_1c_1^{-2}b_1c_1^{-1}b_1^3c_1^{-1}b_1^2c_1^{-1}b_1^{-1}c_1b_1^2c_1^{-1} &= 1, \\
b_1^{-1}c_1^{-2}b_1^{-1}c_1^2b_1c_1^{-1}b_1^3c_1b_1^{-1}c_1^{-2}b_1c_1^2b_1^{-2} &= 1, \\
b_1^{-2}c_1^2b_1^{-2}c_1^{-2}b_1^3c_1^{-1}b_1^{-1}c_1^{-2}b_1c_1^{-1}b_1^{-1}c_1^{-2}d_1 &= 1, \\
c_1b_1c_1b_1^{-1}c_1b_1^{-1}c_1^{-2}b_1^3c_1^2b_1^{-1}c_1b_1^{-1}c_1^2b_1c_1d_1 &= 1.
\end{aligned}$$

Using the faithful permutation representation PH_{11} of H_1 and the MAGMA command `HasComplement(K_1, K_1')` we see that K'_1 has a complement $\langle f_1 \rangle$ of order 2 such that $b_1^{f_1} \in \{b_1, b_1^{-1}\}$. In order to find a generator f_1 of a suitable complement we searched for an involution f_1 of K_1 so that at least one of the elements $c_1^{f_1}$, $d_1 \cdot c_1^{f_1}$, or $d_1^2 \cdot c_1^{f_1}$ is in the collection of all words in b_1 and c_1 of length at most 16. This search was successful. The involution $f_1 = [(b_1a_1^3)^5 \cdot (a_1^4b_1^2a_1)^9 \cdot (a_1^2b_1^3a_1^4b_1)^3 \cdot (b_1a_1^3)^5]^3$ of $K_1 = \langle a_1, b_1 \rangle$ satisfies the following set $\mathcal{R}(f_1)$ of relations: $f_1^2 = 1$, $b_1^{f_1} = b_1^5$, $c_1^{f_1} = d_1(b_1^3c_1b_1^2c_1^6b_1c_1b_1c_1)$, $d_1^{f_1} = d_1^2$. Hence $K_1 = \langle a_1, b_1 \rangle = \langle b_1, c_1, d_1, f_1 \rangle$ has a set $\mathcal{R}(K_1)$ of defining relations consisting of $\mathcal{R}(K'_1)$ and $\mathcal{R}(f_1)$.

The elementary abelian normal subgroup $V = \alpha(O)$ is generated by the involutions $q_i = \alpha(p_i)$ which are the images of the twelve generating involutions r_i of the Fitting subgroup O of H . Using the faithful permutation representation PH_{11} we calculated the images q_i^x in V for all 4 generators $x \in \{b_1, c_1, d_1, f_1\}$ of K_1 . Thus we obtained the following set of essential relations $\mathcal{R}_2(V \rtimes K_1)$ of the semi-direct product $(V \rtimes K_1)$:

$$\begin{aligned}
q_1^{b_1} q_1 q_5 q_6 q_7 q_8 q_{12} &= q_2^{b_1} q_5 q_8 q_9 q_{12} = q_3^{b_1} q_2 q_3 q_4 q_5 q_6 q_7 q_8 q_9 q_{10} q_{11} = 1, \\
q_4^{b_1} q_1 q_2 q_4 q_5 q_8 q_9 q_{10} q_{11} q_{12} &= q_5^{b_1} q_7 q_8 q_9 q_{12} = q_6^{b_1} q_1 q_3 q_6 q_7 q_8 q_9 = 1, \\
q_7^{b_1} q_2 q_5 q_6 q_9 q_{10} q_{11} &= q_8^{b_1} q_1 q_2 q_5 q_7 q_{10} q_{11} q_{12} = q_9^{b_1} q_3 q_5 q_7 q_{10} q_{11} q_{12} = 1, \\
q_{10}^{b_1} q_1 q_4 q_5 q_7 q_8 q_9 q_{10} &= q_{11}^{b_1} q_2 q_4 q_6 q_7 q_8 q_9 q_{11} = q_{12}^{b_1} q_1 q_2 q_4 q_5 q_9 q_{10} q_{11} = 1, \\
q_1^{c_1} q_1 q_2 q_4 q_6 q_7 q_8 &= q_2^{c_1} q_1 q_3 q_4 q_7 q_8 q_9 = q_3^{c_1} q_1 q_3 q_6 q_7 q_8 q_9 q_{12} = 1, \\
q_4^{c_1} q_3 q_4 q_5 q_6 q_8 q_{10} q_{11} &= q_5^{c_1} q_1 q_2 q_5 q_7 q_9 q_{12} = q_6^{c_1} q_3 q_6 q_{11} q_{12} = 1, \\
q_7^{c_1} q_1 q_2 q_3 q_6 q_7 q_8 q_9 q_{10} &= q_8^{c_1} q_2 q_3 q_4 q_5 q_6 q_9 q_{12} = q_9^{c_1} q_1 q_2 q_3 q_4 q_7 q_{11} = 1, \\
q_{10}^{c_1} q_2 q_3 q_5 q_7 q_9 q_{10} q_{12} &= q_{11}^{c_1} q_1 q_4 q_7 q_8 q_{11} q_{12} = q_{12}^{c_1} q_1 q_2 q_8 q_9 = 1, \\
q_1^{d_1} q_2 q_6 q_9 q_{10} &= q_2^{d_1} q_1 q_4 q_5 q_8 q_9 q_{12} = q_3^{d_1} q_1 q_2 q_3 q_4 q_6 q_9 = 1,
\end{aligned}$$

$$\begin{aligned}
q_4^{d_1} q_1 q_2 q_3 q_4 q_9 q_{10} &= q_5^{d_1} q_1 q_3 q_5 q_7 q_{10} q_{12} = q_6^{d_1} q_2 q_3 q_6 q_9 = 1, \\
q_7^{d_1} q_3 q_4 q_5 q_7 q_8 q_{10} q_{11} &= q_8^{d_1} q_2 q_3 q_4 q_5 q_6 q_8 q_{11} q_{12} = q_9^{d_1} q_2 q_3 q_5 q_8 q_{12} = 1, \\
q_{10}^{d_1} q_2 q_4 q_9 q_{10} &= q_{11}^{d_1} q_1 q_8 q_9 q_{10} q_{11} = q_{12}^{d_1} q_1 q_2 q_4 q_7 q_8 q_9 q_{10} q_{11} = 1, \\
q_1^{f_1} q_3 q_9 q_{11} q_{12} &= q_2^{f_1} q_1 q_5 q_6 q_9 q_{12} = q_3^{f_1} q_1 q_2 q_3 q_7 q_8 q_9 = 1, \\
q_4^{f_1} q_1 q_2 q_4 q_6 q_7 q_8 q_9 q_{10} &= q_5^{f_1} q_1 q_5 q_6 q_8 q_{10} q_{12} = q_6^{f_1} q_4 q_5 q_7 q_9 = 1, \\
q_7^{f_1} q_3 q_4 q_5 q_8 q_{10} q_{11} &= q_8^{f_1} q_3 q_4 q_8 q_{11} q_{12} = q_9^{f_1} q_2 q_3 q_6 q_7 q_8 q_9 q_{10} q_{11} q_{12} = 1, \\
q_{10}^{f_1} q_4 q_5 q_6 q_7 q_9 q_{10} &= q_{11}^{f_1} q_1 q_4 q_6 q_9 q_{10} q_{11} = q_{12}^{f_1} q_1 q_4 q_9 q_{12} = 1.
\end{aligned}$$

Hence the set $\mathcal{R}(H_1)$ of defining relations of the semi-direct product $H_1 = (V \rtimes K_1)$ consists $\mathcal{R}(K_1)$, $\mathcal{R}_2(V \rtimes K_1)$ and the following relations:

$$\begin{aligned}
q_j^2 &= 1 \quad \text{for all } 1 \leq j \leq 12, \\
q_k \cdot q_j &= q_j \cdot q_k \quad \text{for all } 1 \leq j, k \leq 12.
\end{aligned}$$

In order to get a presentation of H we lift the generators a_1 and b_1 of K_1 to H . Clearly, $a = r_1 r_3^3 v r_3$ and $b = (r_1 r_3 r_1 r_3^2 v)^6 (r_1 r_3 r_1 r_3 v r_3 v)^{12}$ of H map onto a_1 and b_1 of H_1 , respectively. Let $c = (ab)^2$, $d = (bc)^7$ and $f = [(ba^3)^5 \cdot (a^4 b^2 a)^9 \cdot (a^2 b^3 a^4 b)^3 \cdot (ba^3)^5]^3$. Then c_1, d_1, f_1 in H_1 are images of c, d, f in H under α , respectively. Since $z = (p_1 p_5)^2$ generates the center $Z(O)$ of the Frattini subgroup $O = \langle p_i \mid 1 \leq i \leq 12 \rangle$ it follows that

$$H = \langle a, b, p_i \mid 1 \leq i \leq 12 \rangle = \langle b, c, d, f, p_i \mid 1 \leq i \leq 12 \rangle.$$

The set of defining relations $\mathcal{R}(H)$ of $H = \langle b, c, d, f, p_i \mid 1 \leq i \leq 12 \rangle$ has been obtained by evaluating the lifted equations of the presentation $\mathcal{R}(H_1)$ of H_1 in the permutation representation PH with stabilizer U_1 defined in the proof of (c). The resulting equations of $\mathcal{R}(H)$ are stated in assertion (k).

The map $\mathfrak{H} \rightarrow H$ sending each generator \mathfrak{x} of \mathfrak{H} in (i) to the corresponding generator $x \in H$ in (j) is an isomorphism by (a) and the order of H .

(k) The given stabilizer of the group $H = \langle b, c, d, f, p_i \mid 1 \leq i \leq 12 \rangle$ has been found as follows. In the original permutation representation of the finitely presented group G of degree 306936 we checked that the subgroup $L = \langle b, c^3, f \rangle$ has index 1032192 in H and that $z \notin L$. Using then MAGMA and the command `MyCosetAction(H, L: maxsize:=10000000)` we verified that L has the same index in the finitely presented group $H = \langle b, c, d, f, p_i \mid 1 \leq i \leq 12 \rangle$. In the corresponding permutation representation pH of this group we searched then for an element $p \in O = \langle p_i \rangle$ such that $z \notin L_1 = \langle L, p \rangle$. MAGMA found an involution $p \in O$ with these properties. Since O is extra-special of order 2^{13} the command `LookupWord(0, p)` worked well. The word of p is stated in the assertion. Using the command `MyCosetAction(H, L_1: maxsize:=10000000)` MAGMA established in 70 seconds the index $|H : L_1| = 258048$.

(l) Both elements r and p of H have order 6. Using the faithful permutation representation PH of H with stabilizer L_1 and MAGMA it has been verified that $H = \langle r, p, b \rangle$. Since $H_1 = H / \langle z \rangle$ has a faithful permutation representation of degree 504 we used it and Kratzer's Algorithm 5.3.18 of [12] to calculate a system of representatives of the classes of $H_1 = \langle \alpha(r), \alpha(p), \alpha(b) \rangle$. H_1 has 123 conjugacy classes. Their representatives have been lifted to H . Using PH we have checked the

conjugacy of the lifted representatives and the products with the central involution z of H .

(m) The character table of H was calculated automatically by MAGMA using *PH*. \square

8. GROUP ORDER

In this section we check the group order of \mathfrak{G} by means of Thompson's group order formula and Theorem 6.1.4 of [14].

Proposition 8.1. *Let $\mathfrak{G} = \langle \mathfrak{q}, \eta, \mathfrak{t}, \mathfrak{w} \rangle$ be the subgroup of $\text{GL}_{8671}(13)$ constructed in Proposition 5.2. Let $\mathfrak{x} = [(\eta \mathfrak{q}^2 \eta \mathfrak{q} \eta \mathfrak{q}^2)^{11} (\mathfrak{q}^2 \eta^2 \mathfrak{q} \eta \mathfrak{q} \eta)^{11} (\mathfrak{q} \eta^2 \mathfrak{q} \eta \mathfrak{q} \eta \mathfrak{q})^4]^{12}$ and $\mathfrak{s} = (\eta^5 \mathfrak{t})^7$.*

Let $\mathfrak{r}_1 = (\mathfrak{s} \eta^3)^3$, $\mathfrak{r}_2 = (\eta^2 \mathfrak{w} \eta \mathfrak{s})^6$, $\mathfrak{r}_3 = (\mathfrak{s} \eta \mathfrak{w} \eta \mathfrak{s})^2$, $\mathfrak{r}_4 = (\mathfrak{s} \mathfrak{w} \eta \mathfrak{s} \mathfrak{w})^6$, and $\mathfrak{v} = (\mathfrak{w} \mathfrak{q} \mathfrak{w} \eta \mathfrak{q} \eta)^7$. Let $\mathfrak{a} = \mathfrak{r}_1 \mathfrak{r}_3^3 \mathfrak{v} \mathfrak{r}_3$, $\mathfrak{b} = [\mathfrak{r}_1 \mathfrak{r}_3 \mathfrak{r}_1 \mathfrak{r}_3^2 \mathfrak{v}]^6 [\mathfrak{r}_1 \mathfrak{r}_3 \mathfrak{r}_1 \mathfrak{r}_3 \mathfrak{v} \mathfrak{r}_3 \mathfrak{v}]^{12}$, $\mathfrak{f} = [(\mathfrak{b} \mathfrak{a}^3)^5 (\mathfrak{a}^4 \mathfrak{b}^2 \mathfrak{a})^9 (\mathfrak{a}^2 \mathfrak{b}^3 \mathfrak{a}^4 \mathfrak{b})^3 (\mathfrak{b} \mathfrak{a}^3)^5]^3$, $\mathfrak{r} = [(\mathfrak{f} \mathfrak{r}_2^2)^2 ((\mathfrak{a} \mathfrak{b})^2 \mathfrak{f} \mathfrak{b})^3 (\mathfrak{f} \mathfrak{r}_2^2)^2 (\mathfrak{f} (\mathfrak{a} \mathfrak{b})^8)]^6$, and $\mathfrak{p} = [(\mathfrak{f} \mathfrak{r}_2^2)^2 ((\mathfrak{a} \mathfrak{b})^2 \mathfrak{f} \mathfrak{b})^3 (\mathfrak{f} \mathfrak{r}_2^2)^2 (\mathfrak{b}^2 \mathfrak{f} (\mathfrak{a} \mathfrak{b})^2)]^5$.

Then the following assertions hold:

- (a) $\mathfrak{z} = (\mathfrak{x} \eta \mathfrak{w})^8$ is a 2-central involution of \mathfrak{G} with centralizer $\mathfrak{H} = C_{\mathfrak{G}}(\mathfrak{z}) = \langle \mathfrak{b}, \mathfrak{p}, \mathfrak{r} \rangle$. Furthermore, \mathfrak{H} has 9 conjugacy classes of involutions 2_i , $1 \leq i \leq 9$ with representatives given in Table A.3, and $|\mathfrak{H}| = 2^{21} \cdot 3^7 \cdot 5 \cdot 7$.
- (b) $\mathfrak{u} = (\eta \mathfrak{s})^{14}$ is an involution of \mathfrak{G} with centralizer $\mathfrak{U} = C_{\mathfrak{G}}(\mathfrak{u}) = \mathfrak{A}_1 = \langle \mathfrak{q}, \eta, \mathfrak{s} \rangle$. Furthermore, \mathfrak{U} has 7 conjugacy classes of involutions 2_i , $1 \leq i \leq 7$, with representatives given in Table A.2, and $|\mathfrak{U}| = 2^{19} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$.
- (c) The Fitting subgroup \mathfrak{B} of $\mathfrak{E} = \langle \mathfrak{x}, \eta, \mathfrak{w}, \mathfrak{s} \rangle$ is an elementary abelian group of order 2^{11} such that $\mathfrak{E}/\mathfrak{B} \cong \mathcal{M}_{24}$.
- (d) $\mathfrak{D} = \langle \mathfrak{r}_j \mid 1 \leq j \leq 4 \rangle = N_{\mathfrak{H}}(\mathfrak{B}) = C_{\mathfrak{E}}(\mathfrak{z}_1)$ where $\mathfrak{z}_1 = \mathfrak{x}^2 \in \mathfrak{E}$.
- (e) $\mathfrak{D}_1 = \langle \mathfrak{x}, \eta, \mathfrak{s} \rangle = N_{\mathfrak{A}_1}(\mathfrak{B}) = C_{\mathfrak{E}}(\mathfrak{z}_2)$ where $\mathfrak{z}_2 = (\mathfrak{x} \eta^2)^7 \in \mathfrak{E}$.
- (f) \mathfrak{G} has two conjugacy classes of involutions represented by \mathfrak{z} and \mathfrak{u} .
- (g) The conjugacy classes of involutions $2_1, 2_3, 2_5, 2_6, 2_8$ and 2_9 of \mathfrak{H} fuse with \mathfrak{z} in \mathfrak{G} . Its classes $2_2, 2_4$ and 2_7 fuse with \mathfrak{u} in \mathfrak{G} .
- (h) The conjugacy classes of involutions $2_1, 2_2, 2_3$ and 2_4 of \mathfrak{U} fuse with \mathfrak{u} in \mathfrak{G} . Its classes $2_5, 2_6$ and 2_7 fuse with \mathfrak{z} in \mathfrak{G} .
- (i) $|\mathfrak{G}| = 2132400816 \cdot |\mathfrak{U}| + 4388805476055 \cdot |\mathfrak{H}| = 2^{21} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29$.

Proof. (a) holds by Proposition 7.1(a), (j), (l) and Table A.3.

(b) By Proposition 6.1(c) we know that $\mathfrak{A}_1 = \langle \mathfrak{q}, \eta, \mathfrak{s} \rangle$. Its conjugacy classes are classified in Table A.2. It asserts that the involution $\mathfrak{u} = (\eta \mathfrak{s})^{14}$ generates the center of \mathfrak{A}_1 . The equation $C_{\mathfrak{G}}(\mathfrak{u}) = \mathfrak{A}_1$ is a consequence of Lemma 6.2(e) and (f). All other assertions hold by Table A.2.

(c) and (d) hold by Proposition 7.1(d) and (g), respectively.

(e) This is true by Proposition 6.1(d), (f) and Table A.1.

(f) Table A.1 shows that E has 5 conjugacy classes of involutions and that $z_1 = x^2$ is the representative of the unique 2-central conjugacy class of $E = \langle x, y, e \rangle$. By (d) the subgroup $\mathfrak{D} = \langle \mathfrak{x}, \eta, \mathfrak{s} \rangle = \mathfrak{H} \cap \mathfrak{E}$. It has 18 conjugacy classes 2_k , $1 \leq k \leq 18$ of involutions. Using MAGMA and the faithful permutation representations $P\mathfrak{H}$ and $P\mathfrak{E}$ of Propositions 7.1(c) and 6.1(h), respectively, we calculated the fusion of the classes 2_k of \mathfrak{D} in \mathfrak{H} and also in \mathfrak{E} . Thus we obtained a fusion graph $\mathcal{G}(H)$ of the fusion of the \mathfrak{H} -classes and \mathfrak{E} -classes of order 2 in the matrix group \mathfrak{G} . It follows

that \mathfrak{G} has 2 conjugacy classes of involutions represented by \mathfrak{z} and \mathfrak{u} belonging to the classes 2_1 and 2_2 of \mathfrak{H} , respectively.

(g) This statement follows also from the fusion graph \mathcal{G} .

(h) In view of (e) we now study the fusion of the 11 classes of involutions of $\mathfrak{R} = N_{\mathfrak{A}_1}(\mathfrak{B}) = \mathfrak{A}_1 \cap \mathfrak{E}$ in the two over groups \mathfrak{A}_1 and \mathfrak{E} . Using MAGMA and the faithful permutation representation PA_1 of Lemma 3.1(c) it follows that the classes $2_1, 2_2, 2_3$ and 2_4 of \mathfrak{U} fuse with \mathfrak{u} , and that the remaining three classes of \mathfrak{U} fuse in \mathfrak{G} with \mathfrak{z} .

(i) In order to simplify notation we replace the Gothic letters by Roman ones. Let $r(z, u, z) = |\{(x, y) \in (z^G \cap H) \times (u^G \cap H) \mid z \in \langle xy \rangle\}|$ and $r(z, u, u) = |\{(x, y) \in (z^G \cap U) \times (u^G \cap U) \mid u \in \langle xy \rangle\}|$.

By Table B.4 H has 45 real z -special conjugacy classes. Here their representatives are denoted by $\{t_j \mid 1 \leq j \leq 45\}$. Let z_i and u_k be representatives of the H -classes of involutions fusing to z and u in G , respectively. For each triple (z_i, u_k, t_j) let

$$d(z_i, u_k, t_j) = \frac{|H|^2}{|C_H(z_i)| \cdot |C_H(u_k)| \cdot |C_H(t_j)|} \cdot \sum_{\psi \in \text{Irr}_{\mathbb{C}}(H)} \psi(z_i) \psi(u_k) \psi(t_j) \psi(1)^{-1}.$$

Then Theorem 1.6.4 of [14] and (g) imply that

$$r(z, u, z) = \sum_{i=1}^6 \sum_{k=1}^3 \sum_{j=1}^{45} d(z_i, u_k, t_j).$$

Using (g) and the values of the character Table B.4 of H these formulas yield that $r(z, u, z) = 2132400816$.

By Table B.5 $U = C_G(u) = A_1$ has 22 real u -special conjugacy classes. Denote their representatives by $\{s_n \mid 1 \leq n \leq 22\}$. Let u_i and z_k be representatives of the U -classes of involutions fusing to u and z in G , respectively. For each triple (u_i, z_k, s_n) let

$$d(u_i, z_k, s_n) = \frac{|U|^2}{|C_U(u_i)| \cdot |C_U(z_k)| \cdot |C_U(s_n)|} \cdot \sum_{\psi \in \text{Irr}_{\mathbb{C}}(U)} \psi(u_i) \psi(z_k) \psi(s_n) \psi(1)^{-1}.$$

Then Theorem 1.6.4 of [14] and (h) imply that

$$r(z, u, u) = \sum_{i=1}^4 \sum_{k=1}^3 \sum_{n=1}^{22} d(u_i, z_k, s_n).$$

Using (h) and the values of the character Table B.5 of U these formulas yields that $r(z, u, u) = 4388805476055$. Now Theorem 4.2.1 of [12] due to J. G. Thompson implies

$$|G| = r(z, u, z) \cdot |C_G(u)| + r(z, u, u) \cdot |C_G(z)| = 2^{21} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23 \cdot 29.$$

□

APPENDIX A. REPRESENTATIVES OF CONJUGACY CLASSES

A.1. Conjugacy classes of $E(\text{Fi}'_{24}) = \langle x, y, e \rangle$

Class	Representative	Class	Centralizer	2P	3P	5P	7P	11P	23P
1	1	1	$2^{21} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 23$	1	1	1	1	1	1
2 ₁	$(e)^2$	276	$2^{19} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$	1	2 ₁	2 ₁	2 ₁	2 ₁	2 ₁
2 ₂	$(x)^2$	1771	$2^{21} \cdot 3^3 \cdot 5$	1	2 ₂	2 ₂	2 ₂	2 ₂	2 ₂
2 ₃	$(y)^3$	182160	$2^{17} \cdot 3 \cdot 7$	1	2 ₃	2 ₃	2 ₃	2 ₃	2 ₃
2 ₄	$(ye^2)^3$	1275120	$2^{17} \cdot 3$	1	2 ₄	2 ₄	2 ₄	2 ₄	2 ₄
2 ₅	$(xey)^3$	2040192	$2^{14} \cdot 3 \cdot 5$	1	2 ₅	2 ₅	2 ₅	2 ₅	2 ₅
3 ₁	$(y)^2$	14508032	$2^8 \cdot 3^3 \cdot 5$	3 ₁	1	3 ₁	3 ₁	3 ₁	3 ₁
3 ₂	$(xy)^4$	124354560	$2^6 \cdot 3^2 \cdot 7$	3 ₂	1	3 ₂	3 ₂	3 ₂	3 ₂
4 ₁	$(xy)^3$	1020096	$2^{15} \cdot 3 \cdot 5$	2 ₂	4 ₁	4 ₁	4 ₁	4 ₁	4 ₁
4 ₂	$(xyey)^3$	1020096	$2^{15} \cdot 3 \cdot 5$	2 ₂	4 ₂	4 ₂	4 ₂	4 ₂	4 ₂
4 ₃	$(xe)^3$	5100480	$2^{15} \cdot 3$	2 ₂	4 ₃	4 ₃	4 ₃	4 ₃	4 ₃
4 ₄	$(xye)^6$	5100480	$2^{15} \cdot 3$	2 ₂	4 ₄	4 ₄	4 ₄	4 ₄	4 ₄
4 ₅	e	11658240	$2^{11} \cdot 3 \cdot 7$	2 ₁	4 ₅	4 ₅	4 ₅	4 ₅	4 ₅
4 ₆	x	61205760	2^{13}	2 ₂	4 ₆	4 ₆	4 ₆	4 ₆	4 ₆
4 ₇	$(x^2yxy^2)^3$	81607680	$2^{11} \cdot 3$	2 ₃	4 ₇	4 ₇	4 ₇	4 ₇	4 ₇
4 ₈	$(x^2yxy^2e^2)^3$	81607680	$2^{11} \cdot 3$	2 ₃	4 ₈	4 ₈	4 ₈	4 ₈	4 ₈
4 ₉	$(xyeyey)^2$	244823040	2^{11}	2 ₄	4 ₉	4 ₉	4 ₉	4 ₉	4 ₉
4 ₁₀	$(x^3exy)^2$	244823040	2^{11}	2 ₄	4 ₁₀	4 ₁₀	4 ₁₀	4 ₁₀	4 ₁₀
4 ₁₁	x^3y^2xe	489646080	2^{10}	2 ₃	4 ₁₁	4 ₁₁	4 ₁₁	4 ₁₁	4 ₁₁
4 ₁₂	$xyxyxey$	489646080	2^{10}	2 ₄	4 ₁₂	4 ₁₂	4 ₁₂	4 ₁₂	4 ₁₂
5	$(xy)^2$	1044578304	$2^5 \cdot 3 \cdot 5$	5	5	1	5	5	5
6 ₁	$(xye^2y)^5$	14508032	$2^8 \cdot 3^3 \cdot 5$	3 ₁	2 ₂	6 ₁	6 ₁	6 ₁	6 ₁
6 ₂	$(xe)^2$	217620480	$2^8 \cdot 3^2$	3 ₁	2 ₂	6 ₂	6 ₂	6 ₂	6 ₂
6 ₃	x^2y^2	217620480	$2^8 \cdot 3^2$	3 ₁	2 ₁	6 ₃	6 ₃	6 ₃	6 ₃
6 ₄	$(xy)^2$	870481920	$2^6 \cdot 3^2$	3 ₂	2 ₂	6 ₄	6 ₄	6 ₄	6 ₄
6 ₅	y	2611445760	$2^6 \cdot 3$	3 ₁	2 ₃	6 ₅	6 ₅	6 ₅	6 ₅
6 ₆	ye^2	2611445760	$2^6 \cdot 3$	3 ₁	2 ₄	6 ₆	6 ₆	6 ₆	6 ₆
6 ₇	xey	10445783040	$2^3 \cdot 3$	3 ₂	2 ₅	6 ₇	6 ₇	6 ₇	6 ₇
7 ₁	$(xyxe)^2$	2984509440	$2^3 \cdot 3 \cdot 7$	7 ₁	7 ₂	1	7 ₁	7 ₁	7 ₁
7 ₂	$(xyex)^2$	2984509440	$2^3 \cdot 3 \cdot 7$	7 ₂	7 ₁	7 ₁	1	7 ₂	7 ₂
8 ₁	$(xyc)^3$	652861440	$2^8 \cdot 3$	4 ₄	8 ₁	8 ₁	8 ₁	8 ₁	8 ₁
8 ₂	$(xey)^3$	652861440	$2^8 \cdot 3$	4 ₁	8 ₂	8 ₂	8 ₂	8 ₂	8 ₂
8 ₃	$(xyxy)^3$	652861440	$2^8 \cdot 3$	4 ₂	8 ₃	8 ₃	8 ₃	8 ₃	8 ₃
8 ₄	$(xexey)^2$	979292160	2^9	4 ₄	8 ₄	8 ₄	8 ₄	8 ₄	8 ₄
8 ₅	xy^2xe	1958584320	2^8	4 ₃	8 ₅	8 ₅	8 ₅	8 ₅	8 ₅
8 ₆	$xyxy^2xy^2$	3917168640	2^7	4 ₆	8 ₆	8 ₆	8 ₆	8 ₆	8 ₆
8 ₇	$xyeyey$	7834337280	2^6	4 ₉	8 ₇	8 ₇	8 ₇	8 ₇	8 ₇
8 ₈	x^3exey	7834337280	2^6	4 ₁₀	8 ₈	8 ₈	8 ₈	8 ₈	8 ₈
10 ₁	$(xye^2y)^3$	1044578304	$2^5 \cdot 3 \cdot 5$	5	10 ₁	2 ₂	10 ₁	10 ₁	10 ₁
10 ₂	y^2e	6267469824	$2^4 \cdot 5$	5	10 ₂	2 ₁	10 ₂	10 ₂	10 ₂
10 ₃	$xyxeyey$	6267469824	$2^4 \cdot 5$	5	10 ₄	2 ₅	10 ₄	10 ₃	10 ₄
10 ₄	$xyxex^3$	6267469824	$2^4 \cdot 5$	5	10 ₃	2 ₅	10 ₃	10 ₄	10 ₃
11	$(ye)^2$	22790799360	$2 \cdot 11$	11	11	11	11	1	11
12 ₁	xe	2611445760	$2^6 \cdot 3$	6 ₂	4 ₃	12 ₁	12 ₁	12 ₁	12 ₁
12 ₂	$(xye)^2$	2611445760	$2^6 \cdot 3$	6 ₂	4 ₄	12 ₂	12 ₂	12 ₂	12 ₂
12 ₃	xy	5222891520	$2^5 \cdot 3$	6 ₄	4 ₁	12 ₃	12 ₃	12 ₃	12 ₃
12 ₄	$xyey$	5222891520	$2^5 \cdot 3$	6 ₄	4 ₂	12 ₄	12 ₄	12 ₄	12 ₄
12 ₅	y^3e	10445783040	$2^3 \cdot 3$	6 ₃	4 ₅	12 ₅	12 ₅	12 ₅	12 ₅
12 ₆	x^2yxy^2	10445783040	$2^4 \cdot 3$	6 ₅	4 ₇	12 ₆	12 ₆	12 ₆	12 ₆
12 ₇	$x^2yxy^2e^2$	10445783040	$2^4 \cdot 3$	6 ₅	4 ₈	12 ₇	12 ₇	12 ₇	12 ₇
14 ₁	$(xyey)^2$	8953528320	$2^3 \cdot 7$	7 ₂	14 ₂	14 ₂	2 ₁	14 ₁	14 ₁
14 ₂	$(xye^2xe)^2$	8953528320	$2^3 \cdot 7$	7 ₁	14 ₁	14 ₁	2 ₁	14 ₂	14 ₂
14 ₃	$xyxe$	17907056640	$2^2 \cdot 7$	7 ₁	14 ₄	14 ₄	2 ₃	14 ₃	14 ₃
14 ₄	$xyex$	17907056640	$2^2 \cdot 7$	7 ₂	14 ₃	14 ₃	2 ₃	14 ₄	14 ₄
15 ₁	xy^2	16713252864	$2 \cdot 3 \cdot 5$	15 ₁	5	3 ₁	15 ₂	15 ₂	15 ₁
15 ₂	xy^3	16713252864	$2 \cdot 3 \cdot 5$	15 ₂	5	3 ₁	15 ₁	15 ₁	15 ₂
16	$xexey$	15668674560	2^5	8 ₄	16	16	16	16	16
20 ₁	$xyxex$	6267469824	$2^3 \cdot 5$	10 ₁	20 ₁	4 ₂	20 ₁	20 ₁	20 ₁
20 ₂	xy^2xy^4	6267469824	$2^4 \cdot 5$	10 ₁	20 ₂	4 ₁	20 ₂	20 ₂	20 ₂

Conjugacy classes of $E(\text{Fi}'_{24}) = \langle x, y, e \rangle$ (continued)

Class	Representative	Class	Centralizer	2P	3P	5P	7P	11P	23P
21 ₁	$xeye$	23876075520	$3 \cdot 7$	21 ₁	7 ₁	21 ₂	3 ₂	21 ₁	21 ₁
21 ₂	xy^2eye	23876075520	$3 \cdot 7$	21 ₂	7 ₂	21 ₁	3 ₂	21 ₂	21 ₂
22	ye	22790799360	$2 \cdot 11$	11	22	22	22	2 ₁	22
23 ₁	xy^2e	21799895040	23	23 ₁	23 ₁	23 ₂	23 ₂	23 ₂	1
23 ₂	xye^3	21799895040	23	23 ₂	23 ₂	23 ₁	23 ₁	23 ₁	1
24 ₁	xye	10445783040	$2^4 \cdot 3$	12 ₂	8 ₁	24 ₂	24 ₂	24 ₁	24 ₁
24 ₂	xy^3	10445783040	$2^4 \cdot 3$	12 ₂	8 ₁	24 ₁	24 ₁	24 ₂	24 ₂
24 ₃	xye^2	20891566080	$2^3 \cdot 3$	12 ₃	8 ₂	24 ₃	24 ₃	24 ₃	24 ₃
24 ₄	$xyxy^2$	20891566080	$2^3 \cdot 3$	12 ₄	8 ₃	24 ₄	24 ₄	24 ₄	24 ₄
28 ₁	xye^2y^2	17907056640	$2^2 \cdot 7$	14 ₁	28 ₂	28 ₂	4 ₅	28 ₁	28 ₁
28 ₂	xye^2xe	17907056640	$2^2 \cdot 7$	14 ₂	28 ₁	28 ₁	4 ₅	28 ₂	28 ₂
30 ₁	x^2ye^2y	16713252864	$2 \cdot 3 \cdot 5$	15 ₁	10 ₁	6 ₁	30 ₂	30 ₂	30 ₁
30 ₂	$x^2y^2xy^2$	16713252864	$2 \cdot 3 \cdot 5$	15 ₂	10 ₁	6 ₁	30 ₁	30 ₁	30 ₂

A.2. Conjugacy classes of $A_1 = \text{Aut}(2\text{Fi}_{22}) = \langle y, q, s \rangle$

Class	Representative	Centralizer	2P	3P	5P	7P	11P	13P
1	1	$2^{19} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	1	1	1	1	1	1
2 ₁	$(ys)^{14}$	$2^{19} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	1	2 ₁	2 ₁	2 ₁	2 ₁	2 ₁
2 ₂	$(y)^7$	$2^{17} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	1	2 ₂	2 ₂	2 ₂	2 ₂	2 ₂
2 ₃	s	$2^{14} \cdot 3^6 \cdot 5^2 \cdot 7$	1	2 ₃	2 ₃	2 ₃	2 ₃	2 ₃
2 ₄	$(yq)^{10}$	$2^{19} \cdot 3^4 \cdot 5$	1	2 ₄	2 ₄	2 ₄	2 ₄	2 ₄
2 ₅	$(y^2q)^8$	$2^{19} \cdot 3^4 \cdot 5$	1	2 ₅	2 ₅	2 ₅	2 ₅	2 ₅
2 ₆	$(y^3qs)^9$	$2^{14} \cdot 3^4 \cdot 5$	1	2 ₆	2 ₆	2 ₆	2 ₆	2 ₆
2 ₇	$(ysq)^{12}$	$2^{17} \cdot 3^3$	1	2 ₇	2 ₇	2 ₇	2 ₇	2 ₇
3 ₁	$(qs)^4$	$2^{10} \cdot 3^7 \cdot 5 \cdot 7$	3 ₁	1	3 ₁	3 ₁	3 ₁	3 ₁
3 ₂	q	$2^9 \cdot 3^9$	3 ₂	1	3 ₂	3 ₂	3 ₂	3 ₂
3 ₃	$(ysq)^8$	$2^8 \cdot 3^7$	3 ₃	1	3 ₃	3 ₃	3 ₃	3 ₃
3 ₄	$(y^3qs)^6$	$2^5 \cdot 3^7$	3 ₄	1	3 ₄	3 ₄	3 ₄	3 ₄
4 ₁	$(ys)^7$	$2^{11} \cdot 3^4 \cdot 5 \cdot 7$	2 ₁	4 ₁	4 ₁	4 ₁	4 ₁	4 ₁
4 ₂	$(yqs)^9$	$2^{14} \cdot 3^4$	2 ₅	4 ₂	4 ₂	4 ₂	4 ₂	4 ₂
4 ₃	$(y^3sq^2)^5$	$2^{14} \cdot 3 \cdot 5$	2 ₅	4 ₃	4 ₃	4 ₃	4 ₃	4 ₃
4 ₄	$(yqqyq^2)^6$	$2^{13} \cdot 3^3$	2 ₅	4 ₄	4 ₄	4 ₄	4 ₄	4 ₄
4 ₅	$(yq)^5$	$2^{11} \cdot 3^2 \cdot 5$	2 ₄	4 ₅	4 ₅	4 ₅	4 ₅	4 ₅
4 ₆	$(y^2q)^4$	$2^{13} \cdot 3$	2 ₅	4 ₆	4 ₆	4 ₆	4 ₆	4 ₆
4 ₇	$(qs)^3$	$2^{11} \cdot 3^2$	2 ₄	4 ₇	4 ₇	4 ₇	4 ₇	4 ₇
4 ₈	$(ysq)^6$	$2^{11} \cdot 3^2$	2 ₇	4 ₈	4 ₈	4 ₈	4 ₈	4 ₈
4 ₉	y^2sys	$2^{11} \cdot 3^2$	2 ₇	4 ₉	4 ₉	4 ₉	4 ₉	4 ₉
4 ₁₀	$(y^4sq)^3$	$2^{12} \cdot 3$	2 ₅	4 ₁₀	4 ₁₀	4 ₁₀	4 ₁₀	4 ₁₀
4 ₁₁	$(y^2sy^2sq)^3$	$2^{11} \cdot 3$	2 ₄	4 ₁₁	4 ₁₁	4 ₁₁	4 ₁₁	4 ₁₁
4 ₁₂	$(y^2qsq)^3$	$2^{10} \cdot 3$	2 ₇	4 ₁₂	4 ₁₂	4 ₁₂	4 ₁₂	4 ₁₂
5	$(yq)^4$	$2^5 \cdot 3 \cdot 5^2$	5	5	1	5	5	5
6 ₁	$(y^3q^2)^5$	$2^{10} \cdot 3^7 \cdot 5 \cdot 7$	3 ₁	2 ₁	6 ₁	6 ₁	6 ₁	6 ₁
6 ₂	$(y^4qys)^6$	$2^9 \cdot 3^9$	3 ₂	2 ₁	6 ₂	6 ₂	6 ₂	6 ₂
6 ₃	$(y^2s)^2$	$2^8 \cdot 3^7$	3 ₃	2 ₁	6 ₃	6 ₃	6 ₃	6 ₃
6 ₄	$(yqsys)^5$	$2^8 \cdot 3^5 \cdot 5$	3 ₁	2 ₂	6 ₄	6 ₄	6 ₄	6 ₄
6 ₅	q^2sqsqs	$2^8 \cdot 3^5 \cdot 5$	3 ₁	2 ₃	6 ₅	6 ₅	6 ₅	6 ₅
6 ₆	$(y^3sq^2s)^3$	$2^8 \cdot 3^6$	3 ₂	2 ₂	6 ₆	6 ₆	6 ₆	6 ₆
6 ₇	$(y^3q^2s)^7$	$2^8 \cdot 3^4 \cdot 7$	3 ₁	2 ₃	6 ₈	6 ₇	6 ₈	6 ₇
6 ₈	$(y^2qysq^2)^7$	$2^8 \cdot 3^4 \cdot 7$	3 ₁	2 ₃	6 ₇	6 ₈	6 ₇	6 ₈
6 ₉	$(y^2sqsq^2)^3$	$2^5 \cdot 3^7$	3 ₄	2 ₁	6 ₉	6 ₉	6 ₉	6 ₉
6 ₁₀	$(yqs)^6$	$2^9 \cdot 3^4$	3 ₂	2 ₅	6 ₁₀	6 ₁₀	6 ₁₀	6 ₁₀
6 ₁₁	$(y^2qsy^2s)^3$	$2^9 \cdot 3^4$	3 ₂	2 ₄	6 ₁₁	6 ₁₁	6 ₁₁	6 ₁₁
6 ₁₂	$(qs)^2$	$2^{10} \cdot 3^3$	3 ₁	2 ₄	6 ₁₂	6 ₁₂	6 ₁₂	6 ₁₂
6 ₁₃	$(yqqyq^2)^4$	$2^{10} \cdot 3^3$	3 ₁	2 ₅	6 ₁₃	6 ₁₃	6 ₁₃	6 ₁₃
6 ₁₄	$(y^2sq^2)^3$	$2^5 \cdot 3^6$	3 ₂	2 ₃	6 ₁₄	6 ₁₄	6 ₁₄	6 ₁₄
6 ₁₅	y^3s	$2^5 \cdot 3^5$	3 ₃	2 ₃	6 ₁₅	6 ₁₅	6 ₁₅	6 ₁₅
6 ₁₆	y^7q	$2^5 \cdot 3^5$	3 ₃	2 ₂	6 ₁₆	6 ₁₆	6 ₁₆	6 ₁₆
6 ₁₇	$(ysq)^4$	$2^8 \cdot 3^3$	3 ₃	2 ₇	6 ₁₇	6 ₁₇	6 ₁₇	6 ₁₇
6 ₁₈	y^2qsys	$2^8 \cdot 3^3$	3 ₃	2 ₇	6 ₁₈	6 ₁₈	6 ₁₈	6 ₁₈
6 ₁₉	$(yqqsys)^2$	$2^8 \cdot 3^3$	3 ₂	2 ₇	6 ₁₉	6 ₁₉	6 ₁₉	6 ₁₉
6 ₂₀	$y^3sqy^2sq^2s$	$2^8 \cdot 3^3$	3 ₁	2 ₆	6 ₂₀	6 ₂₀	6 ₂₀	6 ₂₀
6 ₂₁	$y^2q^2sqsq^2yq$	$2^8 \cdot 3^3$	3 ₁	2 ₇	6 ₂₁	6 ₂₁	6 ₂₁	6 ₂₁
6 ₂₂	$(y^4sq)^2$	$2^7 \cdot 3^3$	3 ₃	2 ₅	6 ₂₂	6 ₂₂	6 ₂₂	6 ₂₂
6 ₂₃	$(y^3q^2ys)^2$	$2^7 \cdot 3^3$	3 ₃	2 ₄	6 ₂₃	6 ₂₃	6 ₂₃	6 ₂₃
6 ₂₄	$(y^3qs)^3$	$2^5 \cdot 3^4$	3 ₄	2 ₆	6 ₂₅	6 ₂₄	6 ₂₅	6 ₂₄
6 ₂₅	$(y^3sqqyq)^3$	$2^5 \cdot 3^4$	3 ₄	2 ₆	6 ₂₄	6 ₂₅	6 ₂₄	6 ₂₅
6 ₂₆	y^3q^2syq	$2^5 \cdot 3^4$	3 ₄	2 ₃	6 ₂₇	6 ₂₆	6 ₂₇	6 ₂₆
6 ₂₇	y^4q^2sqsys	$2^5 \cdot 3^4$	3 ₄	2 ₃	6 ₂₆	6 ₂₇	6 ₂₆	6 ₂₇
6 ₂₈	$(y^2qsq)^2$	$2^5 \cdot 3^3$	3 ₄	2 ₇	6 ₂₈	6 ₂₈	6 ₂₈	6 ₂₈

Conjugacy classes of $A_1 = \text{Aut}(2\text{Fi}_{22}) = \langle y, q, s \rangle$ (continued)

Class	Representative	Centralizer	2P	3P	5P	7P	11P	13P
6 ₂₉	$y^3 qsq$	$2^5 \cdot 3^3$	3 ₄	2 ₇	6 ₂₉	6 ₂₉	6 ₂₉	6 ₂₉
6 ₃₀	$y^4 qy^2 q$	$2^5 \cdot 3^3$	3 ₃	2 ₇	6 ₃₀	6 ₃₀	6 ₃₀	6 ₃₀
6 ₃₁	$y^3 sqsqys$	$2^5 \cdot 3^3$	3 ₃	2 ₆	6 ₃₁	6 ₃₁	6 ₃₁	6 ₃₁
7	$(y)^2$	$2^3 \cdot 3 \cdot 7$	7	7	7	1	7	7
8 ₁	$y^2 s$	$2^8 \cdot 3$	4 ₆	8 ₁	8 ₁	8 ₁	8 ₁	8 ₁
8 ₂	$(yqqy^2)^3$	$2^8 \cdot 3$	4 ₄	8 ₂	8 ₂	8 ₂	8 ₂	8 ₂
8 ₃	$(y^4 qs)^3$	$2^8 \cdot 3$	4 ₄	8 ₃	8 ₃	8 ₃	8 ₃	8 ₃
8 ₄	$(ysq^2 sq)^3$	$2^8 \cdot 3$	4 ₄	8 ₄	8 ₄	8 ₄	8 ₄	8 ₄
8 ₅	$y^4 qy^2 s$	$2^8 \cdot 3$	4 ₄	8 ₅	8 ₅	8 ₅	8 ₅	8 ₅
8 ₆	$(y^2 q)^2$	2^8	4 ₆	8 ₆	8 ₆	8 ₆	8 ₆	8 ₆
8 ₇	$(ysq)^3$	$2^6 \cdot 3$	4 ₈	8 ₇	8 ₇	8 ₇	8 ₇	8 ₇
8 ₈	$y^2 sqys$	2^6	4 ₉	8 ₈	8 ₈	8 ₈	8 ₈	8 ₈
9 ₁	$(yqs)^4$	$2^4 \cdot 3^4$	9 ₁	3 ₂	9 ₁	9 ₁	9 ₁	9 ₁
9 ₂	$(y^2 sq^2)^2$	$2^3 \cdot 3^4$	9 ₂	3 ₂	9 ₂	9 ₂	9 ₂	9 ₂
9 ₃	$(y^3 qs)^2$	$2^2 \cdot 3^3$	9 ₃	3 ₄	9 ₃	9 ₃	9 ₃	9 ₃
10 ₁	$(y^3 q^2)^3$	$2^5 \cdot 3 \cdot 5^2$	5	10 ₁	2 ₁	10 ₁	10 ₁	10 ₁
10 ₂	$(y^4 qysq)^3$	$2^4 \cdot 3 \cdot 5^2$	5	10 ₂	2 ₃	10 ₂	10 ₂	10 ₂
10 ₃	$(yq)^2$	$2^5 \cdot 5$	5	10 ₃	2 ₄	10 ₃	10 ₃	10 ₃
10 ₄	$(y^3 sq^2)^2$	$2^5 \cdot 5$	5	10 ₄	2 ₅	10 ₄	10 ₄	10 ₄
10 ₅	yq^2	$2^3 \cdot 3 \cdot 5$	5	10 ₅	2 ₂	10 ₅	10 ₅	10 ₅
10 ₆	$y^2 q^2 s$	$2^4 \cdot 5$	5	10 ₆	2 ₆	10 ₆	10 ₆	10 ₆
11	$(y^3 q)^2$	$2^2 \cdot 11$	11	11	11	11	1	11
12 ₁	$(yq^2 sq)^5$	$2^6 \cdot 3^3 \cdot 5$	6 ₁	4 ₁	12 ₁	12 ₁	12 ₁	12 ₁
12 ₂	$yq^2 ysqsq^2 ys$	$2^8 \cdot 3^3$	6 ₁₃	4 ₂	12 ₂	12 ₂	12 ₂	12 ₂
12 ₃	$(yqs)^3$	$2^5 \cdot 3^4$	6 ₁₀	4 ₂	12 ₃	12 ₃	12 ₃	12 ₃
12 ₄	$(y^4 qys)^3$	$2^5 \cdot 3^4$	6 ₂	4 ₁	12 ₄	12 ₄	12 ₄	12 ₄
12 ₅	$(yqqy^2)^2$	$2^8 \cdot 3^2$	6 ₁₃	4 ₄	12 ₅	12 ₅	12 ₅	12 ₅
12 ₆	$(y^4 qs)^2$	$2^8 \cdot 3^2$	6 ₁₃	4 ₄	12 ₆	12 ₆	12 ₆	12 ₆
12 ₇	$y^2 qsqysqys$	$2^6 \cdot 3^3$	6 ₁₀	4 ₄	12 ₇	12 ₇	12 ₇	12 ₇
12 ₈	$yq^2 sqysq^2$	$2^7 \cdot 3^2$	6 ₁₃	4 ₄	12 ₈	12 ₈	12 ₈	12 ₈
12 ₉	$y^2 sqsqsqyq$	$2^7 \cdot 3^2$	6 ₁₃	4 ₂	12 ₉	12 ₉	12 ₉	12 ₉
12 ₁₀	$y^4 qy^3 qys$	$2^5 \cdot 3^3$	6 ₂₂	4 ₂	12 ₁₀	12 ₁₀	12 ₁₀	12 ₁₀
12 ₁₁	$y^2 qy^2 qy^2 sq$	$2^8 \cdot 3$	6 ₁₃	4 ₃	12 ₁₁	12 ₁₁	12 ₁₁	12 ₁₁
12 ₁₂	qs	$2^6 \cdot 3^2$	6 ₁₂	4 ₇	12 ₁₂	12 ₁₂	12 ₁₂	12 ₁₂
12 ₁₃	$(ysq)^2$	$2^6 \cdot 3^2$	6 ₁₇	4 ₈	12 ₁₃	12 ₁₃	12 ₁₃	12 ₁₃
12 ₁₄	$y^2 sqsqyq$	$2^6 \cdot 3^2$	6 ₁₂	4 ₅	12 ₁₄	12 ₁₄	12 ₁₄	12 ₁₄
12 ₁₅	$y^3 sqsyq^2$	$2^6 \cdot 3^2$	6 ₁₇	4 ₉	12 ₁₅	12 ₁₅	12 ₁₅	12 ₁₅
12 ₁₆	$y^4 s$	$2^4 \cdot 3^3$	6 ₃	4 ₁	12 ₁₆	12 ₁₆	12 ₁₆	12 ₁₆
12 ₁₇	$yqysys$	$2^5 \cdot 3^2$	6 ₁₉	4 ₉	12 ₁₇	12 ₁₇	12 ₁₇	12 ₁₇
12 ₁₈	$y^4 sqysq$	$2^5 \cdot 3^2$	6 ₁₉	4 ₈	12 ₁₈	12 ₁₈	12 ₁₈	12 ₁₈
12 ₁₉	$y^2 sqsqsq$	$2^5 \cdot 3^2$	6 ₁₁	4 ₇	12 ₁₉	12 ₁₉	12 ₁₉	12 ₁₉
12 ₂₀	$(yqys)^2$	$2^6 \cdot 3$	6 ₁₀	4 ₆	12 ₂₀	12 ₂₀	12 ₂₀	12 ₂₀
12 ₂₁	$y^2 sy^2 sq$	$2^6 \cdot 3$	6 ₁₂	4 ₁₁	12 ₂₁	12 ₂₁	12 ₂₁	12 ₂₁
12 ₂₂	$y^3 q^2 ys$	$2^4 \cdot 3^2$	6 ₂₃	4 ₇	12 ₂₂	12 ₂₂	12 ₂₂	12 ₂₂
12 ₂₃	$y^2 sqysq$	$2^4 \cdot 3^2$	6 ₂₃	4 ₅	12 ₂₃	12 ₂₃	12 ₂₃	12 ₂₃
12 ₂₄	$yqysqys$	$2^4 \cdot 3^2$	6 ₂₈	4 ₉	12 ₂₄	12 ₂₄	12 ₂₄	12 ₂₄
12 ₂₅	$yqysyq^2 s$	$2^4 \cdot 3^2$	6 ₂₈	4 ₈	12 ₂₅	12 ₂₅	12 ₂₅	12 ₂₅
12 ₂₆	$y^4 sq$	$2^5 \cdot 3$	6 ₂₂	4 ₁₀	12 ₂₆	12 ₂₆	12 ₂₆	12 ₂₆
12 ₂₇	$y^2 qsq$	$2^4 \cdot 3$	6 ₂₈	4 ₁₂	12 ₂₈	12 ₂₇	12 ₂₈	12 ₂₇
12 ₂₈	$y^3 sqsq$	$2^4 \cdot 3$	6 ₂₈	4 ₁₂	12 ₂₇	12 ₂₈	12 ₂₇	12 ₂₈
13	$y^4 q$	$2 \cdot 13$	13	13	13	13	13	1
14 ₁	$(ys)^2$	$2^3 \cdot 3 \cdot 7$	7	14 ₁	14 ₁	2 ₁	14 ₁	14 ₁
14 ₂	$yqsq$	$2^2 \cdot 3 \cdot 7$	7	14 ₂	14 ₂	2 ₃	14 ₂	14 ₂

Conjugacy classes of $A_1 = \text{Aut}(2\text{Fi}_{22}) = \langle y, q, s \rangle$ (continued)

Class	Representative	Centralizer	2P	3P	5P	7P	11P	13P
14 ₃	y	$2^2 \cdot 7$	7	14 ₃	14 ₃	2 ₂	14 ₃	14 ₃
15	$(y^3 q^2)^2$	$2^3 \cdot 3 \cdot 5$	15	5	3 ₁	15	15	15
16 ₁	$y^2 q$	2^5	8 ₆	16 ₁	16 ₁	16 ₁	16 ₁	16 ₁
16 ₂	$y^2 qs$	2^5	8 ₆	16 ₂	16 ₂	16 ₂	16 ₂	16 ₂
18 ₁	$y^4 syqyqs$	$2^4 \cdot 3^4$	9 ₁	6 ₂	18 ₁	18 ₁	18 ₁	18 ₁
18 ₂	$(y^4 qys)^2$	$2^3 \cdot 3^4$	9 ₂	6 ₂	18 ₂	18 ₂	18 ₂	18 ₂
18 ₃	$y^4 qsq$	$2^3 \cdot 3^4$	9 ₁	6 ₁₄	18 ₆	18 ₃	18 ₆	18 ₃
18 ₄	$y^3 sq^2 s$	$2^3 \cdot 3^3$	9 ₁	6 ₆	18 ₅	18 ₄	18 ₅	18 ₄
18 ₅	$y^4 qsqs$	$2^3 \cdot 3^3$	9 ₁	6 ₆	18 ₄	18 ₅	18 ₄	18 ₅
18 ₆	$y^2 q^2 syq^2$	$2^3 \cdot 3^3$	9 ₁	6 ₁₄	18 ₃	18 ₆	18 ₃	18 ₆
18 ₇	$(yqs)^2$	$2^4 \cdot 3^2$	9 ₁	6 ₁₀	18 ₇	18 ₇	18 ₇	18 ₇
18 ₈	$y^2 qs y^2 s$	$2^4 \cdot 3^2$	9 ₁	6 ₁₁	18 ₈	18 ₈	18 ₈	18 ₈
18 ₉	$y^2 sq^2$	$2^2 \cdot 3^3$	9 ₂	6 ₁₄	18 ₉	18 ₉	18 ₉	18 ₉
18 ₁₀	$y^2 sqsq^2$	$2^2 \cdot 3^3$	9 ₃	6 ₉	18 ₁₀	18 ₁₀	18 ₁₀	18 ₁₀
18 ₁₁	$ysqysq^2$	$2^2 \cdot 3^3$	9 ₂	6 ₆	18 ₁₁	18 ₁₁	18 ₁₁	18 ₁₁
18 ₁₂	$y^3 qs$	$2^2 \cdot 3^2$	9 ₃	6 ₂₄	18 ₁₃	18 ₁₂	18 ₁₃	18 ₁₂
18 ₁₃	$y^3 sqyq$	$2^2 \cdot 3^2$	9 ₃	6 ₂₅	18 ₁₂	18 ₁₃	18 ₁₂	18 ₁₃
20 ₁	$yq^2 ys$	$2^3 \cdot 3 \cdot 5$	10 ₁	20 ₁	4 ₁	20 ₁	20 ₁	20 ₁
20 ₂	yq	$2^3 \cdot 5$	10 ₃	20 ₂	4 ₅	20 ₂	20 ₂	20 ₂
20 ₃	$y^3 sq^2$	$2^3 \cdot 5$	10 ₄	20 ₃	4 ₃	20 ₃	20 ₃	20 ₃
21	$(y^3 q^2 s)^2$	$2^2 \cdot 3 \cdot 7$	21	7	21	3 ₁	21	21
22 ₁	$y^3 q$	$2^2 \cdot 11$	11	22 ₁	22 ₁	22 ₃	2 ₂	22 ₃
22 ₂	$ysqs$	$2^2 \cdot 11$	11	22 ₂	22 ₂	22 ₂	2 ₁	22 ₂
22 ₃	$y^2 sqs$	$2^2 \cdot 11$	11	22 ₃	22 ₃	22 ₁	2 ₂	22 ₁
24 ₁	$yqyq^2$	$2^5 \cdot 3$	12 ₅	8 ₂	24 ₁	24 ₁	24 ₁	24 ₁
24 ₂	$y^4 qs$	$2^5 \cdot 3$	12 ₆	8 ₃	24 ₂	24 ₂	24 ₂	24 ₂
24 ₃	$ysq^2 sq$	$2^5 \cdot 3$	12 ₆	8 ₄	24 ₃	24 ₃	24 ₃	24 ₃
24 ₄	$y^4 qysyq$	$2^5 \cdot 3$	12 ₅	8 ₅	24 ₄	24 ₄	24 ₄	24 ₄
24 ₅	ysq	$2^4 \cdot 3$	12 ₁₃	8 ₇	24 ₇	24 ₇	24 ₅	24 ₅
24 ₆	$yqys$	$2^4 \cdot 3$	12 ₂₀	8 ₁	24 ₈	24 ₈	24 ₆	24 ₆
24 ₇	ysq^2	$2^4 \cdot 3$	12 ₁₃	8 ₇	24 ₅	24 ₅	24 ₇	24 ₇
24 ₈	$y^2 qys$	$2^4 \cdot 3$	12 ₂₀	8 ₁	24 ₆	24 ₆	24 ₈	24 ₈
26	$y^5 q$	$2 \cdot 13$	13	26	26	26	26	2 ₁
28	ys	$2^2 \cdot 7$	14 ₁	28	28	4 ₁	28	28
30 ₁	$y^3 q^2$	$2^3 \cdot 3 \cdot 5$	15	10 ₁	6 ₁	30 ₁	30 ₁	30 ₁
30 ₂	$yqsqs$	$2^2 \cdot 3 \cdot 5$	15	10 ₅	6 ₄	30 ₂	30 ₂	30 ₂
30 ₃	$y^4 qysq$	$2^2 \cdot 3 \cdot 5$	15	10 ₂	6 ₅	30 ₃	30 ₃	30 ₃
36 ₁	yqs	$2^3 \cdot 3^2$	18 ₇	12 ₃	36 ₁	36 ₁	36 ₁	36 ₁
36 ₂	$ysysqs$	$2^3 \cdot 3^2$	18 ₇	12 ₃	36 ₂	36 ₂	36 ₂	36 ₂
36 ₃	$y^4 qys$	$2^2 \cdot 3^2$	18 ₂	12 ₄	36 ₃	36 ₃	36 ₃	36 ₃
42 ₁	$y^3 q^2 s$	$2^2 \cdot 3 \cdot 7$	21	14 ₂	42 ₃	6 ₇	42 ₃	42 ₁
42 ₂	$y^3 sqs$	$2^2 \cdot 3 \cdot 7$	21	14 ₁	42 ₂	6 ₁	42 ₂	42 ₂
42 ₃	$y^2 qysq^2$	$2^2 \cdot 3 \cdot 7$	21	14 ₂	42 ₁	6 ₈	42 ₁	42 ₃
60	$yq^2 sq$	$2^2 \cdot 3 \cdot 5$	30 ₁	20 ₁	12 ₁	60	60	60

A.3. Conjugacy classes of $H(\text{Fi}'_{24}) = \langle r, p, b \rangle$

No.	Class	Representative	Centralizer	2P	3P	5P	7P
1	1	1	$2^{21} \cdot 3^7 \cdot 5^1 \cdot 7^1$	1	1	1	1
2	2 ₁	$z = (p^4 r^2)^6$	$2^{21} \cdot 3^7 \cdot 5^1 \cdot 7^1$	1	2 ₁	2 ₁	2 ₁
3	2 ₂	$(pr)^6$	$2^{19} \cdot 3^4 \cdot 5^1$	1	2 ₂	2 ₂	2 ₂
4	2 ₃	r^6	$2^{20} \cdot 3^2 \cdot 5^1$	1	2 ₃	2 ₃	2 ₃
5	2 ₄	$(p^2 br)^9$	$2^{14} \cdot 3^4 \cdot 5^1$	1	2 ₄	2 ₄	2 ₄
6	2 ₅	$z(p^2 br)^9$	$2^{14} \cdot 3^4 \cdot 5^1$	1	2 ₅	2 ₅	2 ₅
7	2 ₆	p^6	$2^{17} \cdot 3^3$	1	2 ₆	2 ₆	2 ₆
8	2 ₇	zp^6	$2^{17} \cdot 3^3$	1	2 ₇	2 ₇	2 ₇
9	2 ₈	$(rb^3)^4$	$2^{16} \cdot 3^1$	1	2 ₈	2 ₈	2 ₈
10	2 ₉	$(p^2 r)^3$	$2^{13} \cdot 3^2$	1	2 ₉	2 ₉	2 ₉
11	3 ₁	p^4	$2^8 \cdot 3^7 \cdot 5^1 \cdot 7^1$	3 ₁	1	3 ₁	3 ₁
12	3 ₂	$z(prpb^2)^4$	$2^{12} \cdot 3^6$	3 ₂	1	3 ₂	3 ₂
13	3 ₃	$z(pr^2 b)^4$	$2^{10} \cdot 3^7$	3 ₃	1	3 ₃	3 ₃
14	3 ₄	$z(pb^2)^3$	$2^5 \cdot 3^7$	3 ₄	1	3 ₄	3 ₄
15	3 ₅	b^2	$2^8 \cdot 3^5$	3 ₅	1	3 ₅	3 ₅
16	3 ₆	$(prbr)^2$	$2^5 \cdot 3^6$	3 ₆	1	3 ₆	3 ₆
17	3 ₇	r^4	$2^6 \cdot 3^4$	3 ₇	1	3 ₇	3 ₇
18	4 ₁	$(p^3 br)^6$	$2^{15} \cdot 3^5 \cdot 5^1$	2 ₁	4 ₁	4 ₁	4 ₁
19	4 ₂	$(rb)^4$	$2^{15} \cdot 3^2$	2 ₁	4 ₂	4 ₂	4 ₂
20	4 ₃	$(prb)^5$	$2^{13} \cdot 3^1 \cdot 5^1$	2 ₃	4 ₃	4 ₃	4 ₃
21	4 ₄	$(pr)^3$	$2^{11} \cdot 3^2 \cdot 5^1$	2 ₂	4 ₄	4 ₄	4 ₄
22	4 ₅	r^3	$2^{13} \cdot 3^2$	2 ₃	4 ₅	4 ₅	4 ₅
23	4 ₆	$(p^2 brb)^2$	$2^{14} \cdot 3^1$	2 ₃	4 ₆	4 ₆	4 ₆
24	4 ₇	p^3	$2^{11} \cdot 3^2$	2 ₆	4 ₇	4 ₇	4 ₇
25	4 ₈	$(p^4 rprb)^3$	$2^{11} \cdot 3^2$	2 ₂	4 ₈	4 ₈	4 ₈
26	4 ₉	zp^3	$2^{11} \cdot 3^2$	2 ₆	4 ₉	4 ₉	4 ₉
27	4 ₁₀	$(p^2 rpr^2)^2$	2^{14}	2 ₃	4 ₁₀	4 ₁₀	4 ₁₀
28	4 ₁₁	$(p^2 r^2 bprb)^3$	$2^{11} \cdot 3^1$	2 ₂	4 ₁₁	4 ₁₁	4 ₁₁
29	4 ₁₂	$p^4 r^3$	2^{12}	2 ₃	4 ₁₂	4 ₁₂	4 ₁₂
30	4 ₁₃	$z(p^2 b^3 r)^3$	$2^{10} \cdot 3^1$	2 ₈	4 ₁₃	4 ₁₃	4 ₁₃
31	4 ₁₄	$z(p^2 r b)^3$	$2^{10} \cdot 3^1$	2 ₆	4 ₁₄	4 ₁₄	4 ₁₄
32	4 ₁₅	$(p^2 b^3 r)^3$	$2^{10} \cdot 3^1$	2 ₈	4 ₁₅	4 ₁₅	4 ₁₅
33	4 ₁₆	$(p^2 b^3)^3$	$2^9 \cdot 3^1$	2 ₇	4 ₁₆	4 ₁₆	4 ₁₆
34	4 ₁₇	$(p^2 bprb)^3$	$2^9 \cdot 3^1$	2 ₇	4 ₁₇	4 ₁₇	4 ₁₇
35	4 ₁₈	$(rb^3)^2$	2^{10}	2 ₈	4 ₁₈	4 ₁₈	4 ₁₈
36	4 ₁₉	$p^2 r^3$	2^{10}	2 ₈	4 ₁₉	4 ₁₉	4 ₁₉
37	4 ₂₀	$prpr^3$	2^9	2 ₈	4 ₂₀	4 ₂₀	4 ₂₀
38	5	$(p^2 b)^6$	$2^6 \cdot 3^1 \cdot 5^1$	5	5	1	5
39	6 ₁	$(p^2 b)^5$	$2^8 \cdot 3^7 \cdot 5^1 \cdot 7^1$	3 ₁	2 ₁	6 ₁	6 ₁
40	6 ₂	$(prpb^2)^4$	$2^{12} \cdot 3^6$	3 ₂	2 ₁	6 ₂	6 ₂
41	6 ₃	$(p^2 brpr)^3$	$2^{10} \cdot 3^7$	3 ₃	2 ₁	6 ₃	6 ₃
42	6 ₄	$(pb^2)^3$	$2^5 \cdot 3^7$	3 ₄	2 ₁	6 ₄	6 ₄
43	6 ₅	$(p^4 r b^2 r)^2$	$2^8 \cdot 3^5$	3 ₅	2 ₁	6 ₅	6 ₅
44	6 ₆	$(prbpbr)^3$	$2^9 \cdot 3^4$	3 ₃	2 ₂	6 ₆	6 ₆
45	6 ₇	$(p^2 r^2 bprb)^2$	$2^{10} \cdot 3^3$	3 ₂	2 ₂	6 ₇	6 ₇
46	6 ₈	$(pr^2 b^2 pb^2)^2$	$2^5 \cdot 3^6$	3 ₆	2 ₁	6 ₈	6 ₈
47	6 ₉	$(p^4 bprpb)^2$	$2^{11} \cdot 3^2$	3 ₂	2 ₃	6 ₉	6 ₉
48	6 ₁₀	$p^2 r b p b r^2 b p r$	$2^8 \cdot 3^3$	3 ₂	2 ₆	6 ₁₀	6 ₁₀
49	6 ₁₁	$(p^2 b^3)^2$	$2^8 \cdot 3^3$	3 ₁	2 ₇	6 ₁₁	6 ₁₁
50	6 ₁₂	$z(p^3 b p b r b)$	$2^8 \cdot 3^3$	3 ₂	2 ₅	6 ₁₂	6 ₁₂
51	6 ₁₃	$p^3 b p b r b$	$2^8 \cdot 3^3$	3 ₂	2 ₄	6 ₁₃	6 ₁₃
52	6 ₁₄	p^2	$2^8 \cdot 3^3$	3 ₁	2 ₆	6 ₁₄	6 ₁₄
53	6 ₁₅	$(p^2 b^2 pb^2)^2$	$2^8 \cdot 3^3$	3 ₃	2 ₆	6 ₁₅	6 ₁₅
54	6 ₁₆	$z(p^2 b^2 pb^2)^2$	$2^8 \cdot 3^3$	3 ₃	2 ₇	6 ₁₆	6 ₁₆
55	6 ₁₇	b	$2^8 \cdot 3^3$	3 ₅	2 ₇	6 ₁₇	6 ₁₇

Conjugacy classes of $H(\text{Fi}'_{24}) = \langle r, p, b \rangle$ (continued)

No	Class	Representative	Centralizer	2P	3P	5P	7P
56	6 ₁₈	zb	$2^8 \cdot 3^3$	3 ₅	2 ₆	6 ₁₈	6 ₁₈
57	6 ₁₉	$z(p^2rbpbr^2bpr)$	$2^8 \cdot 3^3$	3 ₂	2 ₇	6 ₁₉	6 ₁₉
58	6 ₂₀	$(p^3br)^4$	$2^6 \cdot 3^4$	3 ₇	2 ₁	6 ₂₀	6 ₂₀
59	6 ₂₁	$(pr)^2$	$2^7 \cdot 3^3$	3 ₅	2 ₂	6 ₂₁	6 ₂₁
60	6 ₂₂	$(p^2br)^3$	$2^5 \cdot 3^4$	3 ₄	2 ₄	6 ₂₃	6 ₂₂
61	6 ₂₃	$(p^2br)^{15}$	$2^5 \cdot 3^4$	3 ₄	2 ₄	6 ₂₂	6 ₂₃
62	6 ₂₄	$z(p^2br)^{15}$	$2^5 \cdot 3^4$	3 ₄	2 ₅	6 ₂₅	6 ₂₄
63	6 ₂₅	$z(p^2br)^3$	$2^5 \cdot 3^4$	3 ₄	2 ₅	6 ₂₄	6 ₂₅
64	6 ₂₆	p^3r^2prpb	$2^7 \cdot 3^2$	3 ₂	2 ₉	6 ₂₆	6 ₂₆
65	6 ₂₇	p^5r	$2^5 \cdot 3^3$	3 ₅	2 ₄	6 ₂₇	6 ₂₇
66	6 ₂₈	$z(p^2rb)^2$	$2^5 \cdot 3^3$	3 ₄	2 ₇	6 ₂₈	6 ₂₈
67	6 ₂₉	p^4b	$2^5 \cdot 3^3$	3 ₅	2 ₇	6 ₂₉	6 ₂₉
68	6 ₃₀	$z(p^4b)$	$2^5 \cdot 3^3$	3 ₅	2 ₆	6 ₃₀	6 ₃₀
69	6 ₃₁	$(p^2r^2bpb)^2$	$2^5 \cdot 3^3$	3 ₄	2 ₆	6 ₃₁	6 ₃₁
70	6 ₃₂	$z(p^5r)$	$2^5 \cdot 3^3$	3 ₅	2 ₅	6 ₃₂	6 ₃₂
71	6 ₃₃	$prbr$	$2^5 \cdot 3^3$	3 ₆	2 ₇	6 ₃₃	6 ₃₃
72	6 ₃₄	$z(prbr)$	$2^5 \cdot 3^3$	3 ₆	2 ₆	6 ₃₄	6 ₃₄
73	6 ₃₅	$(p^2b^3r)^2$	$2^7 \cdot 3^1$	3 ₅	2 ₈	6 ₃₅	6 ₃₅
74	6 ₃₆	r^2	$2^5 \cdot 3^2$	3 ₇	2 ₃	6 ₃₆	6 ₃₆
75	6 ₃₇	p^2r	$2^3 \cdot 3^2$	3 ₇	2 ₉	6 ₃₇	6 ₃₇
76	7	$(pb)^3$	$2^1 \cdot 3^1 \cdot 7^1$	7	7	7	1
77	8 ₁	$(p^3br)^3$	$2^8 \cdot 3^2$	4 ₁	8 ₁	8 ₁	8 ₁
78	8 ₂	$(pr^2b)^3$	$2^9 \cdot 3^1$	4 ₂	8 ₂	8 ₂	8 ₂
79	8 ₃	$(prbpb^2)^3$	$2^8 \cdot 3^1$	4 ₂	8 ₃	8 ₃	8 ₃
80	8 ₄	p^2bpbpr	2^8	4 ₁₀	8 ₄	8 ₄	8 ₄
81	8 ₅	p^2rpr^2	2^8	4 ₁₀	8 ₅	8 ₅	8 ₅
82	8 ₆	p^2r^2brbr	2^8	4 ₆	8 ₆	8 ₆	8 ₆
83	8 ₇	p^3br^4	2^8	4 ₁₀	8 ₇	8 ₇	8 ₇
84	8 ₈	p^2brb	2^8	4 ₆	8 ₈	8 ₈	8 ₈
85	8 ₉	$(pr^2)^3$	$2^6 \cdot 3^1$	4 ₇	8 ₉	8 ₉	8 ₉
86	8 ₁₀	rb^3	2^6	4 ₁₈	8 ₁₀	8 ₁₀	8 ₁₀
87	8 ₁₁	p^2bpb	2^6	4 ₁₈	8 ₁₁	8 ₁₁	8 ₁₁
88	8 ₁₂	pr^3	2^6	4 ₉	8 ₁₂	8 ₁₂	8 ₁₂
89	9 ₁	$(prbpbpr)^2$	$2^5 \cdot 3^4$	9 ₁	3 ₃	9 ₁	9 ₁
90	9 ₂	$(p^2b^2rbr)^4$	$2^3 \cdot 3^4$	9 ₂	3 ₃	9 ₂	9 ₂
91	9 ₃	$(p^2brpr)^2$	$2^4 \cdot 3^4$	9 ₃	3 ₃	9 ₃	9 ₃
92	9 ₄	$(pb^2)^{10}$	$2^2 \cdot 3^3$	9 ₅	3 ₄	9 ₅	9 ₄
93	9 ₅	$(pb^2)^2$	$2^2 \cdot 3^3$	9 ₄	3 ₄	9 ₄	9 ₅
94	10 ₁	$(p^2b)^3$	$2^6 \cdot 3^1 \cdot 5^1$	5	10 ₁	2 ₁	10 ₁
95	10 ₂	$(prb)^6$	$2^5 \cdot 5^1$	5	10 ₃	2 ₃	10 ₃
96	10 ₃	$(prb)^2$	$2^5 \cdot 5^1$	5	10 ₂	2 ₃	10 ₂
97	10 ₄	$(p^3rb)^2$	$2^5 \cdot 5^1$	5	10 ₄	2 ₂	10 ₄
98	10 ₅	$pbrb$	$2^4 \cdot 5^1$	5	10 ₅	2 ₄	10 ₅
99	10 ₆	$z(pbrb)$	$2^4 \cdot 5^1$	5	10 ₆	2 ₅	10 ₆
100	12 ₁	$(prpb^2)^2$	$2^9 \cdot 3^5$	6 ₂	4 ₁	12 ₁	12 ₁
101	12 ₂	$(pbpb^5)^3$	$2^7 \cdot 3^5$	6 ₃	4 ₁	12 ₂	12 ₂
102	12 ₃	$(p^2rbrprb)^2$	$2^9 \cdot 3^3$	6 ₂	4 ₁	12 ₃	12 ₃
103	12 ₄	p^4rb^2r	$2^6 \cdot 3^4$	6 ₅	4 ₁	12 ₄	12 ₄
104	12 ₅	$pr^2b^2pb^2$	$2^4 \cdot 3^5$	6 ₈	4 ₁	12 ₅	12 ₅
105	12 ₆	$z(p^4bprpb)$	$2^8 \cdot 3^2$	6 ₉	4 ₅	12 ₆	12 ₆
106	12 ₇	p^4bprpb	$2^8 \cdot 3^2$	6 ₉	4 ₅	12 ₇	12 ₇
107	12 ₈	$(pr^2b)^2$	$2^7 \cdot 3^2$	6 ₃	4 ₂	12 ₈	12 ₈
108	12 ₉	$(p^3br)^2$	$2^5 \cdot 3^3$	6 ₂₀	4 ₁	12 ₉	12 ₉
109	12 ₁₀	$(pb^2rb)^2$	$2^5 \cdot 3^3$	6 ₂₀	4 ₁	12 ₁₀	12 ₁₀
110	12 ₁₁	$p^2rpr^2b^2$	$2^6 \cdot 3^2$	6 ₇	4 ₄	12 ₁₁	12 ₁₁
111	12 ₁₂	p	$2^6 \cdot 3^2$	6 ₁₄	4 ₇	12 ₁₂	12 ₁₂

Conjugacy classes of $H(\text{Fi}_{24}) = \langle r, p, b \rangle$ (continued)

No	Class	Representative	Centralizer	2P	3P	5P	7P
112	12 ₁₃	$p^4 r p r b$	$2^6 \cdot 3^2$	6 ₇	4 ₈	12 ₁₃	12 ₁₃
113	12 ₁₄	$(p r b p b^2)^2$	$2^6 \cdot 3^2$	6 ₅	4 ₂	12 ₁₄	12 ₁₄
114	12 ₁₅	$z p$	$2^6 \cdot 3^2$	6 ₁₄	4 ₉	12 ₁₅	12 ₁₅
115	12 ₁₆	$p^2 r^2 b p r$	$2^7 \cdot 3^1$	6 ₉	4 ₃	12 ₁₆	12 ₁₆
116	12 ₁₇	$p^2 r p^2 b r$	$2^7 \cdot 3^1$	6 ₉	4 ₆	12 ₁₇	12 ₁₇
117	12 ₁₈	$z(p^2 b^2 p b^2)$	$2^5 \cdot 3^2$	6 ₁₅	4 ₇	12 ₁₈	12 ₁₈
118	12 ₁₉	$p^2 r p^2 b^2 r$	$2^5 \cdot 3^2$	6 ₆	4 ₈	12 ₁₉	12 ₁₉
119	12 ₂₀	$p^2 b^2 p b^2$	$2^5 \cdot 3^2$	6 ₁₅	4 ₉	12 ₂₀	12 ₂₀
120	12 ₂₁	$p^2 r^2 b p r b$	$2^6 \cdot 3^1$	6 ₇	4 ₁₁	12 ₂₁	12 ₂₁
121	12 ₂₂	$p r$	$2^4 \cdot 3^2$	6 ₂₁	4 ₄	12 ₂₂	12 ₂₂
122	12 ₂₃	$p^2 b^2 p r^2$	$2^4 \cdot 3^2$	6 ₂₁	4 ₈	12 ₂₃	12 ₂₃
123	12 ₂₄	r	$2^4 \cdot 3^2$	6 ₃₆	4 ₅	12 ₂₄	12 ₂₄
124	12 ₂₅	$z(p^2 r^2 b p b)$	$2^4 \cdot 3^2$	6 ₃₁	4 ₇	12 ₂₅	12 ₂₅
125	12 ₂₆	$p^2 r^2 b p b$	$2^4 \cdot 3^2$	6 ₃₁	4 ₉	12 ₂₆	12 ₂₆
126	12 ₂₇	$z r$	$2^4 \cdot 3^2$	6 ₃₆	4 ₅	12 ₂₇	12 ₂₇
127	12 ₂₈	$p^5 r b r$	$2^4 \cdot 3^2$	6 ₈	4 ₂	12 ₂₈	12 ₂₈
128	12 ₂₉	$p^2 b^3 r$	$2^5 \cdot 3^1$	6 ₃₅	4 ₁₅	12 ₂₉	12 ₂₉
129	12 ₃₀	$z(p^2 b^3 r)$	$2^5 \cdot 3^1$	6 ₃₅	4 ₁₃	12 ₃₀	12 ₃₀
130	12 ₃₁	$p^2 b p r b$	$2^4 \cdot 3^1$	6 ₁₇	4 ₁₇	12 ₃₁	12 ₃₁
131	12 ₃₂	$z(p^2 r b)^5$	$2^4 \cdot 3^1$	6 ₃₁	4 ₁₄	12 ₃₂	12 ₃₂
132	12 ₃₃	$p^2 b^3$	$2^4 \cdot 3^1$	6 ₁₁	4 ₁₆	12 ₃₃	12 ₃₃
133	12 ₃₄	$z(p^2 r b)$	$2^4 \cdot 3^1$	6 ₃₁	4 ₁₄	12 ₃₂	12 ₃₄
134	14	$z(p b)^3$	$2^1 \cdot 3^1 \cdot 7^1$	7	14	14	2 ₁
135	15	$(p^2 b)^2$	$2^1 \cdot 3^1 \cdot 5^1$	15	5	3 ₁	15
136	16	$r b$	2^5	8 ₂	16	16	16
137	18 ₁	$(p b p b^5)^2$	$2^5 \cdot 3^4$	9 ₁	6 ₃	18 ₁	18 ₁
138	18 ₂	$(p^2 b^2 r b r)^2$	$2^3 \cdot 3^4$	9 ₂	6 ₃	18 ₂	18 ₂
139	18 ₃	$p^2 b r p r$	$2^1 \cdot 3^4$	9 ₃	6 ₃	18 ₃	18 ₃
140	18 ₄	$p r b p b r$	$2^4 \cdot 3^2$	9 ₁	6 ₆	18 ₄	18 ₄
141	18 ₅	$p b^2$	$2^2 \cdot 3^3$	9 ₅	6 ₄	18 ₆	18 ₅
142	18 ₆	$(p b^2)^5$	$2^2 \cdot 3^3$	9 ₄	6 ₄	18 ₅	18 ₆
143	18 ₇	$p^2 b r$	$2^2 \cdot 3^2$	9 ₅	6 ₂₂	18 ₉	18 ₇
144	18 ₈	$z(p^2 b r)^5$	$2^2 \cdot 3^2$	9 ₄	6 ₂₄	18 ₁₀	18 ₈
145	18 ₉	$(p^2 b r)^5$	$2^2 \cdot 3^2$	9 ₄	6 ₂₃	18 ₇	18 ₉
146	18 ₁₀	$z(p^2 b r)$	$2^2 \cdot 3^2$	9 ₅	6 ₂₅	18 ₈	18 ₁₀
147	20 ₁	$p r b r b$	$2^4 \cdot 5^1$	10 ₁	20 ₁	4 ₁	20 ₁
148	20 ₂	$(p r b)^3$	$2^3 \cdot 5^1$	10 ₂	20 ₄	4 ₃	20 ₄
149	20 ₃	$p^3 r b$	$2^3 \cdot 5^1$	10 ₄	20 ₃	4 ₄	20 ₃
150	20 ₄	$p r b$	$2^3 \cdot 5^1$	10 ₃	20 ₂	4 ₃	20 ₂
151	21 ₁	$p b$	$2^1 \cdot 3^1 \cdot 7^1$	21 ₂	7	21 ₁	3 ₁
152	21 ₂	$(p b)^2$	$2^1 \cdot 3^1 \cdot 7^1$	21 ₁	7	21 ₂	3 ₁
153	24 ₁	$p^2 r b r p r b$	$2^5 \cdot 3^2$	12 ₃	8 ₁	24 ₁	24 ₁
154	24 ₂	$p r p b^2$	$2^5 \cdot 3^2$	12 ₁	8 ₁	24 ₂	24 ₂
155	24 ₃	$p b^2 r b$	$2^3 \cdot 3^2$	12 ₁₀	8 ₁	24 ₃	24 ₃
156	24 ₄	$p^3 b r$	$2^3 \cdot 3^2$	12 ₉	8 ₁	24 ₄	24 ₄
157	24 ₅	$(p r^2)^5$	$2^4 \cdot 3^1$	12 ₁₂	8 ₉	24 ₉	24 ₉
158	24 ₆	$p r b p b^2$	$2^4 \cdot 3^1$	12 ₁₄	8 ₃	24 ₇	24 ₇
159	24 ₇	$(p r b p b^2)^5$	$2^4 \cdot 3^1$	12 ₁₄	8 ₃	24 ₆	24 ₆
160	24 ₈	$p r^2 b$	$2^4 \cdot 3^1$	12 ₈	8 ₂	24 ₈	24 ₈
161	24 ₉	$p r^2$	$2^4 \cdot 3^1$	12 ₁₂	8 ₉	24 ₅	24 ₅
162	30	$p^2 b$	$2^1 \cdot 3^1 \cdot 5^1$	15	10 ₁	6 ₁	30
163	36 ₁	$p b p b^5$	$2^4 \cdot 3^3$	18 ₁	12 ₂	36 ₁	36 ₁
164	36 ₂	$p^3 r^2 b$	$2^4 \cdot 3^3$	18 ₁	12 ₂	36 ₂	36 ₂
165	36 ₃	$p^2 b^2 r b r$	$2^2 \cdot 3^3$	18 ₂	12 ₂	36 ₃	36 ₃
166	42 ₁	$z(p b)^2$	$2^1 \cdot 3^1 \cdot 7^1$	21 ₁	14	42 ₁	6 ₁
167	42 ₂	$z(p b)$	$2^1 \cdot 3^1 \cdot 7^1$	21 ₂	14	42 ₂	6 ₁

Character table of $E(Fi'_{24})$ (continued)

2	5	8	8	8	6	6	6	4	3	3	8	8	8	9	8	7	6	6	5	4	4	4	1	6	6	5	5
3	1	3	2	2	2	1	1	1	1	1	1	1	1	.	.	7	6	6	5	4	4	4	1	6	6	5	5
5	1	1	1	1	1	1	1	1	.	1	1	1	1
7	1	1	1	1	1	1	.	1	1	1	1
11	1
23
	5a	6a	6b	6c	6d	6e	6f	6g	7a	7b	8a	8b	8c	8d	8e	8f	8g	8h	10a	10b	10c	10d	11a	12a	12b	12c	12d
2P	5a	3a	3a	3a	3b	3a	3a	3b	7a	7b	4d	4a	4b	4d	4c	4f	4i	4j	5a	5a	5a	5a	11a	6b	6b	6d	6d
3P	5a	2b	2b	2a	2b	2c	2d	2e	7b	7a	8a	8b	8c	8d	8e	8f	8g	8h	10a	10b	10d	10c	11a	4c	4d	4a	4b
5P	1a	6a	6b	6c	6d	6e	6f	6g	7b	7a	8a	8b	8c	8d	8e	8f	8g	8h	2b	2a	2e	2e	11a	12a	12b	12c	12d
7P	5a	6a	6b	6c	6d	6e	6f	6g	1a	1a	8a	8b	8c	8d	8e	8f	8g	8h	10a	10b	10d	10c	11a	12a	12b	12c	12d
11P	5a	6a	6b	6c	6d	6e	6f	6g	7a	7b	8a	8b	8c	8d	8e	8f	8g	8h	10a	10b	10c	10d	1a	12a	12b	12c	12d
23P	5a	6a	6b	6c	6d	6e	6f	6g	7a	7b	8a	8b	8c	8d	8e	8f	8g	8h	10a	10b	10d	10c	11a	12a	12b	12c	12d
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.2	3	5	5	5	-1	1	1	-1	2	-1	-1	-1	1	3	-1	1	1	3	3	-1	-1	1	1	1	-1	-1	
X.3	3	.	.	-1	A	A	-3	1	1	1	1	-1	-1	1	.	.	-1	-1	
X.4	3	.	.	-1	A	A	-3	1	1	1	1	-1	-1	1	.	.	-1	-1	
X.5	1	-3	-3	-3	1	1	1	.	-1	3	3	-1	-1	3	-1	-1	1	1	1	1	1	1	1	1	1	1	
X.6	1	-3	-3	-3	1	1	1	.	-1	3	3	-1	-1	3	-1	-1	1	1	1	1	1	1	1	1	1	1	
X.7	2	9	9	9	1	1	1	.	4	.	4	4	4	.	.	.	2	2	2	2	2	-1	1	1	1	1	
X.8	3	10	10	10	-2	-2	1	1	1	1	1	1	1	1	1	-1	-1	3	3	-1	-1	-2	-2	1	1	1	
X.9	-2	6	6	6	2	2	.	.	3	3	3	3	3	3	3	-1	-1	-2	-2	-2	-2	-1	2	2	2	2	
X.10	4	9	-3	1	5	1	3	3	-1	3	3	-1	-1	1	1	-4	1	1	1	1	
X.11	.	5	5	5	-7	1	1	1	.	2	-2	-2	-2	-2	-2	1	1	1	1	
X.12	.	5	5	5	-7	1	1	1	.	2	-2	-2	-2	-2	-2	1	1	1	1	
X.13	3	.	-1	A	A	6	-2	-2	2	2	-2	-1	-1	
X.14	3	.	-1	A	A	6	-2	-2	2	2	-2	-1	-1	
X.15	6	.	2	-1	-1	3	3	3	-1	-1	3	1	1	1	.	.	2	2	
X.16	-3	.	.	1	D	D	3	-1	-1	3	3	-1	-1	1	.	.	1	1	
X.17	-3	.	.	1	D	D	3	-1	-1	3	3	-1	-1	1	.	.	1	1	
X.18	.	5	5	5	8	1	1	.	-2	-2	-7	-3	-3	1	1	-3	1	1	1	1	1	1	
X.19	3	-10	2	-2	-1	2	-2	-1	.	.	-4	4	.	.	-2	2	3	-1	-1	-1	-1	1	-2	2	3	-1	
X.20	3	-10	2	-2	-1	2	-2	-1	.	.	-4	4	.	.	-2	2	3	-1	-1	-1	-1	1	-2	2	3	-1	
X.21	1	16	16	16	7	.	-1	.	.	3	-1	-1	-5	-5	-1	-1	-1	1	1	1	1	.	-1	-1	-1	-1	
X.22	-1	-1	-1	-1	8	1	-1	1	1	8	-1	-1	-1	-1	.	-1	-1	-1	-1	
X.23	-3	.	.	.	6	.	2	2	2	-3	-3	-3	1	1	-3	-1	-1	-3	-3	1	1	
X.24	-3	.	.	.	-6	.	-2	1	1	-3	-3	1	1	
X.25	10	10	10	-8	-2	-2	.	-1	-1	-3	-3	1	1	
X.26	3	-15	-15	-15	.	1	1	.	.	1	-3	-3	-3	-3	-3	-1	-1	3	3	-1	-1	
X.27	8	-9	-9	-9	7	-1	-1	.	.	1	-3	-3	5	-3	1	1	1	-8	
X.28	-1	9	9	9	1	1	1	.	.	-8	-1	-1	-1	-1	
X.29	1	-9	-9	-9	-1	-1	.	.	.	-4	.	4	4	.	.	.	1	1	1	1	-1	-1	-1	-1	-1	-1	
X.30	3	3	3	-1	-1	3	1	1	
X.31	-4	36	.	-4	.	4	.	.	-6	6	6	-2	-2	-2	.	.	4	
X.32	.	45	-3	-3	.	1	1	.	3	3	1	-3	-3	-7	1	1	-1	-1	-3	-3	.	
X.33	.	-45	-9	11	.	3	-1	.	-3	-3	-4	.	.	4	-4	-1	-5	.	
X.34	4	-36	-12	12	.	-4	4	.	3	3	3	-1	-1	-1	-1	-1	-1	-4	-4	4	.	
X.35	4	18	6	-6	.	2	-2	.	3	3	3	-1	-1	-1	-1	-1	-1	-4	2	-2	.	
X.36	4	18	6	-6	.	2	-2	.	3	3	3	-1	-1	-1	-1	-1	-1	-4	2	-2	.	
X.37	-8	9	-3	1	1	-3	.	.	.	4	8	5	1	.	
X.38	.	-12	8	.	4	.	.	.	-3	-3	-3	-3	5	1	-1	-1	4	.	.	.	
X.39	6	-40	8	-8	2	.	2	6	-2	2	2	.	.	.	-2	-2	
X.40	E	E	3	3	3	-1	-1	-1	-1	
X.41	E	E	3	3	3	-5	3	-1	1	1	
X.42	E	E	3	3	3	-5	3	-1	1	1	
X.43	E	E	3	3	3	-1	-1	-1	-1	
X.44	6	40	-8	8	-2	.	-2	6	-2	2	2	-1	.	.	2	2	
X.45	4	9	9	-7	-3	-3	.	.	-8	-4	1	1	.	
X.46	-4	-54	-6	10	-2	-2	.	3	3	4	-2	-2	.	
X.47	.	45	9	-11	.	1	5	.	.	2	-6	-6	6	-2	2	-3	1	.	
X.48	.	.	.	-3	.	1	.	.	.	4	-4	.	.	.	2	-2	1	.	1	-3	
X.49	.	.	.	-3	.	1	.	.	.	4	-4	.	.	.	2	-2	1	.	-3	1	
X.50	-3	-3	6	-6	-6	-6	2	2	
X.51	-3	-3	1	F	G	-1	
X.52	-3	-3	1	G	F	-1	
X.53	-10	2	-2	-1	2	-2	-1	.	.	.	4	-4	2	-2	-2	2	-1	3
X.54	-10	2	-2	-1	2	-2	-1	.	.	.	4	-4	2	-2	-2	2	-1	3
X.55	45	-3	-3	-3	-3	.	.	.	-1	3	3	-1	-1	-1	1	1	1	1	.	
X.56	45	-3	-3	-3	-3	.	.	.	7	3	3	7	-1	-1	1	1	-3	-3	.	
X.57	45	-3	-3	-3	-3	.	.	.	-1	3	3	3	3	-1	-1	-1	1	1	.	
X.58	3	-30	6	-6	-2	2	3	-1	-1	-1	.	.	2	-2	.	
X.59	3	-30	6	-6	-2	2	3	-1	-1	-1	.	.	2	-2	.	
X.60	-45	3	3	-1	-1	.	3	3	8	3	3	.	
X.61	-3	.	.	-3	.	1	.	.	-4	4	.	.	.	-2	2	-3	1	1	1	1	.	.	.	-3	1	.	
X.62	-3	.	.	-3	.	1	.	.	-4	4	.	.	.	-2	2	-3	1	1	1	1	.	.	.	-3	1	.	
X.63	-20	4	-4	-2	-4	4	-2	4	-4	2	2
X.64	-30	6	-6	.	2	-2	-1	-2	2	2
X.65	-30	6	-6	.	2	-2	-1	-2	2	2
X.66	-45	3	3	.	3	3	.	.	-2	6	6	2	2	-2	-1	-1	.	
X.67	-3	.	.	.	3	.	-1	.	.	-8	8	-3	1	1	1	1	.	.	3	-1	
X																											

Character table of $E(\text{Fi}'_{24})$ (continued)

2	4	4	4	3	3	2	2	1	1	5	4	4	.	.	1	.	.	4	4	3	3	2	2	1	1
3	1	1	1	1	1	.	.	.	1	1	.	.	.	1	1	1	1	.	.	1	1
5
7	.	.	.	1	1	1	1	1	1	1	1	.	.
11	1
23	1	1
2P	6c	6e	6e	7b	7a	7a	7b	15a	15b	8d	10a	10a	21a	21b	11a	23a	23b	12b	12b	12c	12d	14a	14b	15a	15b
3P	4e	4g	4h	14b	14a	14d	14c	5a	5a	16a	20a	20b	7a	7b	22a	23a	23b	8a	8a	8b	8c	28b	28a	10a	10a
5P	12e	12f	12g	14b	14a	14d	14c	3a	3a	16a	4b	4a	21b	21a	22a	23b	23a	24b	24a	24c	24d	28b	28a	6a	6a
7P	12e	12f	12g	2a	2a	2c	2c	15b	15a	16a	20a	20b	3b	3b	22a	23b	23a	24b	24a	24c	24d	4e	4e	30b	30a
11P	12e	12f	12g	14a	14b	14c	14d	15b	15a	16a	20a	20b	21a	21b	2a	23b	23a	24a	24b	24c	24d	28a	28b	30b	30a
23P	12e	12f	12g	14a	14b	14c	14d	15a	15b	16a	20a	20b	21a	21b	22a	1a	1a	24a	24b	24c	24d	28a	28b	30a	30b
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	-1	-1	2	2	1	-1	-1	-1	-1	1	1	1	-1	-1	-1	-1	1	1	1	1
X.3	.	.	.	A	A	-A	-A	.	.	-1	.	.	A	A	1	-1	-1	.	.	1	1	-A	-A	.	.
X.4	.	.	.	A	A	-A	-A	.	.	-1	.	.	A	A	1	-1	-1	.	.	1	1	-A	-A	.	.
X.5	1	-1	-1	B	B	-1	1	1	.	.	1	1	-1	-1	B	B	B
X.6	1	-1	-1	B	B	-1	1	1	.	.	1	1	-1	-1	B	B	B
X.7	1	1	1	-1	-1	2	2	.	.	.	-1	-1	-1	1	1	.	.	.	-1	-1	-1
X.8	-2	.	.	1	1	-1	-1	.	.	-1	-1	-1	1	1	1	1	-1	-1	.	.	.
X.9	2	1	1	-1	-2	-2	.	.	-1	1	1	1
X.10	-1	1	1	-1	-1	1	1	1	1	-1	-1	-1	.	.	-1	-1	-1	-1
X.11	1	-1	-1	C	C	-1	-1	1	1
X.12	1	-1	-1	C	C	-1	-1	1	1
X.13	1	-1	-1	A	A	A	A	A	A	1	1	.	.	1	1	A	A	.	.
X.14	.	.	.	A	A	A	A	A	A	1	1	.	.	1	1	A	A	.	.
X.15	.	.	.	-1	-1	-1	-1	.	.	1	.	.	-1	-1	1	1	-1	-1	.	.	.
X.16	.	.	.	D	D	-1	.	.	-A	-A	1	.	.	.	-1	-1
X.17	.	.	.	D	D	-1	.	.	-A	-A	1	.	.	.	-1	-1
X.18	1	-1	-1	-2	-2	1	.	.	1	1	.	.	.	-1	-1
X.19	.	2	-2	-1	3	.	.	.	-1	.	.	.	-1	-1
X.20	.	-2	2	3	-1	.	.	.	-1	.	.	.	-1	-1
X.21	1	1	-1	1	1	-1	-1	.	.	1	1	1	1
X.22	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	1	1	.	.	.	-1	-1	.	.	1	1	-1	-1
X.23	.	.	.	2	2	-1	1	1	-1	-1
X.24	.	.	.	1	1	-1	-1	.	.	1	1	1	1	1	1	-1	-1	.	.
X.25	-2	.	.	-1	-1	1	1	-1	-1	.	.	.	1	1	.	.	1	1	.	.
X.26	1	1	1	-1	-1	-1	-1	-1	1	1	1	1
X.27	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	.	.	1	1
X.28	1	1	1	-1	-1	-1	-1	-1	1	1	1	1	.	.	-1	-1
X.29	-1	-1	-1	1	1	.	1	1	.	.	-1	.	.	-1	-1	-1	-1	.	.	1	1
X.30	1	1	-1	-1
X.31	-2	-1	-1	1	1
X.32	1	-1	-1	-1	-1	-1	-1	.	.	1	1	1	.	.	1	1	.	.
X.33	1	1	1	1	1	-1	-1	-1	-1	.	.	1	1	.	.
X.34	1	1	1	1	1	-1	-1	-1
X.35	B	B	1	1	1	-B	-B	-B
X.36	-1	-1	-1	B	B	1	1	1	-B	-B	-B
X.37	1	1	1	1	-1	-1
X.38	-2	1	1
X.39	-2	-2
X.40	.	.	.	-A	-A	-A	-A	.	.	1	A	A	.
X.41	.	.	.	-A	-A	A	A	.	.	-1	-A	-A	.
X.42	.	.	.	-A	-A	A	A	.	.	-1	-A	-A	.
X.43	.	.	.	-A	-A	-A	-A	.	.	1	A	A	.
X.44	-2	-2	.	.	.	1
X.45	1	-1	-1	1	1	1	1	.	.	.	-1	-1	-1
X.46	2	.	.	-1	-1	1	1	-1	-1	-1	-1	1	1
X.47	-1	1	1	-1	.	.	-1	-1
X.48	-1	1	1	.	.
X.49	-1	-1	-1	.	.
X.50	.	.	.	1	1	1	1	-1	-1	.	.
X.51	1	1	.	.	.	1
X.52	1	1	.	.	.	1
X.53	.	2	-2	1	-1	.	.
X.54	.	-2	2	-1	1	.	.
X.55	1	1	1	-1	-1	-1
X.56	1	-1	-1	-1	1	1
X.57	1	1	1	1	-1	-1
X.58	.	2	-2	3	-1
X.59	-2	-1	3
X.60	-1	1	1	-1	-1	-1	-1	-1	-1	.	.	1	1	.	.
X.61	1	-3	-1	1	.	.
X.62	-3	1	1	-1	.	.
X.63
X.64	1	.	.	H	-H
X.65	1	.	.	-H	H
X.66	-1	-1	-1	1	1
X.67	1	-3	.	.	.	-1	1	-1	.	.
X.68	-3	1	.	.	.	-1	-1	1	.	.
X.69	-1	3	1	-1	.	.
X.70	3	-1	-1	1	.	.
X.71	1
X.72	-1

where $A = \zeta(7)^4 + \zeta(7)^2 + \zeta(7)$, $B = -2\zeta(15)_3\zeta(15)_5^3 - 2\zeta(15)_3\zeta(15)_5^2 - \zeta(15)_3 - \zeta(15)_5^3 - \zeta(15)_5^2 - 1$, $C = -\zeta(23)^{18} - \zeta(23)^{16} - \zeta(23)^{13} - \zeta(23)^{12} - \zeta(23)^9 - \zeta(23)^8 - \zeta(23)^6 - \zeta(23)^4 - \zeta(23)^3 - \zeta(23)^2 - \zeta(23) - 1$, $D = -2\zeta(7)^4 - 2\zeta(7)^2 - 2\zeta(7) - 2$, $E = -3\zeta(7)^4 - 3\zeta(7)^2 - 3\zeta(7) - 3$, $F = -4\zeta(5)^3 - 4\zeta(5)^2 - 3$, $G = 4\zeta(5)^3 + 4\zeta(5)^2 + 1$, $H = -4\zeta(12)_4\zeta(12)_3 - 2\zeta(12)_4$.

B.2. Character table of $mH = \langle r, u, v \rangle$

	2	13	10	11	13	10	8	7	7	6	4	4	4	2	3	9	10	8
3	13	9	6	5	5	13	10	10	7	11	10	9	9	7	4	4	3	
5	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	1a	2a	2b	2c	2d	3a	3b	3c	3d	3e	3f	3g	3h	3i	4a	4b	4c	4d
2P	1a	1a	1a	1a	1a	3a	3b	3c	3d	3e	3f	3g	3h	3i	2b	2c	2b	
3P	1a	2a	2b	2c	2d	1a	1a	1a	1a	1a	1a	1a	1a	1a	4a	4b	4c	
5P	1a	2a	2b	2c	2d	3a	3b	3c	3d	3e	3f	3g	3h	3i	4a	4b	4c	
7P	1a	2a	2b	2c	2d	3a	3b	3c	3d	3e	3f	3g	3h	3i	4a	4b	4c	
13P	1a	2a	2b	2c	2d	3a	3b	3c	3d	3e	3f	3g	3h	3i	4a	4b	4c	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	-1	1	1	-1	1	1	1	1	1	1	1	1	1	-1	1	-1	
X.3	2		2	2		2	2	2	-1	2	2	2	2	-1		2		
X.4	300	-90	20	12	-10	57	30	30	27	3	3	-6	12		-10	8	-6	
X.5	300	90	20	12	10	57	30	30	27	3	3	-6	12		10	8	6	
X.6	600		40	24		114	60	60	-27	6	6	-12	24			16		
X.7	780	78	76	12	14	51	-3	24		51	-3	24	-3		6	12	6	
X.8	780	-104	76	12	-8	51	24	-3		-30	24	24	-3		-16	12	-8	
X.9	780	104	76	12	8	51	24	-3		-30	24	24	-3		16	12	8	
X.10	780	-78	76	12	-14	51	-3	24		51	-3	24	-3		-6	12	-6	
X.11	2275	-455	35	-29	25	88	115	-20	91	7	34	7	7	10	-15	3	7	
X.12	2275	455	35	-29	-25	88	115	-20	91	7	34	7	7	10	15	3	7	
X.13	2457	-273	49	57	-33	270	108	108		27	27	27	27		-29	21	7	
X.14	2457	273	49	57	33	270	108	108		27	27	27	27		29	21	-7	
X.15	2808	468	120	-8	-12	-108	54	54	78	-27	-27	27		-3	12	8	20	
X.16	2808	-468	120	-8	12	-108	54	54	78	-27	-27	27		-3	-12	8	-20	
X.17	4550		70	-58		176	230	-40	-91	14	68	14	14	-10		6		
X.18	5616		240	-16		-216	108	108	-78	-54	-54	54		3		16		
X.19	6825	-455	105	-87	25	264	75	210		102	75	21	21		-15	9	-7	
X.20	6825	455	105	-87	-25	264	75	210		102	75	21	21		15	9	7	
X.21	9450	1260	210	74	60	-27	135	135	168	54	54	27		6	-18	28		
X.22	9450	-1260	210	74	-60	-27	135	135	168	54	54	27		6	-18	-28		
X.23	16380	546	-84	60	2	-387	99	342		18	18	45	-9		-6	36	-14	
X.24	16380	1092	-84	60	36	-387	342	99		18	18	45	-9		44	36	-28	
X.25	16380	-546	-84	60	-2	-387	99	342		18	18	45	-9		6	36	14	
X.26	16380	-1092	-84	60	-36	-387	342	99		18	18	45	-9		-44	36	28	
X.27	17550	1170	-90	46	-30	783	135	135	78	-27	-27		27	-3	30	26	-6	
X.28	17550	-1170	-90	46	30	783	135	135	78	-27	-27		27	-3	-30	26	6	
X.29	18200	1820	280	24	60	461	110	110	182	-79	-79	-7	11	-7	20	8	28	
X.30	18200	-1820	280	24	-60	-461	110	110	182	-79	-79	-7	11	-7	-20	8	-28	
X.31	18200	-1820	280	24	-60	461	110	110	182	-79	-79	-7	11	-7	-20	8	-28	
X.32	18200	1820	280	24	60	-461	110	110	182	-79	-79	-7	11	-7	20	8	28	
X.33	18900		420	148		-54	270	270	-168	108	108	54		-6		-36		
X.34	24192	2016		128	96	864	216	216	168	54	54		27	6				
X.35	24192	-2016		128	-96	864	216	216	168	54	54		27	6				
X.36	27300	2730	140	-124	-70	813	300	300	273	30	30	21	39	3	10		14	
X.37	27300	-2730	140	-124	70	813	300	300	273	30	30	21	39	3	-10		-14	
X.38	35100		-180	92		1566	270	270	-78	-54	-54		54	3		52		
X.39	36400		560	48		-92	220	-320	-182	274	-104	58	4	-20		16		
X.40	36400		560	48		922	220	220	-182	-158	-158	-14	22	7		16		
X.41	48384			256		1728	432	432	-168	108	108		54	-6				
X.42	54600	-5460	280	8	-20	-561	600	-210	546	60	-21	-39	-3	6	-60	16	-28	
X.43	54600	-1820	840	72	-60	-75	210	60		-156	33	87	6		-20	24	-28	
X.44	54600		280	-248		1626	600	600	-273	60	60	42	78	-3				
X.45	54600	1820	840	72	60	-75	210	60		-156	33	87	6		20	24	-28	
X.46	54600	5460	280	8	20	-561	600	-210	546	60	-21	-39	-3	6	60	16	28	
X.47	70200	1560	680	120	120	216	-270	540		216	54	-27	-27			48	8	
X.48	70200	-1560	680	120	-120	216	-270	540		-216	54	-27	-27			48	-8	
X.49	70200	3900	680	120	60	216	540	-270		-27	135	-27	-27		100	48	20	
X.50	70200	-3900	680	120	-60	216	540	-270		-27	135	-27	-27		-100	48	-20	
X.51	87360	4368	448	-192	-48	-120	636	96		204	96	42	-39		80			
X.52	87360	-4368	448	-192	48	-120	636	96		204	96	42	-39					
X.53	87360	2912	448	-192	-32	-120	96	636		204	96	42	-39					
X.54	87360	-2912	448	-192	32	-120	96	636		204	96	42	-39					
X.55	109200	-4368	448	-192	48	-120	636	96		42	150	42	-39		-80			
X.56	122850	5460	-70	-126	-60	2565	945	135		-108	54	-27	54		80	30	-28	
X.57	122850	-5460	-70	-126	60	2565	945	135		108	54	-27	54		80	30	28	
X.58	122850	2730	-70	-126	-90	2565	135	945		135	-27	-27	54		-30	30	14	
X.59	122850	-2730	-70	-126	90	2565	135	945		-108	54	-27	54		-30	30	-14	
X.60	139776	-5824	896		-64	-192	456	456		-192	-192	132	-30		-64			
X.61	139776	5824	896		64	-192	456	456		192	192	132	-30		64			
X.62	147420	-1638	924	-36	186	891	-567	-324		405	-81	81			6	12	-14	
X.63	147420	1638	924	-36	-186	891	-567	-324		-405	81	81			-6	12	14	
X.64	147420	-3276	924	-36	-12	891	-324	-567		-324	162	81			36	12	-28	
X.65	147420	3276	924	-36	12	891	-324	-567		-324	162	81			-36	12	28	
X.66	163800	-5460	840	24	-20	2691	180	180		-63	-63	-117	-9		-60	48	-28	
X.67	163800	5460	840	24	20	-2691	180	180		63	63	-117	-9		60	48	28	
X.68	163800	-5460	840	24	-20	2691	180	180		-63	-63	-117	-9		-60	48	-28	
X.69	163800	5460	840	24	20	-2691	180	180		63	63	-117	-9		60	48	28	
X.70	184275	-12285	315	51	-45	567	810	-405	819	-162	81			9	15	-25	-21	
X.71	184275	12285	315	51	45	567	810	-405	819	-162	81			9	-15	-25	21	
X.72	199017	7371	441	297	171	2187	729	729							51	-63	-21	
X.73	199017	-7371	441	297	-171	2187	729	729							-51	63	21	
X.74	218700	-2430	-540	108	-270	2187			27					27	90		6	
X.75	218700	2430	-540	108	270	2187			-27					-27	-90		-6	
X.76	245700	-8190	-420	-28	210	2943	270	270	273	27	27	27	27	30	90	-8	14	
X.77	245700	8190	-420	-28	-210	2943	270	270	273	27	27	27	27	30	-90	-8	-14	
X.78	291200	-14560		-128	160	-2344	680	680	728	-22	-22	-40	-4	26				
X.79	291200	14560		128	-160	2344	680	680	728	-22	-22	-40	-4	26				
X.80	291200	-14560		-128	-160	1544	680											

Character table of mH (continued)

	2	10	7	9	8	4	6	5	7	7	7	8	4	4	6	6	6	7	7	3	6	4
	3	2	2	2	2	2	9	7	7	5	6	5	7	7	5	5	5	4	4	6	4	5
	5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		4d	4e	4f	4g	5a	61	62	63	64	65	66	67	68	69	610	611	612	613	614	615	616
2P		2c	2b	2b	2c	5a	3a	3b	3c	3c	3a	3a	3e	3f	3a	3b	3c	3b	3c	3g	3c	3e
3P		4d	4e	4f	4g	5a	2a	2a	2a	2d	2b	2c	2a	2a	2d	2b	2c	2c	2c	2a	2d	2c
5P		4d	4e	4f	4g	1a	61	62	63	64	65	66	67	68	69	610	611	612	613	614	615	616
7P		4d	4e	4f	4g	5a	61	62	63	64	65	66	67	68	69	610	611	612	613	614	615	616
13P		4d	4e	4f	4g	5a	61	62	63	64	65	66	67	68	69	610	611	612	613	614	615	616
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.3	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
X.4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.7	12	4	6	2	5	15	6	14	5	3	3	3	5	13	4	3	3	3	3	6	2	3
X.8	12	4	6	2	5	15	6	14	5	3	3	3	5	13	4	3	3	3	3	6	2	3
X.9	12	4	6	2	5	15	6	14	5	3	3	3	5	13	4	3	3	3	3	6	2	3
X.10	12	4	6	2	5	15	6	14	5	3	3	3	5	13	4	3	3	3	3	6	2	3
X.11	3	5	1	1	5	10	5	14	10	8	8	23	4	2	1	8	5	4	5	2	7	7
X.12	3	5	1	1	5	10	5	14	10	8	8	23	4	2	1	8	5	4	5	2	7	7
X.13	3	5	1	1	5	10	5	14	10	8	8	23	4	2	1	8	5	4	5	2	7	7
X.14	3	5	1	1	5	10	5	14	10	8	8	23	4	2	1	8	5	4	5	2	7	7
X.15	8	4	4	4	8	18	36	18	6	12	4	9	9	6	6	6	6	6	6	6	6	6
X.16	8	4	4	4	8	18	36	18	6	12	4	9	9	6	6	6	6	6	6	6	6	6
X.17	6	10	8	8	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.18	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.19	9	15	1	1	5	10	5	14	10	24	24	4	23	2	15	6	3	6	5	2	6	6
X.20	9	15	1	1	5	10	5	14	10	24	24	4	23	2	15	6	3	6	5	2	6	6
X.21	6	10	8	8	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.22	6	10	8	8	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.23	12	4	10	6	5	39	30	33	9	3	3	12	12	9	6	15	6	21	3	3	6	6
X.24	12	4	10	6	5	39	30	33	9	3	3	12	12	9	6	15	6	21	3	3	6	6
X.25	12	4	10	6	5	39	30	33	9	3	3	12	12	9	6	15	6	21	3	3	6	6
X.26	12	4	10	6	5	39	30	33	9	3	3	12	12	9	6	15	6	21	3	3	6	6
X.27	2	10	6	10	4	45	45	63	15	9	31	9	9	21	9	7	7	7	7	9	9	9
X.28	2	10	6	10	4	45	45	63	15	9	31	9	9	21	9	7	7	7	7	9	9	9
X.29	8	4	4	4	8	20	56	37	3	11	11	3	10	10	6	6	6	6	6	6	6	6
X.30	8	4	4	4	8	20	56	37	3	11	11	3	10	10	6	6	6	6	6	6	6	6
X.31	8	4	4	4	8	20	56	37	3	11	11	3	10	10	6	6	6	6	6	6	6	6
X.32	8	4	4	4	8	20	56	37	3	11	11	3	10	10	6	6	6	6	6	6	6	6
X.33	12	20	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.34	12	20	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.35	12	20	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.36	8	14	6	6	8	30	84	20	5	19	30	30	7	4	4	4	4	4	4	4	4	4
X.37	8	14	6	6	8	30	84	20	5	19	30	30	7	4	4	4	4	4	4	4	4	4
X.38	4	20	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.39	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.40	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.41	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.42	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.43	24	4	4	4	43	20	2	30	51	27	16	11	3	6	12	6	12	11	6	4	4	4
X.44	16	4	4	4	43	20	2	30	51	27	16	11	3	6	12	6	12	11	6	4	4	4
X.45	24	4	4	4	43	20	2	30	51	27	16	11	3	6	12	6	12	11	6	4	4	4
X.46	16	4	4	4	43	20	2	30	51	27	16	11	3	6	12	6	12	11	6	4	4	4
X.47	16	4	4	4	43	20	2	30	51	27	16	11	3	6	12	6	12	11	6	4	4	4
X.48	16	4	4	4	43	20	2	30	51	27	16	11	3	6	12	6	12	11	6	4	4	4
X.49	16	4	4	4	43	20	2	30	51	27	16	11	3	6	12	6	12	11	6	4	4	4
X.50	16	4	4	4	43	20	2	30	51	27	16	11	3	6	12	6	12	11	6	4	4	4
X.51	16	4	4	4	43	20	2	30	51	27	16	11	3	6	12	6	12	11	6	4	4	4
X.52	16	4	4	4	43	20	2	30	51	27	16	11	3	6	12	6	12	11	6	4	4	4
X.53	16	4	4	4	43	20	2	30	51	27	16	11	3	6	12	6	12	11	6	4	4	4
X.54	16	4	4	4	43	20	2	30	51	27	16	11	3	6	12	6	12	11	6	4	4	4
X.55	16	4	4	4	43	20	2	30	51	27	16	11	3	6	12	6	12	11	6	4	4	4
X.56	6	10	8	8	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.57	6	10	8	8	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.58	6	10	8	8	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.59	6	10	8	8	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.60	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.61	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
X.62	12	4	10	2	5	63	45	18	6	21	27	9	9	15	3	24	15	12	9	6	9	9
X.63	12	4	10	2	5	63	45	18	6	21	27	9	9	15	3	24	15	12	9	6	9	9
X.64	12	4	10	2	5	63	45	18	6	21	27	9	9	15	3	24	15	12	9	6	9	9
X.65	12	4	10	2	5	63	45	18	6	21	27	9	9	15	3	24	15	12	9	6	9	9
X.66	12	4	10	2	5	63	45	18	6	21	27	9	9	15	3	24	15	12	9	6	9	9
X.67	12	4	10	2	5	63	45	18	6	21	27	9	9	15	3	24	15	12	9			

2	4	4	4	2	3	6	4	4	4	2	3	2	3	3	6	6	6	6	3	3	1	3	1	1	6
3	5	5	5	6	4	3	4	4	4	5	4	4	3	1	1	1	1	1	6	6	7	5	6	6	6
5
7
13
2P	6 ₁₇	6 ₁₈	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂	6 ₂₃	6 ₂₄	6 ₂₅	6 ₂₆	6 ₂₇	6 ₂₈	6 ₂₉	7 _a	8 _a	8 _b	8 _c	8 _d	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f	9 _g
3P	3 _g	3 _e	3 _f	3 _h	3 _b	3 _d	3 _g	3 _f	3 _g	3 _h	3 _g	3 _h	3 _c	7 _a	4 _b	4 _d	4 _b	4 _d	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f	9 _g
5P	2 _c	2 _d	2 _d	2 _c	2 _d	2 _c	2 _c	2 _b	2 _c	2 _c	2 _d	2 _d	2 _c	7 _a	8 _a	8 _b	8 _c	8 _d	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f	9 _g
7P	6 ₁₇	6 ₁₈	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂	6 ₂₃	6 ₂₄	6 ₂₅	6 ₂₆	6 ₂₇	6 ₂₈	6 ₂₉	7 _a	8 _a	8 _b	8 _c	8 _d	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f	9 _g
13P	6 ₁₇	6 ₁₈	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂	6 ₂₃	6 ₂₄	6 ₂₅	6 ₂₆	6 ₂₇	6 ₂₈	6 ₂₉	7 _a	8 _a	8 _b	8 _c	8 _d	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f	9 _g
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	-1	1	1	1	1	1	1	1
X.3	2	-1	2	2	2	2	2	.	-1	2	.	2	2	.	2	2	2	-1	2	-1	-1
X.4	.	-1	-1	.	-4	3	2	3	.	.	2	-4	.	-1	.	-2	2	.	3	3	3	3	3	3	6
X.5	.	1	1	.	4	3	2	3	.	.	-2	4	.	-1	.	-2	2	.	3	3	3	3	3	3	6
X.6	-3	4	6	-2	.	-4	4	.	6	6	6	-3	6	-6	3
X.7	12	5	5	-3	-1	4	4	-3	.	3	2	-1	.	3	2	.	.	2	6	-3	-3
X.8	-6	4	4	-5	-2	.	4	.	6	3	2	-1	.	3	.	.	.	-3	6	-3	
X.9	12	5	5	-3	-1	4	4	-3	.	3	2	-1	.	3	.	.	.	-3	6	-3	
X.10	12	-5	3	.	3	1	4	-3	.	3	2	-1	.	3	-2	.	-2	3	6	-3	
X.11	1	1	-8	-5	7	-5	-1	-2	1	1	1	1	-2	.	1	-1	-1	1	-2	7	7	10	-2	1	1
X.12	1	-1	8	-5	-7	-5	-1	-2	1	1	-1	-1	-2	.	-1	-1	-1	-1	-2	7	7	10	-2	1	1
X.13	3	-3	-3	-3	-3	6	-5	3	3	3	-3	-3	.	-3	3	3	3	1	9	9	.	.	.		

Character table of mH (continued)

	2	1	1	1	4	4	4	7	5	7	5	5	5	5	3	4	5	5	5	5	5	3	4	4	3	3
	3	5	5	4	1	1	1	3	4	2	3	3	3	3	4	3	2	2	2	2	2	3	2	2	2	2
	5
	7
	13
		9h	9i	9j	10a	10b	10c	12a	12b	12c	12d	12e	12f	12g	12h	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	12s
2P	9h	9i	9j	5a	5a	5a	6 ₆	6 ₅	6 ₆	6 ₁₂	6 ₁₃	6 ₁₀	6 ₅	6 ₁₆	6 ₁₁	6 ₂₂	6 ₂₂	6 ₁₀	6 ₆	6 ₁₃	6 ₂₄	6 ₁₀	6 ₁₁	6 ₁₆	6 ₂₃	
3P	3e	3e	3f	10a	10b	10c	4b	4a	4d	4b	4b	4a	4c	4b	4a	4d	4b	4f	4g	4g	4b	4e	4e	4g	4c	
5P	9h	9i	9j	2d	2a	2b	12a	12b	12c	12d	12e	12f	12g	12h	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	12s	
7P	9h	9i	9j	10a	10b	10c	12a	12b	12c	12d	12e	12f	12g	12h	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	12s	
13P	9h	9i	9j	10a	10b	10c	12a	12b	12c	12d	12e	12f	12g	12h	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	12s	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.2	1	1	1	-1	-1	1	1	1	1	1	1	1	-1	-1	1	-1	1	1	-1	-1	-1	1	1	1	-1	
X.3	-1	2	2	.	.	2	2	2	2	2	2	2	.	2	.	-1	-1	.	.	.	2	2	2	.	.	
X.4	5	-1	-3	2	2	2	2	3	-1	-4	3	-1	-2	-1	2	-1	.	.	-1	
X.5	5	1	-3	2	2	-2	-3	-1	4	3	-1	2	1	-2	-1	.	.	1	.	
X.6	10	.	-6	4	4	.	.	-2	.	-3	1	.	.	.	-2	
X.7	.	6	.	-1	3	1	3	-3	3	-3	.	3	-3	3	.	.	3	1	-2	-3	1	-2	1	.	.	
X.8	.	-3	3	-3	1	1	3	-7	3	.	-3	-4	1	-6	-1	.	.	-3	3	.	-2	1	.	-2	.	
X.9	.	-3	3	3	-1	1	3	7	3	.	-3	-4	-1	-6	1	.	.	-3	-3	.	1	-2	-1	.	.	
X.10	.	6	.	1	-3	1	3	3	3	-3	.	-3	3	3	.	.	-3	1	-2	-3	1	-2	-1	.	.	
X.11	1	-2	1	-6	3	.	.	-3	2	3	.	3	3	-1	-2	-2	.	1	-2	-1	-1	
X.12	1	-2	1	6	3	.	.	-3	2	3	.	3	3	-1	-2	-2	.	1	-2	-1	-1	
X.13	.	.	.	-3	-3	-1	6	-2	6	.	.	-2	-2	3	-2	.	.	-2	-2	-2	3	.	1	1	1	
X.14	.	.	.	3	3	-1	6	2	6	.	.	2	2	3	2	.	.	2	2	2	3	.	-1	-1	-1	
X.15	6	.	.	-2	-2	.	-4	-6	-4	2	2	6	2	-1	.	2	2	2	2	-1	.	.	.	-1	-1	
X.16	6	.	.	2	2	.	-4	6	-4	2	2	-6	-2	-1	.	2	2	-2	-2	-2	-1	.	1	1	1	
X.17	-1	-4	2	6	6	.	-3	-3	.	.	.	2	-4	.	.	.	
X.18	-6	-8	-8	4	4	.	-2	.	-2	-2	-2	
X.19	.	3	-6	3	6	-3	2	1	-2	-2	3	-3	.	-2	-1	-1	
X.20	.	3	6	3	6	3	-2	-1	2	2	3	-3	.	2	1	1	
X.21	6	-3	9	-3	3	3	3	1	.	-3	.	-1	-3	3	.	1	1	.	1	1	
X.22	6	-3	9	-3	3	3	-3	-1	.	3	.	1	3	-3	.	1	1	.	-1	-1	
X.23	.	.	.	-3	1	1	-3	3	-3	3	-6	-3	-5	.	.	.	1	3	.	.	1	-2	.	1	1	
X.24	.	.	.	1	-3	1	-3	-1	-3	-6	3	2	-1	.	5	.	2	-1	-1	.	-2	1	2	-1	-1	
X.25	.	.	.	3	-1	1	-3	-3	3	-3	-6	3	5	.	.	.	-1	-3	.	.	1	-2	.	-1	-1	
X.26	.	.	.	-1	3	1	-3	1	-3	-6	3	-2	1	.	-5	.	-2	1	1	.	-2	1	-2	1	1	
X.27	-3	-1	3	-1	-1	-1	3	3	-1	3	2	2	3	-1	-1	-1	-1	-1	-1	-1	
X.28	-3	-1	3	-1	-1	-1	3	3	-1	3	2	2	3	1	1	-1	-1	-1	-1	-1	
X.29	2	-1	-1	.	.	.	5	-7	5	2	2	2	1	-1	2	2	2	-2	1	4	-1	.	1	1	1	
X.30	2	-1	-1	.	.	.	5	-7	5	2	2	-2	-1	-1	-2	2	2	2	-1	2	2	.	-1	-1	-1	
X.31	2	-1	-1	.	.	.	5	-7	5	2	2	-2	-1	-1	-2	2	2	2	-1	-4	-1	.	-1	-1	-1	
X.32	2	-1	-1	.	.	.	-1	-7	-1	2	-4	2	1	-1	2	2	-2	1	-2	2	.	1	1	1	1	
X.33	-6	-6	-6	6	6	2	2	.	.	.	
X.34	-3	.	.	-4	-4	
X.35	-3	.	.	4	4	
X.36	3	9	1	1	.	-2	5	.	4	1	-3	-2	-3	-1	-1	
X.37	3	9	-1	1	.	2	-5	.	-4	1	-3	2	3	1	1	
X.38	3	-2	-2	-2	-2	.	.	-2	.	-2	-2	.	.	.	-2	-2	-2	.	.	.	
X.39	-2	4	-2	.	.	.	-2	.	2	4	-8	.	.	-2	-2	-2	.	.	.	4	
X.40	-2	-2	-2	.	.	.	10	.	10	4	4	.	.	-2	-2	-2	.	.	.	-2	
X.41	3	
X.42	-3	-5	3	3	4	-2	6	-1	-2	.	6	-2	-2	-1	2	1	.	2	-1	-1	
X.43	-3	-18	7	-3	-6	.	-2	-1	6	-2	.	2	-1	2	3	.	2	-1	.	.	
X.44	-3	18	
X.45	-3	-7	-3	-6	.	2	1	6	2	.	-1	3	
X.46	-3	-5	-3	3	4	-2	-6	1	-2	.	6	-2	2	1	-2	1	.	-2	1	1	
X.47	-6	.	.	8	-6	
X.48	10	.	-6	-2	2	3	4	.	.	.	-2	2	2	-3	.	-1	-1	-1	-1	
X.49	-10	.	-6	-2	-2	3	-4	.	.	.	2	-2	-2	-3	.	1	1	1	1	
X.50	-6	.	.	-8	-6	.	.	.	-4	
X.51	.	6	.	2	-2	-2	.	8	.	.	.	2	.	.	-4	.	2	
X.52	-3	3	2	-2	-2	
X.53	-3	3	2	-2	-2	
X.54	.	6	.	-2	-2	-2	.	-8	.	.	.	-2	.	.	4	.	-2	
X.55	3	-10	.	6	8	-4	.	.	-4	.	-6	2	.	.	.	2	
X.56	-3	-1	-3	-3	3	-1	-1	-6	-1	.	-1	3	-3	.	1	1	.	-1	-1	
X.57	-3	3	-3	3	-3	3	-5	3	3	.	-1	-1	-1	-3	1	1	-1	1	1	
X.58	-3	-3	-3	3	-3	-3	5	3	-3	.	1	1	1	-3	1	1	1	-1	-1	
X.59	-3	1	-3	-3	3	1	1	-6	1	.	1	-3	3	.	1	1	.	1	1	
X.60	-3	-3	1	1	1	.	.	8	.	.	.	-4	.	.	-2	-2	-2	.	.	.	
X.61	-3	-3	-1	-1	1	.	.	-8	.	.	.	4	-2	-2	.	.	.	
X.62	.	.	1	-3	-1	3	.	-3	3	-3	.	-3	-5	3	.	.	1	-1	2	-3	.	-1	-2	-1	-1	
X.63	.	.	1	3	-1	3	.	-3	3	-3	.	-3	5	3	.	.	-1	1	-2	-3	.	-1	2	1	-1	
X.64	.	.	3	-1	-1	3	.	-3	9	3	.	-6	-1	-6	-3	.	-2	3	3	.	2	-1	.	-1	-1	
X.65	.	.	-3	1	-1	3	.	3	9	3	.	6	1	-6	3	.	-2	-3	-3	.	2	-1	.	1	1	
X.66	15	3	-9	.	.	6	-1	3	.	.	-2	-1	-4	3	.	.	-1	-1	-1	
X.67	-15	-3	9	.	6	-6	1	3	.	.	2	1	-2	.	.	.	1	1	1	
X.68	15	-3	-9	.	.	-6	1	3	.	.	2	1	4	3	.	.	.	1	1	
X.69	-15	3	9	.	6	6	-1	3	.	.	-2	-1	2	.	.	.	-1	-1	-1	
X.70	-1	-3	-1	2	-1	.	-3	2	3	-1	-1	.	1	1	-1	-2	1	-2	.	
X.71	-1	-3	-1	2	-1	.	3	2	-3	-1	-1	.	-1	-1	-1	-2	1	2	.	
X.72	.	.	.	1	1	1	3	-3	3	-3	-3	-3	-3	.	.											

Character table of mH (continued)

	2i	3	3	2	1	1	3	2	1	1	3	2	3	3	3	3	1	1	1	1	1	1	3	3	.	3
	3	2	2	2	1	1	.	2	2	2	4	4	3	3	3	3	3	3	3	3	3	3	3	3	1	1
	5	1	1	1	1	.
	7	1	1	1	1	.
13	1	1	1	1	.
	12i	12u	12v	13a	13b	14a	14b	15a	15b	18a	18b	18c	18d	18e	18f	18g	18h	18i	18j	18k	18l	20a	20b	21a	24a	
2P	6 ₂₄	6 ₂₃	6 ₂₉	13b	13a	7a	7a	15a	15b	9b	9a	9d	9a	9b	9b	9h	9e	9f	9i	9j	9c	10c	10c	21a	12d	
3P	4g	4f	4d	13a	13b	14a	14b	5a	5a	6 ₁	6 ₁	6 ₁₆	6 ₅	6 ₉	6 ₅	6 ₁₆	6 ₉	6 ₁₆	6 ₇	6 ₈	6 ₉	20a	20b	7a	8a	
5P	12i	12u	12v	13b	13a	14a	14b	3c	3b	18a	18b	18c	18d	18e	18f	18g	18h	18i	18j	18k	18l	4a	4e	21a	24a	
7P	12i	12u	12v	13b	13a	2b	2a	15a	15b	18a	18b	18c	18d	18e	18f	18g	18h	18i	18j	18k	18l	20a	20b	3d	24a	
13P	12i	12u	12v	1a	1a	14a	14b	15a	15b	18a	18b	18c	18d	18e	18f	18g	18h	18i	18j	18k	18l	20a	20b	21a	24a	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.2	-1	-1	1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	-1	1	-1	-1	-1	1	1	1	-1	
X.3	.	.	-1	2	2	2	.	2	2	.	.	-1	2	.	2	-1	.	-1	.	.	.	2	-1	.	.	
X.4	-1	-2	.	1	1	-1	1	.	.	3	-3	3	-1	-1	-1	.	-1	.	.	.	-1	
X.5	1	2	.	1	1	-1	-1	.	.	-3	3	3	-1	1	-1	.	1	.	.	.	1	.	-1	.	.	
X.6	.	.	.	2	2	-2	.	.	.	-3	-2	.	-2	.	-2	1	.	.	
X.7	1	.	.	-1	1	-1	2	3	.	-2	-1	1	.	1	.	-1	.	.	.	-1	1	-1	.	-1	.	
X.8	.	.	.	-1	1	2	-1	-2	1	.	1	-2	-2	.	-2	.	-1	.	1	1	1	-1	-1	.	.	
X.9	.	.	.	-1	-1	2	-1	-2	-1	.	1	2	-2	.	-2	.	-1	.	-1	-1	-1	1	-1	.	.	
X.10	-1	-1	-2	-2	1	1	.	1	1	-1	-1	.	.	
X.11	-2	1	-2	-2	-2	-2	-1	-1	1	-2	1	-2	1	1	
X.12	2	-1	-1	2	-2	2	-1	-1	1	2	1	2	-1	-1	.	.	.	-1	
X.13	1	1	-2	-2	-3	-3	.	1	-3	1	1	-1	.	.	.	
X.14	-1	-1	-2	-2	3	3	.	1	3	1	-1	-1	.	.	.	
X.15	-1	-1	-1	.	.	1	-1	-1	-1	.	1	.	.	.	-2	.	1	2	.	1	.	
X.16	1	1	-1	.	.	1	1	-1	-1	.	1	.	.	.	-2	.	1	.	.	.	-2	.	1	.	.	
X.17	2	4	.	-2	-1	-1	
X.18	.	.	1	.	.	2	.	-2	-2	.	-1	.	.	.	2	.	-1	-1	.	.	
X.19	1	1	1	-2	.	.	.	1	3	.	1	.	1	-2	-2	.	.	1	.	
X.20	-1	-1	-1	2	.	.	-1	3	.	-1	.	-1	2	2	.	.	.	-1	.	
X.21	.	-1	.	-1	-1	2	.	.	.	2	.	-1	1	.	
X.22	.	1	.	-1	-1	2	.	.	.	2	.	-1	-1	.	
X.23	.	1	2	-1	3	.	.	-1	3	.	-1	.	.	.	-1	-1	-1	.	1	.	
X.24	2	-1	-1	2	-3	.	.	3	-1	-1	.	.	.	
X.25	.	-1	2	-1	-3	.	.	1	3	.	1	.	.	.	1	1	-1	.	-1	.	
X.26	-2	1	-1	2	3	.	.	3	1	-1	.	.	.	
X.27	-1	.	-1	.	.	1	1	.	.	.	1	1	.	-2	1	-1	.	
X.28	1	.	-1	.	.	1	-1	1	.	-2	1	1	.	
X.29	1	1	-1	5	-1	.	1	-3	1	.	.	.	-1	-1	
X.30	2	-1	2	4	1	-3	1	.	-2	.	.	.	-2	1	
X.31	-1	-1	-1	-5	1	.	1	3	1	.	.	.	1	1	
X.32	-2	1	2	-4	-1	-3	1	.	-2	.	.	.	2	-1	
X.33	.	.	-2	-2	-2	-2	.	1	
X.34	.	.	-1	-1	.	.	.	1	1	.	2	.	.	.	-1	.	2	
X.35	.	.	-1	-1	.	.	.	1	1	.	2	.	.	.	-1	.	2	
X.36	.	1	1	3	3	2	-1	-1	-1	-1	-1	-1	.	.	-1	
X.37	-1	1	-3	-3	2	-1	1	-1	-1	-1	-1	.	.	1	
X.38	.	1	.	.	.	2	.	.	.	-1	-1	.	2	-1	.	.	.	
X.39	.	-2	3	2	.	-4	.	2	
X.40	.	1	2	.	2	.	1	.	-2	
X.41	.	.	-2	-2	.	.	.	2	2	.	-2	.	.	.	1	1	-1	1	-1	.	-2	
X.42	-1	1	-3	1	-2	1	1	-1	1	-1	1	-1	.	.	-2	
X.43	-1	-1	4	1	-2	.	-3	.	-2	1	.	1	.	1	-2	
X.44	.	.	-1	-4	-1	-2	-3	.	-2	1	.	1	
X.45	1	1	-3	.	-1	-2	-1	1	-1	-1	-1	.	-1	2	
X.46	1	-1	3	.	-1	-2	-1	1	-1	-1	-1	.	.	2	
X.47	1	1	.	.	.	1	-1	.	-3	.	-1	-2	-3	-1	
X.48	-1	1	.	.	.	1	1	.	-3	.	-1	.	2	
X.49	1	-1	.	.	.	1	-1	.	3	.	-1	.	2	
X.50	.	-1	.	.	.	1	1	.	3	.	2	3	-1	
X.51	.	2	1	1	-3	.	1	.	-2	.	.	.	1	1	-1	
X.52	1	1	-5	4	-2	-1	1	.	-1	.	1	1	-1	
X.53	1	1	5	-4	-2	1	1	.	1	.	-1	-1	1	
X.54	-2	1	1	3	.	1	.	-2	
X.55	1	-4	.	2	1	.	1	
X.56	.	-1	3	.	-1	.	2	1	.	
X.57	-1	-1	-3	.	2	-3	-1	1	.	
X.58	1	1	3	.	2	3	-1	-1	.	
X.59	.	1	-3	.	-1	-1	.	
X.60	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	.	.		
X.61	1	1	1	1	-1	1	-1	1	1	1	1	1	1	-1	1	.	.	.	
X.62	-1	1	1	-2	-1	1	.	-1	
X.63	1	1	-2	1	1	1	.	1	
X.64	1	1	-2	1	1	1	.	1	
X.65	.	-1	-2	1	-1	1	.	.	
X.66	-1	1	6	.	.	-2	.	.	.	1	1	
X.67	-2	-1	3	.	.	-1	-3	.	2	-1	
X.68	1	-1	-6	.	.	2	.	.	.	-1	.	.	.	-1	
X.69	2	1	-3	.	.	1	-3	.	-2	1	
X.70	1	.	-1	-3	
X.71	-1	.	-1	-3	
X.72	-1	-1	1	1	.	-1	
X.73	-1	-1	-1	1	.	1	
X.74	.	.	-1	1	1	-1	-1	-1	.	
X.75	.	.	-1	1	1	-1	1	-1	.	
X.76	1	-1	-1	-1	.	-1	
X.77	-1	1	-1	-1	.	-1	
X.78	.	.	.																							

Character table of mH (continued)

2	3	3	1	1	.	2	1	1	2	2	.	.
3	1	1	.	.	3	.	1	1	2	2	1	1
5	1	1
7	1
13	.	.	1	1	1
<hr/>												
	24b	24c	26a	26b	27a	28a	30a	30b	36a	36b	39a	39b
2P	12j	12k	13a	13b	27a	14a	15a	15b	18c	18d	39b	39a
3P	8b	8c	26a	26b	9c	28a	10a	10b	12h	12b	13a	13b
5P	24b	24c	26b	26a	27a	28a	6 ₄	6 ₂	36a	36b	39b	39a
7P	24b	24c	26b	26a	27a	4c	30a	30b	36a	36b	39b	39a
13P	24b	24c	2a	2a	27a	28a	30a	30b	36a	36b	3d	3d
<hr/>												
X.1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	1	-1	-1	1	-1	-1	-1	1	-1	1	1
X.3	-1	-1	.	.	-1	.	.	.	-1	.	-1	-1
X.4	1	-1	1	1	.	1	.	.	-1	-1	1	1
X.5	1	-1	-1	-1	.	-1	.	.	-1	1	1	1
X.6	-1	1	1	.	-1	-1
X.7	-1	-1
X.8	-1	1	.	.	-1	.	.	.
X.9	1	-1	.	.	1	.	.	.
X.10	1	1
X.11	-1	-1	.	.	1
X.12	-1	-1	.	.	1
X.13	1	.	.
X.14	-1	.	.
X.15	-1	1	1	-1
X.16	1	-1	-1	-1
X.17	1	1	.	.	-1
X.18	1	.	.	.
X.19
X.20
X.21	.	.	-1	-1	-1	-1	.
X.22	.	.	1	1	-1	-1	.
X.23	1
X.24	1	.	-1	.	.	.
X.25	-1
X.26	1	-1	.	.	1	.	.
X.27	1	.	-1
X.28	-1	.	.	-1
X.29	.	.	.	-1	.	.	.	2	-1	.	.	.
X.30	.	.	.	-1	.	.	.	-1	1	.	.	.
X.31	.	.	.	-1	.	.	.	2	1	.	.	.
X.32	.	.	.	-1	.	.	.	-1	-1	.	.	.
X.33	1	1	.
X.34	.	.	1	1	.	-1	-1	.	.	-1	-1	.
X.35	.	.	-1	-1	.	1	1	.	.	-1	-1	.
X.36	-1	1	1	.	.	.
X.37	-1	1	-1	.	.	.
X.38	1
X.39	1	.	.	1
X.40	1	.	.	-2
X.41	1	1	.
X.42	1
X.43	1	.	.	.
X.44	1	-1
X.45	-1	.	.	.
X.46	1
X.47	1
X.48	-1	.	.	.	1	.	.	.
X.49	1	.	.	.	-1	.	.	.
X.50	-1	-1	.	.
X.51	-1	1	.	-1	.	.	.
X.52	-1	1
X.53	1	-1
X.54	1	-1	.	1	.	.	.
X.55	-1
X.56	-1	.	.	.
X.57
X.58
X.59	1	.	.	.
X.60	1	1	.	-1	.	.	.
X.61	-1	-1	.	1	.	.	.
X.62	1
X.63	-1
X.64	-1
X.65	1
X.66
X.67
X.68
X.69
X.70	1	1	-1	.	.	.
X.71	1	1	-1	.	.	.
X.72	1	1
X.73	-1	-1
X.74	-1	1	1	1	-1	1	1	.
X.75	-1	1	-1	-1	.	1	.	.	.	1	1	.
X.76	1	-1	1
X.77	1	-1	1
X.78	.	.	.	-1
X.79	.	.	.	-1
X.80	.	.	.	-1
X.81	.	.	.	-1
X.82	.	.	.	1	-1
X.83	.	.	.	1	1
X.84	-1	-1	1
X.85	1	-1	-1	-1	.
X.86	-1	1	-1

Character table of mH (continued)

2	13	10	11	13	10	8	7	7	6	4	4	4	2	3	9	10
3	13	9	6	5	5	13	10	10	7	11	10	9	9	7	4	4
5	2	1	1	.	1	.	1	1	1	.
7	1	1	1	1
13	1	1	1
	1a	2a	2b	2c	2d	3a	3b	3c	3d	3e	3f	3g	3h	3i	4a	4b
2P	1a	1a	1a	1a	1a	3a	3b	3c	3d	3e	3f	3g	3h	3i	2b	2c
3P	1a	2a	2b	2c	2d	1a	1a	1a	1a	1a	1a	1a	1a	1a	4a	4b
5P	1a	2a	2b	2c	2d	3a	3b	3c	3d	3e	3f	3g	3h	3i	4a	4b
7P	1a	2a	2b	2c	2d	3a	3b	3c	3d	3e	3f	3g	3h	3i	4a	4b
13P	1a	2a	2b	2c	2d	3a	3b	3c	3d	3e	3f	3g	3h	3i	4a	4b
X.87	491400	-16380	-840	200	-60	-675	540	540	546	54	54	-27	-27	-21	60	.
X.88	491400	16380	-840	200	60	-675	540	540	546	54	54	-27	-27	-21	-60	.
X.89	491400	5460	-280	264	180	3699	-270	-270	.	54	54	135	-27	.	-20	72
X.90	491400	-5460	-280	264	-180	3699	-270	-270	.	54	54	135	-27	.	20	72
X.91	531441	-19683	729	81	-243	.	.	.	729	81	-27
X.92	531441	19683	729	81	243	.	.	.	729	-81	-27
X.93	552825	12285	945	153	45	1701	.	1215	.	243	-15	-75
X.94	552825	-12285	945	153	-45	1701	.	1215	.	243	15	-75
X.95	568620	2106	-36	108	-198	-2187	-729	1458	-6	36
X.96	568620	-2106	-36	108	198	-2187	-729	1458	6	36
X.97	568620	16848	-36	108	144	-2187	1458	-729	96
X.98	568620	-16848	-36	108	-144	-2187	1458	-729	-96
X.99	582400	.	.	-256	.	3088	1360	-800	-728	388	-152	-8	-26	28	.	.
X.100	582400	.	.	-256	.	-4688	1360	1360	-728	-44	-44	-80	-8	-26	.	.
X.101	665600	.	2560	.	.	-3136	320	320	-832	-112	-112	68	32	32	.	.
X.102	698880	17472	896	.	-64	-960	552	-1176	.	336	-96	12	12	.	64	.
X.103	698880	5824	896	.	-192	-960	-1176	552	.	-312	120	12	12	.	-64	.
X.104	698880	-17472	896	.	64	-960	552	-1176	.	336	-96	12	12	.	-64	.
X.105	698880	-5824	896	.	192	-960	-1176	552	.	-312	120	12	12	.	64	.
X.106	716800	17920	.	.	.	1408	-320	-320	448	-104	-104	-32	-14	16	.	.
X.107	716800	-17920	.	.	.	1408	-320	-320	448	-104	-104	-32	-14	16	.	.
X.108	716800	-17920	.	.	.	1408	-320	-320	448	-104	-104	-32	-14	16	.	.
X.109	716800	17920	.	.	.	1408	-320	-320	448	-104	-104	-32	-14	16	.	.
X.110	786240	4368	-448	-192	-48	-1080	-756	864	.	378	54	54	-27	.	-80	.
X.111	786240	-17472	-448	-192	192	-1080	864	-756	.	-108	216	54	-27	.	.	.
X.112	786240	-4368	-448	-192	48	-1080	-756	864	.	378	54	54	-27	.	80	.
X.113	786240	17472	-448	-192	-192	-1080	864	-756	.	-108	216	54	-27	.	.	.
X.114	873600	-14560	.	-384	160	4632	-120	960	.	-228	42	-12	-39	.	.	.
X.115	873600	14560	.	-384	-160	4632	-120	960	.	-228	42	-12	-39	.	.	.
X.116	982800	.	-1680	400	.	-1350	1080	1080	-546	108	108	-54	-54	21	.	.
X.117	998400	16640	-1280	.	.	-4704	480	480	.	-168	-168	102	48	.	.	.
X.118	998400	-16640	-1280	.	.	-4704	480	480	.	-168	-168	102	48	.	.	.
X.119	1062882	.	1458	162	-729	-54
X.120	1257984	17472	896	.	192	-1728	-1080	-1080	.	216	216	-108	54	.	-64	.
X.121	1257984	-17472	896	.	-192	-1728	-1080	-1080	.	216	216	-108	54	.	64	.
X.122	1397760	-11648	-1792	.	128	-1920	-624	-624	.	24	24	24	24	.	.	.
X.123	1397760	11648	-1792	.	-128	-1920	-624	-624	.	24	24	24	24	.	.	.
X.124	1433600	2816	-640	-640	-448	-208	-208	-64	-28	-16	.	.
X.125	1433600	2816	-640	-640	-448	-208	-208	-64	-28	-16	.	.

Character table of mH (continued)

2	3	4	4	4	5	2	5	6	4	4	4	4	2	3	2	3	3	6	6	6	6	3	3	1	3	1	6
3	5	5	5	5	6	6	3	3	4	4	4	4	5	4	4	3	1	1	1	1	.	6	6	7	5	6	6
5
7	1
13
2P	6 ₁₆	6 ₁₇	6 ₁₈	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂	6 ₂₃	6 ₂₄	6 ₂₅	6 ₂₆	6 ₂₇	6 ₂₈	6 ₂₉	7 _a	8 _a	8 _b	8 _c	8 _d	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f		
3P	3 _e	3 _g	3 _e	3 _f	3 _h	3 _b	3 _d	3 _g	3 _f	3 _g	3 _h	3 _g	3 _h	3 _i	7 _a	8 _a	8 _b	8 _c	8 _d	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f		
5P	2 _c	2 _c	2 _d	2 _d	2 _a	2 _d	2 _c	2 _b	2 _c	2 _c	2 _c	2 _d	2 _d	2 _c	7 _a	8 _a	8 _b	8 _c	8 _d	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f		
7P	6 ₁₆	6 ₁₇	6 ₁₈	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂	6 ₂₃	6 ₂₄	6 ₂₅	6 ₂₆	6 ₂₇	6 ₂₈	6 ₂₉	7 _a	8 _a	8 _b	8 _c	8 _d	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f		
13P	6 ₁₆	6 ₁₇	6 ₁₈	6 ₁₉	6 ₂₀	6 ₂₁	6 ₂₂	6 ₂₃	6 ₂₄	6 ₂₅	6 ₂₆	6 ₂₇	6 ₂₈	6 ₂₉	7 _a	8 _a	8 _b	8 _c	8 _d	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f		
X.87	2	-1	-6	-6	9	-6	2	-3	2	-1	-1	3	3	-1	6	.	-3	
X.88	2	-1	6	6	-9	6	2	-3	2	-1	-1	-3	-3	-1	6	.	-3	
X.89	-6	3	-6	-6	-3	.	.	-1	-6	3	3	3	3	-9	-9	
X.90	-6	3	6	6	3	.	.	-1	-6	3	3	-3	-3	-9	-9	
X.91	9	1	3	-1	-1	-1	
X.92	9	1	-3	-1	-1	1	
X.93	-9	.	9	-18	3	3	3	-1	
X.94	-9	.	-9	18	-3	3	3	1	
X.95	9	3	2	.	-2	
X.96	-9	3	-2	.	-2	
X.97	3	
X.98	3	
X.99	-4	-16	8	.	8	8	2	.	.	-4	-8	-8	10	-2	-8	-5	.	
X.100	-4	8	8	.	-4	8	8	.	.	2	-8	-8	10	4	10	7	.	
X.101	4	-2	-4	-4	-4	8	-4	2	.	
X.102	.	.	8	8	-6	2	-4	.	.	.	-4	2	3	3	-15	3	.	.	.	
X.103	-8	6	-4	3	3	12	.	-6	.	.	
X.104	.	.	-8	-8	6	-2	-4	4	-2	3	3	-15	3	.	.	.	
X.105	8	-6	-4	3	3	12	.	-6	.	.	
X.106	10	4	4	4	-8	4	4	.	
X.107	-10	4	4	4	-8	4	4	.	
X.108	-10	4	4	4	-8	4	4	.	
X.109	10	4	4	4	-8	4	4	.	
X.110	6	-12	6	6	3	-6	.	2	-6	.	-3	.	3	-9	18	
X.111	-12	6	.	.	-3	.	.	2	-6	-3	-6	-3	18	-9	
X.112	6	-12	-6	-6	-3	6	.	2	-6	.	-3	.	-3	-9	18	
X.113	-12	6	.	.	3	.	.	2	-6	-3	6	3	18	-9	
X.114	12	12	4	-14	11	4	.	.	6	-3	-2	1	-12	-12	-12	.	-3	.	.	
X.115	12	12	-4	14	-11	-4	.	.	6	-3	2	-1	-12	-12	-12	.	-3	.	.	
X.116	4	-2	-2	-6	4	-2	-2	.	1	-6	.	3	.	
X.117	8	.	-2	-3	-6	-6	-6	.	-6	.	.	
X.118	-8	.	-2	-3	-6	-6	-6	.	-6	.	.	
X.119	-9	2	.	-2	-2	
X.120	-6	-6	.	-4	-9	-9	
X.121	6	6	.	-4	-9	-9	
X.122	.	.	8	8	-2	-4	.	8	.	.	-4	2	6	6	-3	.	-3	.	.	
X.123	.	.	-8	-8	2	4	.	8	.	.	4	-2	6	6	-3	.	-3	.	.	
X.124	8	8	8	8	8	-4	.	
X.125	8	8	8	8	8	-4	.	

2	3	3	3	3	2	1	1	3	2	1	1	3	2	3	3	3	3	1	1	1	1	1	1	3	3	1
3	3	2	2	2	1	1				2	2	4	4	3	3	3	3	3	3	3	3	3				
5										1	1													1	1	
7								1	1																	1
13						1	1																			
	12s	12t	12u	12v	13a	13b	14a	14b	15a	15b	18a	18b	18c	18d	18e	18f	18g	18h	18i	18j	18k	18l	20a	20b	21a	
2P	623	624	623	629	13b	13a	7a	7a	15a	15b	9b	9a	9d	9a	9b	9b	9h	9e	9f	9i	9j	9c	10c	10c	21a	
3P	4c	4g	4f	4d	13a	13b	14a	14b	5a	5a	6i	6i	616	65	69	65	616	69	616	67	68	69	20a	20b	7a	
5P	12s	12t	12u	12v	13b	13a	14a	14b	3c	3b	18a	18b	18c	18d	18e	18f	18g	18h	18i	18j	18k	18l	4a	4e	21a	
7P	12s	12t	12u	12v	13b	13a	2b	2a	15a	15b	18a	18b	18c	18d	18e	18f	18g	18h	18i	18j	18k	18l	20a	20b	3d	
13P	12s	12t	12u	12v	1a	1a	14a	14b	15a	15b	18a	18b	18c	18d	18e	18f	18g	18h	18i	18j	18k	18l	20a	20b	21a	
X.87	1		-1	1									2				-1		-1							
X.88	-1		1	1									2				-1		-1							
X.89	-1		-1									-3	-3		-1	-3	-1									
X.90	1		1									3	3		-1	3	-1									
X.91					1	1	1	1															1	-1	1	
X.92					1	1	1	-1															-1	-1	1	
X.93		2																								
X.94	-2																									
X.95																										
X.96							-1	-1	-2	1													-1	1		
X.97							-1	-1	1	-2													1	1		
X.98							-1	1	1	-2													-1	1		
X.99													2				-1		-1							
X.100													-4				-1		-1							
X.101							-2							4		4									1	
X.102								-1	2	3	3		-1	-1	-1			-1				-1	-1	-1		
X.103								2	-1	1	1		-1	-3	-1					1	1		1	-1		
X.104								-1	2	-3	-3		-1	1	-1			1								

Character table of mH (continued)

2	3	3	3	1	1	.	2	1	1	2	2	.	.
3	1	1	1	.	.	3	.	1	1	2	2	1	1
5	1	1
7
13	.	.	.	1	1	.	1	1	1
	$24a$	$24b$	$24c$	$26a$	$26b$	$27a$	$28a$	$30a$	$30b$	$36a$	$36b$	$39a$	$39b$
$2P$	$12d$	$12j$	$12k$	$13a$	$13b$	$27a$	$14a$	$15a$	$15b$	$18c$	$18d$	$39b$	$39a$
$3P$	$8a$	$8b$	$8c$	$26a$	$26b$	$9c$	$28a$	$10a$	$10b$	$12h$	$12b$	$13a$	$13b$
$5P$	$24a$	$24b$	$24c$	$26b$	$26a$	$27a$	$28a$	6_4	6_2	$36a$	$36b$	$39b$	$39a$
$7P$	$24a$	$24b$	$24c$	$26b$	$26a$	$27a$	$4c$	$30a$	$30b$	$36a$	$36b$	$39b$	$39a$
$13P$	$24a$	$24b$	$24c$	$2a$	$2a$	$27a$	$28a$	$30a$	$30b$	$36a$	$36b$	$3d$	$3d$
X.87
X.88
X.89	1	.	.	.
X.90	-1	.	.	.
X.91	.	-1	-1	-1	-1	.	1	1	1
X.92	.	-1	-1	1	1	.	-1	1	1
X.93
X.94
X.95	-1	1	.	1
X.96	1	-1	.	-1
X.97	1	-1
X.98	-1	1
X.99	1
X.100	1
X.101	-1
X.102	1	.	.	.	1	.	.
X.103	-1	.	-1	.	.	.
X.104	-1	.	.	-1	.	.	.
X.105	1	.	1	.	.	.
X.106	.	.	.	B	A	1	A	B	.
X.107	.	.	.	$-B$	$-A$	1	A	B	.
X.108	.	.	.	$-A$	$-B$	1	B	A	.
X.109	.	.	.	A	B	1	B	A	.
X.110	-1	1	.	1	.	.	.
X.111	-1	1
X.112	1	-1	.	-1	.	.	.
X.113	1	-1
X.114
X.115
X.116
X.117	-1
X.118	1
X.119	.	1	1	-1	-1	.
X.120	-1	.	.	.
X.121	1	.	.	.
X.122	1	-1
X.123	-1	-1	1
X.124	-1	$-B$	$-A$.
X.125	-1	$-A$	$-B$.

, where $A = -\zeta(13)^{11} - \zeta(13)^8 - \zeta(13)^7 - \zeta(13)^6 - \zeta(13)^5 - \zeta(13)^2 - 1$, $B = -A - 1$, $C = 2B$, $D = 2A$.

B.3. Character table of $mE = \langle u, v, s \rangle$

2	10	10	10	10	10	10	10	10	9	6	5	5	7	4	6	4	4	3	3	3
3	10	9	7	6	5	4	4	3	10	10	10	8	8	8	8	8	8	8	8	7
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2P	1a	1a	1a	1a	1a	1a	1a	1a	3a	3b	3c	3d	3e	3f	3g	3h	3i	3j	3k	3l
3P	1a	2a	2b	2c	2d	2e	2f	2g	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a
5P	1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	3e	3f	3g	3h	3i	3j	3k	3l
7P	1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	3e	3f	3g	3h	3i	3j	3k	3l
13P	1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	3e	3f	3g	3h	3i	3j	3k	3l
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	-1	1	-1	1	-1	1	-1	1	1	1	1	1	1	1	1	1	1	1	1
X.3	2	2	2	2	2	2	2	2	-1	2	-1	2	2	-1	2	2	-1	2	-1	2
X.4	78	78	-34	-34	14	14	-2	-2	78	-3	-3	15	6	15	6	-3	-3	-3	-3	6
X.5	78	-78	-34	34	14	-14	-2	2	78	-3	-3	15	6	15	6	-3	-3	-3	-3	6
X.6	91	-91	-21	21	11	-11	-5	5	91	10	10	1	19	1	19	10	10	10	10	1
X.7	91	91	-21	-21	11	11	-5	-5	91	10	10	1	19	1	19	10	10	10	10	1
X.8	105	105	-35	-35	5	5	1	1	105	24	24	15	6	15	6	-3	-3	-3	-3	-3
X.9	105	-105	-35	35	5	-5	1	-1	105	24	24	15	6	15	6	-3	-3	-3	-3	-3
X.10	156	156	-68	-68	28	28	-4	-4	156	-6	3	30	12	-15	-6	-6	6	6	6	12
X.11	168	168	56	56	24	24	8	8	168	6	6	15	24	15	24	6	6	6	6	6
X.12	168	-168	56	-56	24	-24	8	-8	168	6	6	15	24	15	24	6	6	6	6	6
X.13	182	182	70	70	22	22	6	6	182	20	20	29	11	29	11	-7	-7	-7	-7	2
X.14	182	-182	70	-70	22	-22	6	-6	182	20	20	29	11	29	11	-7	-7	-7	-7	2
X.15	182	182	-42	-42	22	22	-10	-10	182	20	-10	2	38	-1	-19	20	20	-10	-10	2
X.16	195	-195	55	-55	15	-15	11	-11	195	33	33	15	24	15	24	6	6	6	6	-3
X.17	195	195	55	55	15	15	11	11	195	33	33	15	24	15	24	6	6	6	6	-3
X.18	210	210	-70	-70	10	10	2	2	-105	48	-24	30	12	-15	-6	-6	-6	3	3	-6
X.19	260	260	20	20	20	20	4	4	260	17	17	-10	-10	-10	-10	-10	17	-10	17	8
X.20	260	-260	20	-20	20	-20	4	-4	260	17	17	-10	-10	-10	-10	-10	17	-10	17	8
X.21	260	260	20	20	20	20	4	4	260	17	17	-10	-10	-10	-10	-10	17	-10	17	8
X.22	260	-260	20	-20	20	-20	4	-4	260	17	17	-10	-10	-10	-10	-10	17	-10	17	8
X.23	273	273	-91	-91	29	29	-7	-7	273	30	30	30	30	30	30	3	3	3	3	3
X.24	273	-273	-91	91	29	-29	-7	7	273	30	30	30	30	30	30	3	3	3	3	3
X.25	336	336	112	112	48	48	16	16	-168	12	-6	30	48	-15	-24	12	12	-6	-6	12
X.26	364	364	140	140	44	44	12	12	-182	40	-20	58	22	-29	-11	-14	-14	7	7	4
X.27	390	390	110	110	30	30	22	22	-195	66	-33	30	48	-15	-24	12	12	-6	-6	-6
X.28	520	520	40	40	8	8	8	8	-260	34	-17	-20	-20	10	10	34	-20	-17	10	16
X.29	520	-520	40	-40	8	-8	8	-8	-260	34	-17	-20	-20	10	10	34	-20	-17	10	16
X.30	546	546	-182	-182	58	58	-14	-14	-273	60	-30	60	60	-30	-30	6	6	-3	-3	6
X.31	546	-546	154	-154	26	-26	2	-2	546	-21	-21	51	-12	51	-12	6	6	6	6	-3
X.32	546	546	154	154	26	26	2	2	-546	-21	-21	51	-12	51	-12	6	6	6	6	-3
X.33	819	-819	-21	21	-21	21	19	-19	819	90	90	9	9	9	9	9	9	9	9	9
X.34	819	819	-21	-21	-21	-21	19	19	819	90	90	9	9	9	9	9	9	9	9	9
X.35	910	910	-210	-210	30	30	-2	-2	910	19	19	55	-17	55	-17	19	-8	19	-8	1
X.36	910	-910	-210	210	30	-30	-2	2	910	19	19	55	-17	55	-17	-8	19	-8	19	1
X.37	910	910	-210	-210	30	30	-2	-2	910	19	19	55	-17	55	-17	-8	19	-8	19	1
X.38	910	-910	-210	210	30	-30	-2	2	910	19	19	55	-17	55	-17	-8	19	-8	19	1
X.39	1092	1092	308	308	52	52	4	4	-546	-42	21	102	-24	-51	12	12	12	-6	-6	-6
X.40	1092	-1092	-140	-140	52	-52	4	-4	1092	-42	-42	-6	30	-6	30	-15	-15	-15	-15	12
X.41	1092	1092	-140	140	52	52	-4	-4	1092	-42	-42	-6	30	-6	30	-15	-15	-15	-15	12
X.42	1365	1365	-35	-35	45	45	5	5	1365	-12	-12	-30	87	-30	87	15	15	15	15	-3
X.43	1365	-1365	245	-245	5	-5	-27	27	1365	69	69	60	-3	60	-3	15	15	15	15	6
X.44	1365	1365	-35	35	45	-45	5	-5	1365	-12	-12	-30	87	-30	87	15	15	15	15	-3
X.45	1365	-1365	245	-245	5	-5	-27	27	1365	69	69	60	-3	60	-3	15	15	15	15	6
X.46	1560	1560	120	120	-40	-40	-8	-8	1560	-60	-60	30	12	30	12	-6	-6	-6	-6	3
X.47	1560	-1560	120	-120	-40	40	-8	8	1560	-60	-60	30	12	30	12	-6	-6	-6	-6	3
X.48	1560	1560	120	120	-40	-40	-8	-8	1560	-60	-60	30	12	30	12	-6	-6	-6	-6	3
X.49	1560	-1560	120	-120	-40	40	-8	8	1560	-60	-60	30	12	30	12	-6	-6	-6	-6	3
X.50	1638	1638	-42	-42	38	38	-819	180	-90	18	18	-9	-9	18	18	-9	-9	-9	18	18
X.51	1638	-1638	294	-294	54	-54	-10	10	1638	-63	-63	45	18	45	18	-9	-9	-9	-9	9
X.52	1638	1638	294	294	54	54	-10	-10	1638	-63	-63	45	18	45	18	-9	-9	-9	-9	9
X.53	1820	1820	-420	-420	60	60	-4	-4	-910	38	-19	110	-34	-55	17	38	-16	-19	8	2
X.54	1820	-1820	-420	420	60	-60	-4	4	-910	38	-19	110	-34	-55	17	38	-16	-19	8	2
X.55	1820	1820	140	140	-20	-20	-4	-4	1820	-43	-43	20	56	20	56	11	11	11	11	-7
X.56	1820	-1820	140	-140	-20	20	-4	4	1820	-43	-43	20	56	20	56	11	11	11	11	-7
X.57	2106	2106	-414	-414	66	66	-6	-6	2106	-81	-81	81	81	81	81	81	81	81	81	81
X.58	2106	-2106	-414	414	66	-66	-6	6	2106	-81	-81	81	81	81	81	81	81	81	81	81
X.59	2184	2184	56	56	24	24	-24	24	2184	78	78	-21	96	-21	96	24	24	24	24	6
X.60	2184	-2184	56	-56	24	-24	-24	-24	2184	78	78	-21	96	-21	96	24	24	24	24	6
X.61	2457	2457	189	189	21	21	33	33	-1092	-84	42	-12	60	6	-30	-30	15	15	15	24
X.62	2457	-2457	189	-189	21	-21	33	-33	2457	-84	42	-12	60	6	-30	-30	15	15	15	24
X.63	2457	2457	189	189	21	21	33	33	2457	-84	42	-12	60	6	-30	-30	15	15	15	24
X.64	2730	2730	490	490	90	90	-26	-26	2730	-24	-24	75	-15	75	-15	3	3	3	3	12
X.65	2730	-2730	490	-490	90	-90	-26	26	2730	-24	-24	75	-15	75	-15	3	3	3	3	12
X.66	2730	2730	490	490	90	90	26	26	2730	-24	-24	75	-15	75	-15	3	3	3	3	12
X.67	2730	-2730	490	-490	90	-90	26	-26	2730	-24	-24	75	-15	75	-15	3	3	3	3	12
X.68	2730	2730	-70	-70	10	10	-6	-6	2730	138	138	-15	21	-15	21	-24	-24	-24	-24	-6
X.69	2730	-2730	-70	70	10	-10	-6	6	2730	138	138	-15	21	-15	21	-24	-24	-24	-24	-6
X.70	2835	2835	315	315	75	75	3	3	2835	-81	-81	81	81	81	81	81	81	81	81	81
X.71	2835	-2835	315	-315																

Character table of mE (continued)

	2	5	5	6	4	4	7	5	5	5	3	6	6	6	6	7	2	5	5	5	5	5	3
	3	6	6	5	6	6	4	5	5	5	6	4	4	4	4	3	6	4	4	4	4	4	5
	5
	7
13
	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	
2P	3d	3e	3f	3g	3h	3i	3j	3k	3l	3m	3n	3o	3p	3q	3r	3s	3t	3u	3v	3w	3x	3y	3z
3P	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n	2o	2p	2q	2r	2s	2t	2u	2v	2w	2x
5P	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	
7P	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	
13P	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.3	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	
X.4	-7	2	5	2	-7	-2	2	5	-7	6	2	5	1	-2	-2	-3	1	2	-1	1	-2	2	
X.5	-7	2	5	2	-7	-2	2	5	7	-6	2	-5	1	-2	2	3	1	2	-1	1	-2	2	
X.6	-3	-3	-2	-3	-3	-5	3	2	3	-1	-1	-2	-2	-5	-5	-1	-2	-1	5	1	1	-3	
X.7	-3	-3	-2	-3	-3	-5	3	2	3	-1	-1	-2	-2	-5	-5	-1	-2	-1	5	1	1	-3	
X.8	1	-8	-4	-8	1	-2	-8	-4	1	-3	2	-4	4	-2	-2	6	4	2	-1	1	4	1	
X.9	1	-8	-4	-8	1	-2	-8	-4	1	-3	2	-4	4	-2	-2	6	4	2	-1	1	4	1	
X.10	-14	4	10	-2	7	-4	-5	-5	-5	4	2	2	2	2	2	2	-1	-2	-2	2	-4	4	
X.11	11	2	6	2	11	8	2	6	11	6	6	2	8	8	-3	2	2	3	-1	2	2	2	
X.12	11	2	6	2	11	8	-2	6	-11	-6	6	-6	2	8	-8	3	2	3	-1	2	2	2	
X.13	7	7	4	7	7	3	7	4	7	2	7	4	3	3	3	2	7	1	3	3	-2	2	
X.14	7	7	4	7	7	3	-7	4	-7	-2	7	-4	3	-3	-2	7	7	1	3	3	-2	2	
X.15	-6	-6	4	3	3	-10	-2	-2	-2	-2	-2	-2	-4	5	5	2	1	10	2	2	-6	2	
X.16	1	10	-3	10	1	8	-10	-3	-1	3	3	5	8	-8	-6	5	3	5	2	1	2	1	
X.17	1	10	-3	10	1	8	-10	-3	1	-3	-3	5	8	8	6	5	3	5	2	1	2	1	
X.18	2	-16	-8	8	-1	-4	4	4	4	4	8	2	2	2	2	-4	-2	-2	2	8	2	2	
X.19	2	-7	2	2	-2	2	-7	2	2	8	2	-7	1	-2	-2	-1	1	2	2	-2	-2	2	
X.20	2	2	-7	2	2	-2	-2	-2	-7	-2	-8	2	7	1	-2	2	1	1	2	2	-2	-2	
X.21	2	2	-7	2	2	-2	-2	-2	-7	-2	-8	2	-7	1	-2	-2	1	1	2	2	-2	-2	
X.22	2	2	-7	2	2	-2	-2	-2	-7	-2	-8	2	-7	1	-2	2	1	1	2	2	-2	-2	
X.23	-10	-10	2	-10	-10	-10	-10	2	-10	3	2	2	2	2	-10	-10	3	2	2	2	2	-1	
X.24	-10	-10	2	-10	-10	-10	10	2	10	-3	2	-2	2	10	-10	-3	2	2	2	2	2	-1	
X.25	22	4	12	-7	-11	16	-6	-6	-6	14	4	-8	4	-8	4	-2	6	-2	6	-2	4	4	
X.26	14	14	8	-7	-7	6	-3	3	3	14	10	-8	10	-8	10	-5	-7	2	6	10	4	2	
X.27	2	20	-6	-10	-1	16	7	7	7	4	4	2	2	2	2	-1	-2	4	-4	-4	4	4	
X.28	4	4	-14	-2	-2	-4	7	7	7	4	4	2	2	2	2	-1	-2	4	-4	-4	4	4	
X.29	4	4	-14	-2	-2	-4	7	7	7	4	4	2	2	2	2	-1	-2	4	-4	-4	4	4	
X.30	-20	-20	4	10	10	-20	-2	-2	-2	4	4	10	4	10	4	-2	-2	4	4	4	-2	2	
X.31	1	-8	-1	-8	1	-4	-8	-1	1	-3	8	-1	-1	-4	-4	-3	-1	8	-1	5	-4	1	
X.32	1	-8	-1	-8	1	-4	-8	-1	1	-3	8	-1	-1	-4	-4	-3	-1	8	-1	5	-4	1	
X.33	-3	-3	6	-3	-3	1	3	6	-3	-9	-3	-6	10	1	-1	-9	10	-3	-3	1	1	-3	
X.34	-3	-3	6	-3	-3	1	3	6	-3	9	-3	6	10	1	1	9	10	-3	-3	1	1	-3	
X.35	-3	-3	3	-3	-3	7	-3	3	-3	1	3	3	-5	7	7	1	-5	3	3	1	1	-3	
X.36	-3	-3	3	-3	-3	7	-3	3	-3	1	3	-3	-5	7	-7	-1	-5	3	3	1	1	-3	
X.37	-3	-3	3	-3	-3	7	-3	3	-3	1	3	3	-5	7	7	1	-5	3	3	1	1	-3	
X.38	-3	-3	3	-3	-3	7	-3	3	-3	1	3	-3	-5	7	-7	-1	-5	3	3	1	1	-3	
X.39	2	-16	-2	8	-1	-8	1	1	1	16	-2	4	2	4	2	2	1	-8	-2	10	-8	2	
X.40	-14	4	-2	4	-14	-2	4	-2	-14	12	-2	-2	-2	2	-2	-3	-2	-2	-2	4	4	4	
X.41	-14	4	-2	4	-14	-2	4	-2	-14	12	-2	-2	-2	2	-2	-3	-2	-2	-2	4	4	4	
X.42	-8	1	5	11	-3	-1	1	-8	-3	3	-4	-1	-1	-3	-4	-3	-4	3	6	-4	5	1	
X.43	-8	1	5	11	-3	-1	1	-8	-3	3	-4	-1	-1	-3	-4	-3	-4	3	6	-4	5	1	
X.44	-8	1	5	11	-3	-1	1	-8	-3	3	-4	-1	-1	-3	-4	-3	-4	3	6	-4	5	1	
X.45	2	11	5	11	2	-3	11	5	2	6	5	5	-3	-3	-3	6	-3	5	-4	-6	3	2	
X.46	-6	-6	-4	-6	-6	4	-6	-4	-6	3	8	-4	4	4	4	3	4	8	2	-2	-2	3	
X.47	-6	-6	-4	-6	-6	4	-6	-4	-6	3	8	-4	4	4	4	3	4	8	2	-2	-2	3	
X.48	-6	-6	-4	-6	-6	4	-6	-4	-6	3	8	-4	4	4	4	3	4	8	2	-2	-2	3	
X.49	-6	-6	-4	-6	-6	4	-6	-4	-6	3	8	-4	4	4	4	3	4	8	2	-2	-2	3	
X.50	-6	-6	12	3	3	2	-6	-6	-6	20	-1	2	2	2	2	2	-10	3	-6	2	2	-6	
X.51	15	-12	9	-12	15	2	-12	9	15	9	-6	9	5	2	2	2	5	-6	-3	-1	-4	-3	
X.52	15	-12	9	-12	15	2	-12	9	15	-9	-6	-9	5	2	-2	2	5	-6	-3	-1	-4	-3	
X.53	-6	-6	6	3	3	14	-3	3	3	6	-10	-7	6	-10	-7	5	-3	6	2	2	-6	2	
X.54	-6	-6	6	3	3	14	-3	3	3	6	-10	-7	6	-10	-7	5	-3	6	2	2	-6	2	
X.55	-4	-4	-11	-4	-4	8	-4	-11	-4	-7	-8	-11	5	8	-8	-7	5	-8	4	-4	-4	5	
X.56	-4	-4	-11	-4	-4	8	-4	-11	-4	-7	-8	-11	5	8	-8	-7	5	-8	4	-4	-4	5	
X.57	-9	18	3	18	-9	18	3	18	-9	3	3	3	3	3	3	3	3	3	3	3	3	3	
X.58	-9	18	3	18	-9	18	3	18	-9	3	3	3	3	3	3	3	3	3	3	3	3	3	
X.59	11	2	6	2	11	-2	6	-11	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	2	
X.60	11	2	6	2	11	-2	6	-11	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	2	
X.61	-28	8	-4	-4	14	-4	6	2	2	-4	2	2	2	2	2	2	2	2	2	2	2	8	
X.62	.	.	3	.	.	6	3	.	3	6	3	3	6	6	3	6	3	6	3	6	3	.	
X.63	.	.	3	.	.	6	3	.	3	6	3	3	6	6	3	6	3	6	3	6	3	.	
X.64	13	-5	.	-5	13	-7	5	-13	-12	-3	-4	-7	7	-3	-4	-3	3	5	-1	4	2	4	
X.65	4	22	10	-11	-2	-6	-5	-5	10	10	-6	3	3	3	3	3	3	-5	-8	-12	6	4	
X.66	13	-5	.	-5	13	-7	5	-13	-12	-3	-4	-7	7	-3	-4	-3	3	5	-1	4	2	4	
X.67	11	-7	10	-7	11	-3	-7	10	11	-6	1	10	6	-3	-3	3	6	1	1	3	-3	2	
X.68	-16	2	.	-1	8	-2	.	.	6	8	-1	1	8	1	1	4	-3	12	-8	10	2	2	
X.69	11	-7	10	-7	11	-3	-7	10	11	-6	1	10	6	-3	-3	3	6	1	1	3	-3	2	
X.70	18	-9	3	-9	18	9	9	3	-18	3	9	-9	3	9	-9	3	-3	3	3	3	3	.	
X.71	18	-9	3	-9	18	9	9	3	-18	3	9	-9	3	9	-9	3	-3	3	3	3	3	.	
X.72	-12	-12	-8	6	6	8</																	

Character table of mE (continued)

	2	6	6	4	4	4	4	4	2	5	5	5	3	3	3	3	3	4	4	2	2	2	3	3	1	2
	3	3	3	4	4	4	4	4	5	3	3	3	4	4	4	4	4	3	3	4	4	4	3	3	4	3
	5
	7
	13
	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	
2P	3b	3e	3i	3f	3h	3g	3f	3n	3e	3d	3d	3f	3k	3l	3l	3l	3h	3i	3m	3n	3n	3n	3l	3l	3o	3m
3P	2g	2e	2f	2d	2f	2f	2f	2b	2g	2g	2e	2f	2f	2f	2f	2c	2g	2g	2f	2f	2f	2g	2g	2f	2g	
5P	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	
7P	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	
13P	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.2	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	1	1	1	1	-1	-1	-1	
X.3	.	.	2	-1	2	-1	-1	-1	.	.	-1	-1	2	2	.	.	.	2	-1	-1	.	.	-1	.	.	
X.4	1	2	1	-1	1	-2	1	2	-2	1	-1	1	1	-2	-2	2	1	1	1	-2	-2	-2	-2	1	1	
X.5	-1	-2	1	-1	1	-2	1	2	-2	-1	1	1	1	-2	-2	-2	-1	-1	1	-2	-2	2	2	1	-1	
X.6	2	1	-2	5	-2	1	1	-3	-1	-1	-5	-2	-2	1	1	3	2	2	1	1	1	-1	-1	1	-1	
X.7	-2	-1	-2	5	-2	1	1	-3	1	1	5	-2	-2	1	1	-3	-2	-2	1	1	1	1	1	1	1	
X.8	4	2	1	-1	1	4	1	1	4	1	-1	1	1	1	1	1	1	1	-2	1	1	1	1	-2	-2	
X.9	-4	-2	1	-1	1	4	1	1	-4	-1	1	1	1	1	1	-1	-1	-1	-2	1	1	-1	-1	-2	-2	
X.10	.	.	2	1	2	2	-1	-2	.	.	-1	-1	-4	-4	.	.	.	2	2	2	.	.	-1	.	-1	
X.11	2	.	2	3	2	-1	2	2	-1	3	2	2	2	2	2	2	2	2	-1	2	2	2	-1	-1	-1	
X.12	-2	.	2	3	2	2	-1	2	-2	1	-3	2	2	2	2	-2	-2	-2	-1	2	2	-2	-2	-1	1	
X.13	.	7	-3	1	-3	3	3	-2	3	3	1	-3	-3	.	.	-2	-3	-3	
X.14	.	-7	-3	1	-3	3	3	-2	3	-3	-1	-3	-3	.	.	2	3	3	
X.15	.	.	-4	-5	-4	-1	-1	3	.	.	.	2	2	2	2	.	.	.	2	-1	-1	.	.	-1	.	
X.16	-5	.	2	3	2	2	5	1	-2	-5	-3	2	2	-1	-1	-1	-2	-2	2	-1	-1	1	1	2	-2	
X.17	5	.	2	3	2	2	5	1	2	5	3	2	2	-1	-1	1	2	2	2	-1	-1	-1	-1	2	2	
X.18	.	.	2	1	2	-4	-1	-1	.	.	-1	-1	2	2	.	.	.	-4	-1	-1	.	.	2	.	.	
X.19	1	2	1	2	-2	-2	-2	2	-2	-2	2	-2	1	-2	4	2	-2	1	1	4	-2	-2	4	1	1	
X.20	-1	-2	-2	2	1	-2	-2	2	2	-2	1	-2	4	-2	-2	-1	2	1	-2	4	-4	-2	2	1	-1	
X.21	1	2	-2	2	1	-2	-2	2	-2	-2	2	1	-2	4	-2	2	1	-2	1	-2	4	-4	-2	1	1	
X.22	-1	-2	1	2	-2	-2	-2	2	2	-2	-2	1	-2	4	-2	2	-1	1	4	-2	2	-4	1	-1	-1	
X.23	2	2	-1	2	-1	2	2	-1	2	2	2	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
X.24	-2	-2	-1	2	-1	2	2	-1	-2	-2	-2	-1	-1	-1	-1	1	1	1	-1	-1	-1	1	1	-1	1	
X.25	.	.	4	-3	4	-2	1	-2	.	.	-2	-2	4	4	.	.	.	-2	-2	
X.26	.	.	-4	-1	-6	-3	-3	2	.	.	.	-2	-2	-2	.	.	.	4	1	1	.	.	-2	.	.	
X.27	.	.	4	-3	4	-2	-5	-1	.	.	.	-2	-2	-2	.	.	.	4	1	1	.	.	-2	.	.	
X.28	.	.	-2	2	-4	2	2	-2	.	.	-2	-1	-4	8	.	.	.	-2	-4	2	.	.	-1	.	.	
X.29	.	.	-4	-2	2	2	2	-2	.	.	-1	2	8	-4	.	.	.	2	2	-4	.	.	-1	.	.	
X.30	.	.	-2	-2	-2	-2	-2	1	.	.	1	1	-2	-2	.	.	.	-2	1	1	.	.	1	.	.	
X.31	-1	8	-2	1	2	-4	5	1	-4	5	-1	2	2	-1	-1	1	2	2	-1	-1	-1	-1	-1	-1	-1	
X.32	1	-8	2	-1	2	-4	5	1	4	-5	1	2	2	-1	-1	-1	-2	-2	-1	-1	-1	-1	-1	-1	-1	
X.33	-10	3	1	-3	1	1	1	-3	-1	-1	3	1	1	1	1	3	-1	-1	1	1	-1	-1	-1	-1	-1	
X.34	10	-3	1	-3	1	1	1	-3	1	1	-3	1	1	1	1	-3	1	1	1	1	1	1	1	1	1	
X.35	-5	3	4	3	-5	1	1	-3	1	1	3	-5	4	1	1	-3	-5	4	1	1	1	1	1	1	1	
X.36	5	-3	-5	3	4	1	1	-3	-1	-1	-3	4	-5	1	1	3	-4	5	1	1	1	-1	-1	-1	-1	
X.37	-5	3	-5	3	4	1	1	-3	1	1	3	4	-5	1	1	-3	4	-5	1	1	1	1	1	1	1	
X.38	5	-3	4	3	-5	1	1	-3	-1	-1	-3	-5	4	1	1	3	5	-4	1	1	1	-1	-1	-1	-1	
X.39	.	.	4	1	4	4	-5	-1	.	.	-2	-2	-2	-2	.	.	.	-2	1	1	.	.	1	.	.	
X.40	-2	-2	1	-2	1	4	-2	4	4	-2	-2	1	1	-2	-2	4	1	1	1	-2	-2	-2	-2	1	1	
X.41	2	2	1	-2	1	4	-2	4	-4	2	2	1	1	-2	-2	-4	-1	-1	1	-2	2	2	2	1	-1	
X.42	-4	3	-1	6	-1	5	-4	1	-5	-4	6	-1	3	3	-1	-1	1	-1	-1	-1	-1	-1	-1	-1	-1	
X.43	3	-5	3	-4	3	3	-6	1	-3	6	4	3	3	-2	-3	-3	
X.44	4	-3	-1	6	-1	5	-4	1	-5	-4	6	-1	1	-1	-1	-1	1	1	-1	-1	-1	1	1	-1	1	
X.45	-3	5	3	-4	3	3	-6	2	3	-6	-4	3	3	.	.	2	3	3	
X.46	4	8	-2	2	-2	-2	-2	3	-2	-2	2	-2	-2	1	1	3	-2	-2	1	1	1	1	1	1	1	
X.47	-4	-8	-2	2	-2	-2	-2	3	2	-2	-2	-2	-2	1	1	-3	2	2	1	1	-1	-1	-1	-1	-1	
X.48	4	8	-2	2	-2	-2	-2	3	-2	-2	2	-2	-2	1	1	3	-2	-2	1	1	1	1	1	1	1	
X.49	-4	-8	-2	2	-2	-2	-2	3	2	2	-2	-2	-2	1	1	-3	2	2	1	1	-1	-1	-1	-1	-1	
X.50	.	.	2	3	2	-1	-1	3	.	.	-1	-1	2	2	.	.	.	2	-1	-1	.	.	-1	.	.	
X.51	5	-6	-1	3	-1	-4	-1	-3	-4	-1	-3	-1	-1	-1	-1	-3	-1	-1	2	-1	-1	-1	-1	2	2	
X.52	-5	6	-1	3	-1	-4	-1	-3	4	1	3	-1	-1	-1	-1	3	1	1	2	-1	-1	1	1	2	-2	
X.53	.	.	8	-3	-10	-1	-1	3	.	.	5	-4	2	2	.	.	.	2	-1	-1	.	.	-1	.	.	
X.54	.	.	-10	-3	8	-1	-1	3	.	.	-4	5	2	2	.	.	.	2	-1	-1	.	.	-1	.	.	
X.55	5	-8	-1	4	-1	-4	-4	5	-4	-4	4	-1	-1	-1	-1	5	-1	-1	-1	-1	-1	-1	-1	-1	-1	
X.56	-5	8	-1	4	-1	-4	-4	5	4	4	-4	-1	-1	-1	-1	-5	1	1	-1	-1	-1	-1	-1	-1	-1	
X.57	3	.	.	-3	.	-6	3	.	-6	3	-3	
X.58	-3	.	.	-3	.	-6	3	.	-6	3	-3	
X.59	6	.	.	-3	.	-6	3	.	-6	3	-3	-2	
X.60	-6	.	.	3	.	-6	3	.	-6	3	3	2	
X.61	.	.	2	3	2	-4	2	-4	.	.	-1	-1	-4	-4	.	.	.	2	2	2	.	.	-1	.	.	
X.62	3	6	3	.	3	3	3	.	.	.	3	3	
X.63	-3	-6	3	.	3	3	3	.	.	.	-3	-3	
X.64	4	3	-1	3	-1	-1	5	4	1	-5	-3	-1	-1	2	2	-4	1	1	-1	2	2	-2	-2	-1	1	
X.65	.	.	6	4	6	-3	6	-2	.	.	-3	-3	
X.66	-4	-3	-1	3	-1	-1	5	4	-1	5	3	-1	-1	2	2	4	-1	-1	-1	2	2	2	2	-1	-1	
X.67	6	1	.	.	-3	3	2	-3	3	1	2	.	-3	-3	-3	.	

3	2	5	5	5	5	3	2	2	1	1	1	4	4	4	3	4	3	6	7	7	5	5	6	4	3	4		
3	1	1	1	.	.	3	5	5	4	4	4	4	4	1	1	1	1	3	2	3	3	3	2	3	4	3		
5	1	1	1	1	1	1	1	1		
7	1	1	1	1	1	1		
13		
2P	7a	4e	4b	4b	4e	9a	9b	9c	9d	9e	9f	9g	9h	5a	5a	5a	5a	5a	63	69	69	62a	617	63	628	625	622	
3	7a	4e	4b	4b	4e	9a	9b	9c	9d	9e	9f	9g	9h	5a	5a	5a	5a	5a	4a	4a	4e	4e	4b	4b	4b	4b	4a	
5P	7a	8a	8b	8c	8d	9a	9b	9c	9d	9e	9f	9g	9h	2d	2a	2b	2c	2c	121	121	123	123	124	125	126	127	128	129
7P	1a	8a	8b	8c	8d	9a	9b	9c	9d	9e	9f	9g	9h	10a	10b	10c	10d	10e	121	122	123	124	125	126	127	128	129	
13P	7a	8a	8b	8c	8d	9a	9b	9c	9d	9e	9f	9g	9h	10a	10b	10c	10d	10e	121	125	123	124	125	126	127	128	129	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.2	1	1	1	1	-1	-1	1	1	1	1	1	1	1	1	-1	1	-1	-1	1	1	1	1	1	1	1	1	1	
X.3	2	2	2	.	.	.	2	2	-1	2	2	-1	-1	2	2	2	2	2	-1	-1	2	2	-1	-1	2	2	2	
X.4	1	3	3	-1	3	1	1	1	-6	2	2	-1	2	2	-1	2	2	
X.5	1	3	3	-1	3	1	1	1	-6	2	2	-1	2	2	-1	2	2	
X.6	-1	-1	-1	1	1	-2	4	-2	4	1	1	1	1	1	-1	-1	-1	-1	1	3	3	3	-1	3	-1	3	-1	
X.7	-1	-1	-1	-1	-2	4	-2	4	1	1	1	1	1	1	-1	-1	-1	-1	-1	3	3	3	-1	3	-1	3	-1	
X.8	-1	1	1	1	-1	3	1	1	1	1	1	-5	5	1	2	2	1	2	2	-2	
X.9	-1	1	1	-1	1	3	3	3	-5	5	1	2	2	2	2	2	-2	
X.10	2	6	-3	-2	3	2	.	.	6	-2	-2	-2	4	-2	1	-2	.	
X.11	3	3	-1	3	1	-1	1	4	.	.	.	4	.	.	.	-2	
X.12	3	3	-1	-3	1	1	-1	4	.	.	.	4	.	.	.	-2	
X.13	5	-1	5	-1	-1	-1	-1	-1	2	2	.	2	.	10	2	2	2	-1	2	2	-1	1	
X.14	5	-1	5	-1	-1	-1	-1	-1	2	2	-2	.	-2	10	2	2	2	-1	2	2	-1	1	
X.15	-2	-2	.	.	.	-4	8	2	-4	2	2	-1	-1	2	.	-2	.	.	1	-3</								

Character table of mE (continued)

	2	4	5	5	5	3	3	3	3	4	4	4	4	2	2	5	3	3	3	4	1
	3	3	2	2	2	3	3	3	3	2	2	2	2	3	3	1	2	2	2	1	1
	5
	7
	13	1
		12 ₁₀	12 ₁₁	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	12 ₁₇	12 ₁₈	12 ₁₉	12 ₂₀	12 ₂₁	12 ₂₂	12 ₂₃	12 ₂₄	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈	13 _a
2P		6 ₃₀	6 ₂₄	6 ₂₄	6 ₁₇	6 ₂₉	6 ₃₇	6 ₃₆	6 ₃₈	6 ₂₈	6 ₂₂	6 ₃₀	6 ₃₀	6 ₄₅	6 ₄₆	6 ₅₄	6 ₃₆	6 ₃₇	6 ₃₈	6 ₃₀	13 _b
3P		4 _a	4 _d	4 _e	4 _d	4 _a	4 _a	4 _b	4 _b	4 _e	4 _c	4 _c	4 _f	4 _b	4 _b	4 _g	4 _d	4 _f	4 _d	4 _h	13 _a
5P		12 ₁₀	12 ₁₁	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	12 ₁₇	12 ₁₈	12 ₁₉	12 ₂₀	12 ₂₁	12 ₂₂	12 ₂₃	12 ₂₄	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈	13 _b
7P		12 ₁₀	12 ₁₁	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	12 ₁₇	12 ₁₈	12 ₁₉	12 ₂₀	12 ₂₁	12 ₂₂	12 ₂₃	12 ₂₄	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈	13 _b
13P		12 ₁₀	12 ₁₁	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	12 ₁₇	12 ₁₈	12 ₁₉	12 ₂₀	12 ₂₁	12 ₂₂	12 ₂₃	12 ₂₄	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈	1 _a
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	-1	1	-1	1	1	1	1	1	1	-1	-1	1	1	1	-1	-1	1	-1	-1	1
X.3	2	.	2	.	-1	-1	2	2	-1	.	.	2	-1	-1	-1	.	-1	.	.	.	2
X.4	-3	-1	-1	2	.	-3	-1	-1	-1	.	-3	-1	-1	-1	-1	-1	-1	-1	-1	-1	.
X.5	-3	1	-1	-2	.	-3	-1	-1	-1	.	3	-1	-1	-1	-1	1	1	-1	1	1	.
X.6	-1	.	.	-3	-1	-1	.	.	.	1	1	-1	-1	.	1	.
X.7	-1	.	.	3	-1	-1	.	.	.	-1	-1	-1	-1	.	-1	.
X.8	1	2	-2	2	-2	1	-1	-1	-2	-2	-2	-1	-1	-1	-1	-2	-1	-1	-1	-1	1
X.9	1	-2	-2	-2	-2	1	-1	-1	-2	2	-1	-1	-1	-1	-1	2	1	-1	1	1	1
X.10	-6	.	-2	.	.	3	-2	-2	1	.	.	-2	1	1	1	.	.	1	.	.	.
X.11	1	.	.	.	-2	1	2	-1	1	1	.	-1	-1
X.12	1	.	.	.	-2	1	2	-1	1	1	.	-1	-1
X.13	1	2	2	-1	1	1	-1	-1	2	1	1	-1	-1	-1	-1	2	-1	-1	-1	-1	.
X.14	1	-2	2	1	1	1	-1	-1	2	-1	-1	-1	-1	-1	-1	-2	1	-1	1	1	.
X.15	-2	.	.	.	1	1	-2	1	.	.	.
X.16	-1	-3	-1	.	2	-1	.	.	-1	-2	1	1	.	.	.	1	.	1	.	-1	.
X.17	-1	3	-1	.	2	-1	.	.	-1	2	-1	1	.	.	.	-1	.	1	.	1	.
X.18	2	.	-4	.	2	-1	-2	-2	2	.	.	-2	1	1	.	.	1	.	1	.	2
X.19	.	1	1	-2	.	.	1	-2	1	.	.	-2	1	1	.	-2	1	1	1	-2	.
X.20	.	-1	1	2	.	-2	1	1	1	-2	-1	2	.	-1	.	.	.
X.21	.	1	1	-2	.	-2	1	1	1	-2	1	-2	.	1	.	.	.
X.22	.	-1	1	2	.	.	1	-2	1	.	.	.	-2	1	-1	-1	.	-1	2	.	.
X.23	.	4	.	-2	.	.	1	1	1	1	.	1	.	1	.	.
X.24	.	-4	.	2	.	.	1	1	1	1	.	-1	.	-1	.	.
X.25	2	.	.	.	2	-1	.	-2	-2	.	.	2	1	1	.	.	.	-1	.	.	-2
X.26	2	.	.	4	.	-1	-2	1	.	-2	.	.	2	1	1	.	.	.	1	.	.
X.27	-2	.	-2	.	-2	1	.	1	.	.	.	-2	1	1	.	.	.	-1	.	.	.
X.28	.	.	2	.	.	2	-4	-1	2	-1
X.29	.	.	2	.	.	.	-4	2	-1	.	.	.	-1	2
X.30	2	2	-1	-1
X.31	-1	-3	1	.	2	-1	.	.	1	2	-1	1	.	.	.	1	.	1	.	1	.
X.32	-1	3	1	.	2	-1	.	.	1	-2	1	1	.	.	.	-1	.	1	.	-1	.
X.33	-1	2	2	-1	-1	-1	1	1	2	1	1	-1	1	1	-2	-1	-1	-1	-1	1	.
X.34	-1	-2	2	1	-1	-1	1	1	2	-1	-1	-1	1	1	2	1	-1	1	-1	1	.
X.35	-1	-1	-1	-1	-1	-1	2	-1	-1	-1	-1	1	-1	2	-1	2	1	-1	1	1	.
X.36	-1	1	-1	1	-1	-1	-1	2	-1	1	1	1	2	-1	1	1	1	-2	-1	.	.
X.37	-1	-1	-1	-1	-1	-1	-1	2	-1	-1	-1	1	2	-1	-1	-1	1	2	1	1	.
X.38	-1	1	-1	1	-1	-1	2	-1	-1	1	1	-1	2	1	-2	1	-2	1	-1	-1	.
X.39	-2	.	2	.	-2	1	.	-1	.	.	2	-1
X.40	.	-2	-2	-2	.	1	1	-2	.	.	.	1	1	-2	1	-2	1	.	1	.	.
X.41	.	2	-2	2	.	1	1	-2	.	.	.	1	1	2	1	.	.	-1	.	.	.
X.42	2	.	-4	.	-1	2	-1	1	.	-1	2	.	.	-1	-1	.	-1	.	-1	.	.
X.43	2	1	1	-1	1	2	-1	1	1	1	-2	.	.	-1	-1	-1	.	-1	.	.	.
X.44	2	4	.	1	-1	2	-1	-1	.	.	-2	.	.	-1	-1	-1	.	1	.	.	.
X.45	2	1	1	1	-1	2	1	1	1	-1	2	.	1	1	1	1	1	.	1	.	.
X.46
X.47
X.48
X.49
X.50	-2	.	4	.	1	1	2	2	-2	.	.	-2	-1	-1	.	.	1
X.51	3	-1	-1	2	.	3	-1	-1	-1	.	3	1	-1	-1	-1	-1	-1	1	-1	1	.
X.52	3	1	-1	-2	.	3	-1	-1	-1	.	-3	1	-1	-1	-1	1	1	1	1	-1	.
X.53	-2	.	-2	.	1	1	4	-2	1	.	.	2	1	-2	.	.	.	-1	.	.	.
X.54	-2	.	-2	.	1	1	-2	4	1	.	.	2	-2	1	.	.	-1
X.55	.	1	1	4	.	1	1	1	1	1	1	1	1	1	1	.	.
X.56	.	-1	1	-4	.	1	1	1	1	1	-1	-1	.	-1	.	.	.
X.57	1	-3	1	.	-2	1	.	1	-2	1	-1	-1	.	.	.	1	.	-1	.	-1	.
X.58	1	3	1	.	-2	1	.	1	.	1	-2	-1	-1	.	.	-1	.	-1	.	1	.
X.59	-1	.	.	.	2	-1	.	.	.	-2	1	-1	-1	-1	.	1	.
X.60	-1	.	.	.	2	-1	.	.	.	2	-1	-1	-1	.	-1	.
X.61	.	.	-4	.	.	2	2	2	-1	-1
X.62	.	-1	1	2	.	-1	-1	-1	-1	-1	-1	-1	.	-1	.	.	.
X.63	.	1	-1	-2	.	-1	-1	-1	-1	-1	-1	1	1	.	1	.	.
X.64	1	2	-2	-1	1	1	1	1	-2	-1	-1	-1	1	1	2	-1	-1	-1	1	1	.
X.65	4	.	2	.	1	-2	2	2	-1	.	.	.	-1	-1
X.66	1	-2	-2	1	1	1	1	-2	1	1	-1	1	-1	1	-2	1	-1	1	-1	.	.
X.67	-1	.	.	-3	-1	-1	.	.	.	-1	-1	1	1	.	1	.
X.68	4	.	.	.	1	-2	-2	-2	1	1
X.69	-1	.	.	3	-1	-1	.	.	.	1	1	1	1	.	-1	.
X.70	-2	3	1	3	1	-2	.	.	1	-1	2	-1	1
X.71	-2	-3	1	-3	1	-2	.	.	1	1	-2	1	1
X.72
X.73
X.74	6	.	-2	.	.	-3	-2	-2	1	.	.	2	1	1	.	.	-1
X.75	.	2	2	2	-1	.	.	.	-1	-1
X.76	-1	-1	-1	2	2	-1	-1	-1	-1	2	-1	1	-1	-1	-1	-1	-1	1	-1	1	.
X.77	-1	.	.	.	2	-1	.	.	.	2	-1	1	1	.	-1	.
X.78	-1	1	-1	-2	2	-1	-1	-1	-1	-2	1	1	-1	-1	-1	1	1	1	1	-1	.
X.79	-1	.	2	.	2	-1	.	.	-1	.	-2	1	1	1	.	.	.
X.80	2	.	2	.	2	-1	.	.	.	-1	.	-2	1	.	.	.
X.81
X.82	-2	.	.	.	-2	1	-2	1	.	.	.
X.83																	

2	3	1	2	2	2	3			3	3	3	2	2	2	2	3	3	3	1	2	2	3	1	1	3	3
3	1	1				2	2	2	4	4	4	4	4	4	4	3	3	3	4	3	3	2	3	3	1	3
5						1	1	1																	1	1
7		1	1	1																						1
13	1																									
	13b	14a	14b	14c	15a	15b	15c	18a	18b	18c	18d	18e	18f	18g	18h	18i	18j	18k	18l	18m	18n	18o	18p	20a	20b	
2P	13a	7a	7a	7a	15a	15b	15c	9a	9a	9c	9b	9b	9c	9a	9a	9a	9d	9b	9c	9a	9e	9f	10a	10a		
3P	13b	14a	14b	14c	5a	5a	5a	6a	6a	6c	6b	6b	6c	6a	614	6a	6b	6a	614	623	610	611	20a	20b		
5P	13a	14a	14b	14c	3a	3d	3f	18c	18b	18a	18g	18e	18f	18d	18j	18i	18h	18k	18l	18m	18n	18o	18p	4a	4c	
7P	13a	2b	2a	2c	15a	15b	15c	18a	18b	18c	18d	18e	18f	18g	18h	18i	18j	18k	18l	18m	18n	18o	18p	20a	20b	
13P	1a	14a	14b	14c	15a	15b	15c	18a	18b	18c	18d	18e	18f	18g	18h	18i	18j	18k	18l	18m	18n	18o	18p	20a	20b	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.2	1	1	-1	-1	1	1	1	1	-1	1	1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	-1	1	-1	
X.3	2	2			-1	2	-1	2	3	2	-1	2		-1	1	-1	2		-1	1	-1		2			
X.4			1	1	1	3		-1	3	-1	-1	2		-1	-1	-1	2	2	-1	1						
X.5			1	-1	-1	1	3		-1	-3	-1	-1	2		-1	1	1	2	-2	-1						
X.6						1	1	1		-2				-4			2			2	2	-1	-1	-1	-1	
X.7					1	1	1			-2				4			2			2	2	1	1	-1	-1	
X.8	1							1	3	1	1	-2		1	1	-1	1	-2	-2	-1	1					
X.9	1							1	-3	1	1	-2		1	-1	-1	-1	-2	2	-1	1					
X.10		2			-3			-2		-2	1	4		1	-1	-1	-2	-2	1					-2		
X.11	-1				3			2		2	-1	3	2	2	2		2	-1	1					-1	-1	
X.12					3			2		2	2	-1	-3	2	-2		-2	-1	1					-1	1	
X.13					2	-1	-1	1	5	1	1	1	-1	1	1	1	1	1	1	1	1	-1	-1	1	1	
X.14					2	-1	-1	1	-5	1	1	1	1	1	1	-1	-1	-1	-1	1	-1	1	1	1	1	
X.15					-1	2	-1									4				-2				-2		

Character table of mE (continued)

	2i	10	10	10	10	10	10	10	10	9	6	5	5	7	4	6	4	4
	3	10	9	7	6	5	4	4	3	10	10	10	8	8	8	8	8	8
	5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	3e	3f	3g	3h	3i
2P		1a	1a	1a	1a	1a	1a	1a	1a	3a	3b	3c	3d	3e	3f	3g	3h	3i
3P		1a	2a	2b	2c	2d	2e	2f	2g	1a	1a	1a	1a	1a	1a	1a	1a	1a
5P		1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	3e	3f	3g	3h	3i
7P		1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	3e	3f	3g	3h	3i
13P		1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	3e	3f	3g	3h	3i
X.88	5265	-5265	225	-225	-15	15	33	-33		5265	162	162		81		81		
X.89	5460	5460	-140	-140	-140	-140	20	20	20	5460	-129	-129	60	60	60	60	6	6
X.90	5460	5460	420	420	100	100	20	20	20	5460	33	33	-30	96	-30	96	-21	-21
X.91	5460		-140		20		-12			-2730	276	-138	-30	42	15	-21	-48	-48
X.92	5460	-5460	420	-420	100	-100	20	-20		5460	33	33	-30	96	-30	96	-21	-21
X.93	5460		980		180		52			-2730	-48	24	150	-30	-75	15	6	6
X.94	5460	-5460	-700	700	100	-100	-12	12		5460	114	114	60	6	60	6	-21	-21
X.95	5460	-5460	-140	140	-140	140	20	-20		5460	-129	-129	60	60	60	60	6	6
X.96	5460	5460	-700	-700	100	-100	-12	12		5460	114	114	60	6	60	6	-21	-21
X.97	5670		630		150		6			-2335	-162	81		162		-81		
X.98	5824	-5824	896	-896	64	-64				5824	-8	-8	154	-8	154	-8	-8	-8
X.99	5824	5824	896	896	64	64				5824	-8	-8	154	-8	154	-8	-8	-8
X.100	5824	5824	-896	-896	64	64				5824	-8	-8	154	-8	154	-8	-8	-8
X.101	5824	-5824	-896	-896	64	-64				5824	-8	-8	154	-8	154	-8	-8	-8
X.102	6552	-6552	-504	504	-24	24	-8	8		6552	234	234	45	72	45	72	18	18
X.103	6552	6552	-504	-504	-24	-24	-8	-8		6552	234	234	45	72	45	72	18	18
X.104	7020	-7020	540	-540	60	-60	12	-12		7020	-27	-27		-54		-54	54	-27
X.105	7020	7020	540	-540	60	-60	12	-12		7020	-27	-27		-54		-54	54	-27
X.106	7020	7020	540	540	60	60	12	12		7020	-27	-27		-54		-54	54	-27
X.107	7020	7020	540	540	60	60	12	12		7020	-27	-27		-54		-54	54	-27
X.108	7280	-7280	-560	560	80	-80	-16	16		7280	-10	-10	-10	-64	-10	-64	-10	44
X.109	7280	7280	-560	-560	80	80	-16	-16		7280	-10	-10	-10	-64	-10	-64	-10	44
X.110	7280	-7280	-560	560	80	-80	-16	16		7280	-10	-10	-10	-64	-10	-64	-10	44
X.111	7280	7280	-560	-560	80	80	-16	-16		7280	-10	-10	-10	-64	-10	-64	-10	44
X.112	7280	-7280	-560	560	80	-80	-16	16		7280	-10	-10	-10	-64	-10	-64	-10	44
X.113	7280	7280	-560	-560	80	80	-16	-16		7280	-10	-10	-10	-64	-10	-64	-10	44
X.114	7371	7371	819	819	51	51	-21	-21		7371	81	81	81	81	81	81	81	
X.115	7371	-7371	819	-819	51	-51	-21	21		7371	81	81	81	81	81	81	81	
X.116	8190	-8190	-210	210	30	-30	-18	18		8190	171	171	-45	-45	-45	-45	-45	36
X.117	8190	8190	-210	-210	30	30	-18	-18		8190	171	171	-45	-45	-45	-45	-45	36
X.118	8190		630		-90		-50			-4095	414	-207	90	36	-45	-18	-18	-18
X.119	8190	8190	-210	-210	30	30	-18	-18		8190	171	171	-45	-45	-45	-45	-45	36
X.120	8190	-8190	-210	210	30	-30	-18	18		8190	171	171	-45	-45	-45	-45	-45	36
X.121	8190		-1050		150		14			-4095	-72	36	90	144	-45	-72	36	36
X.122	8736		1120		96		32			-4368	312	-156	132	-48	-66	24	-12	-12
X.123	9072		-1008		-48		48			-4536	324	-162	162		-81			
X.124	10530		450		-30		66			-5265	324	-162		162		-81		
X.125	10920		840		200		40			-5460	66	-33	-60	192	30	-96	-42	-42
X.126	10920		-1400		200		-24			-5460	228	-114	120	12	-60	-6	-42	-42
X.127	10920		-280		-280		40			-5460	-258	129	120	120	-60	-60	12	12
X.128	11648		-1792		128					-5824	-16	8	308	-16	-154	8	-16	-16
X.129	11648		1792		128					-5824	-16	8	308	-16	-154	8	-16	-16
X.130	11648	11648		128	128					11648	-16	-16	-124	-16	-124	-16	-16	-16
X.131	11648	-11648		128	-128					11648	-16	-16	-124	-16	-124	-16	-16	-16
X.132	13104		-1008		-48		-16			6552	468	-234	90	144	-45	72	36	36
X.133	14040		1080		120		24			-7020	-54	27	-108		54	-54	108	
X.134	14040		1080		120		24			-7020	-54	27	-108		54	108	-54	
X.135	14560		-1120		160		-32			-7280	-20	10	-20	-128	10	64	-20	88
X.136	14560		-1120		160		-32			-7280	-20	10	-20	-128	10	64	88	-20
X.137	14560		-1120		160		-32			-7280	-20	10	-20	304	10	-152	-20	-20
X.138	14742	-14742	630	-630	102	-102	6	-6		14742	162	162	-81	-81	-81	-81		
X.139	14742	14742	630	630	102	102	6	6		14742	162	162	-81	-81	-81	-81		
X.140	14742		1638		102		-42			-7371	162	-81	162	162	-81	-81		
X.141	16380		-420		60		-36			-8190	342	-171	-90	-90	45	45	-90	72
X.142	16380	-16380	-420	420	60	-60	28	-28		16380	99	99	-90	-36	-90	-36	18	18
X.143	16380	16380	-420	-420	60	60	28	28		16380	99	99	-90	-36	-90	-36	18	18
X.144	16380		-420		60		-36			-8190	342	-171	-90	-90	45	45	72	-90
X.145	16640	16640	-1280	-1280						16640	-208	-208	80	-64	80	-64	8	8
X.146	16640	16640	1280	1280						16640	-208	-208	80	-64	80	-64	8	8
X.147	16640	-16640	-1280	1280						16640	-208	-208	80	-64	80	-64	8	8
X.148	16640	-16640	1280	-1280						16640	-208	-208	80	-64	80	-64	8	8
X.149	17472	-17472	896	-896	-64	64				17472	-24	-24	30	-24	30	-24	-24	-24
X.150	17472	17472	896	896	-64	-64				17472	-24	-24	30	-24	30	-24	-24	-24
X.151	17472	-17472	-896	896	-64	64				17472	-24	-24	30	-24	30	-24	-24	-24
X.152	17472	17472	896	896	-64	-64				17472	-24	-24	30	-24	30	-24	-24	-24
X.153	17920	17920								17920	-224	-224	-80	64	-80	64	-8	-8
X.154	17920	17920								17920	-224	-224	-80	64	-80	64	-8	-8
X.155	17920	-17920								17920	-224	-224	-80	64	-80	64	-8	-8
X.156	17920	-17920								17920	-224	-224	-80	64	-80	64	-8	-8
X.157	19683	19683	-729	-729	-81	-81	27	27		19683								
X.158	19683	-19683	-729	729	-81	81	27	-27		19683								
X.159	21840	-21840	560	-560	-80	80	-48	48		21840	-30	-30	-30	24	-30	24	24	24
X.160	21840	21840	560	560	-80	-80	-48	-48		21840	-30	-30	-30	24	-30	24	24	24
X.161	22113	-22113	189	-189	-171	171	9	-9		22113	243	243						
X.162	22113	22113	189	189	-171	-171	9	9		22113	243	243						
X.163	23296				256					-11648	-32	16	-248	-32	124	16	-32	-32
X.164	29484		1260		204		12			-14742	324	-162	-162	-162	81	81		
X.165	32760		-840		120		56			-16380	198	-99	-180	-72	90	36	36	36
X.166	33280	</																

Character table of mE (continued)

	2	3	3	3	2	2	1	7	8	7	8	8	7	8	7	4	9	6	9	5	7	6
	3	8	8	7	7	7	7	3	3	2	2	2	2	1	1	2	7	9	5	7	7	7
	5	1	1	1	.	1	.	.	.
	7	1
13	1
	3j	3k	3l	3m	3n	3o	4a	4b	4c	4d	4e	4f	4g	4h	5a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	
2P	3j	3k	3l	3m	3n	3o	4a	4b	4c	4d	4e	4f	4g	4h	5a	3a	3b	3c	3d	3e	3f	
3P	1a	1a	1a	1a	1a	1a	4a	4b	4c	4d	4e	4f	4g	4h	5a	2b	2a	2d	2a	2a	2b	
5P	3j	3k	3l	3m	3n	3o	4a	4b	4c	4d	4e	4f	4g	4h	5a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	
7P	3j	3k	3l	3m	3n	3o	4a	4b	4c	4d	4e	4f	4g	4h	5a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	
13P	3j	3k	3l	3m	3n	3o	4a	4b	4c	4d	4e	4f	4g	4h	5a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	
X.88	5	-3	-5	3	-3	-3	3	3	.	225	-162	-15	.	-81	-18	
X.89	6	6	15	-3	15	-3	.	12	.	12	-4	-4	.	.	.	-140	-129	-140	60	60	-5	
X.90	-21	-21	-3	-3	-3	-3	.	4	.	4	4	4	.	.	.	420	33	100	-30	96	-39	
X.91	24	24	-12	6	6	-3	-20	12	.	.	12	-4	.	.	.	70	.	-10	.	.	76	
X.92	-21	-21	-3	-3	-3	-3	.	4	.	-4	4	-4	.	.	.	420	-33	100	30	-96	-39	
X.93	-3	-3	24	6	-12	-3	20	-4	.	.	-4	4	.	.	.	-490	.	-90	.	.	8	
X.94	-21	-21	6	6	6	6	.	-4	.	4	-4	4	.	.	.	-700	-114	100	-60	-6	2	
X.95	6	6	15	-3	15	-3	.	12	.	-12	-4	4	.	.	.	-140	129	-140	-60	-60	-5	
X.96	-21	-21	6	6	6	6	.	-4	.	-4	-4	-4	.	.	.	-700	114	100	60	6	2	
X.97	-10	6	.	.	-10	6	.	.	.	-315	.	-75	.	.	-18	
X.98	-8	-8	-8	-8	-8	-8	-16	.	.	-16	896	8	64	-154	8	32	
X.99	-8	-8	-8	-8	-8	-8	-16	.	.	-16	-1	896	-8	64	154	-8	32
X.100	-8	-8	-8	-8	-8	-8	-16	.	.	-16	-1	896	-8	64	154	-8	32
X.101	-8	-8	-8	-8	-8	-8	-16	.	.	-16	-1	896	8	64	-154	8	32
X.102	18	18	.	9	.	9	4	4	.	-4	-3	-504	-234	-24	-45	-72	-18	
X.103	18	18	.	9	.	9	4	4	.	4	-3	-504	234	-24	45	72	-18	
X.104	54	-27	4	.	-4	4	-4	.	.	.	540	27	60	.	54	-27	
X.105	-27	54	4	.	-4	4	-4	.	.	.	540	27	60	.	54	-27	
X.106	-27	54	4	.	4	4	4	.	.	.	540	-27	60	.	-54	-27	
X.107	54	-27	4	.	4	4	4	.	.	.	540	-27	60	.	-54	-27	
X.108	-10	44	8	-1	8	-1	-560	10	80	10	64	34	
X.109	-10	44	8	-1	8	-1	-560	-10	80	-10	-64	34	
X.110	44	-10	8	-1	8	-1	-560	10	80	10	64	34	
X.111	44	-10	8	-1	8	-1	-560	-10	80	-10	-64	34	
X.112	-10	-10	-10	-10	-10	-10	-560	-10	80	10	-152	34	
X.113	-10	-10	-10	-10	-10	-10	-560	-10	80	-10	152	34	
X.114	-1	-9	-1	-9	-1	-1	-1	-1	.	819	81	51	81	9	9	
X.115	-1	-9	1	9	-1	-1	1	1	1	819	81	51	-81	-81	9	
X.116	-45	36	-9	.	-9	.	10	2	-10	-2	2	2	-2	-2	.	-210	-171	30	45	45	-21	
X.117	-45	36	-9	.	-9	.	10	2	10	2	2	2	2	2	.	-210	171	30	-45	-45	-21	
X.118	9	9	.	18	.	-9	10	22	.	.	-2	2	.	.	.	-315	.	45	.	.	-18	
X.119	36	-45	-9	.	-9	.	10	2	10	2	2	2	2	2	.	-210	171	30	-45	-45	-21	
X.120	36	-45	-9	.	-9	.	10	2	-10	-2	2	2	-2	-2	.	-210	-171	30	45	45	-21	
X.121	-18	-18	18	.	-9	.	10	-18	.	.	6	2	.	.	.	525	.	-75	.	.	-24	
X.122	6	6	-12	6	6	-3	-4	-560	.	-48	.	.	40	
X.123	-8	.	.	.	-8	504	.	24	.	.	-36	
X.124	10	-6	.	.	-6	-6	.	.	.	-225	.	15	.	.	-36	
X.125	21	21	-6	-6	3	3	.	8	.	.	8	-420	.	-100	.	.	-78	
X.126	21	21	12	12	-6	-6	.	-8	.	.	-8	700	.	-100	.	.	4	
X.127	-6	-6	30	-6	-15	3	.	24	.	.	-8	140	.	140	.	.	-10	
X.128	8	8	-16	-16	8	8	-32	-2	896	.	-64	.	.	-64	
X.129	8	8	-16	-16	8	8	32	-2	-896	.	-64	.	.	64	
X.130	-16	-16	20	2	20	2	-2	.	-16	128	-124	-16	.	
X.131	-16	-16	20	2	20	2	-2	.	16	128	124	16	.	
X.132	-18	-18	.	18	.	-9	8	.	.	.	8	.	.	.	-6	504	.	24	.	.	-36	
X.133	27	-54	8	.	.	.	8	-540	.	-60	.	.	-54	
X.134	-54	27	8	.	.	.	8	-540	.	-60	.	.	-54	
X.135	10	-44	16	-2	-8	1	560	.	-80	.	.	68	
X.136	-44	10	16	-2	-8	1	560	.	-80	.	.	68	
X.137	10	10	-20	-20	10	10	560	.	-80	.	.	68	
X.138	-10	-6	10	6	-6	-2	6	2	2	630	-162	102	81	81	-18	
X.139	-10	-6	-10	-6	-6	-2	-6	-2	2	630	162	102	-81	-81	-18	
X.140	-2	-18	.	.	-2	-2	.	.	2	-819	.	-51	.	.	18	
X.141	45	-36	-18	.	9	20	4	.	.	.	4	4	.	.	.	210	.	-30	.	.	-42	
X.142	18	18	-9	-9	-9	-9	12	.	-12	.	-4	4	.	.	.	-420	-99	60	90	36	39	
X.143	18	18	-9	-9	-9	-9	12	.	12	.	-4	-4	.	.	.	-420	99	60	-90	-36	39	
X.144	-36	45	-18	.	9	20	4	.	.	.	4	4	.	.	.	210	.	-30	.	.	-42	
X.145	8	8	-10	8	-10	8	-1280	-208	.	80	-64	16	
X.146	8	8	-10	8	-10	8	1280	-208	.	80	-64	-16	
X.147	8	8	-10	8	-10	8	-1280	208	.	-80	64	16	
X.148	8	8	-10	8	-10	8	1280	208	.	-80	64	-16	
X.149	-24	-24	12	-6	12	-6	-16	.	16	-3	896	24	-64	-30	24	32	
X.150	-24	-24	12	-6	12	-6	16	.	16	-3	-896	-24	-64	30	-24	-32	
X.151	-24	-24	12	-6	12	-6	16	.	-16	-3	-896	24	-64	-30	24	-32	
X.152	-24	-24	12	-6	12	-6	-16	.	-16	-3	896	-24	-64	30	-24	32	
X.153	-8	-8	-8	10	-8	10	-224	.	-80	64	.	.	
X.154	-8	-8	-8	10	-8	10	-224	.	-80	64	.	.	
X.155	-8	-8	-8	10	-8	10	224	.	80	-64	.	.	
X.156	-8	-8	-8	10	-8	10	224	.	80	-64	.	.	
X.157	9	-9	9	-9	3	-3	3	-3	3	-729	.	-81	.	.	.	
X.158	9	-9	-9	9	3	-3	-3	3	3	-729	.	-81	.	.	.	
X.159	24	24	6	-12	6	-12	560	30	-80	30	-24	-34	
X.160	24	24	6	-12	6	-12	560	-30	-80	-30	24	-34	
X.161	-9	-3	9	3	1	3	-1	-3	3	189	-243	-171	.	.	27	
X.162	-9	-3	-9	-3	1	3	1	3	3	189	243	-171	.	.	27	
X.163	16	16	40	4	-20	-2	-4	.	.	-128	.	.	-36	
X.164	-20	-12	.	.	-12	-4	.	.	4	-630	.	-102	.	.	-78	
X.165	-18	-18	-18	-18	9	9	.	24	420	.	-60	.	.	32	
X.166	-8	-8	-20	16	10	-8	-1280	-32	
X.167	-8	-8	-20	16	10	-8	1280	32	

Character table of mE (continued)

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	3	7	6	4	7	7	6	6	5	6	6	4	5	5	5	6	4	4	4	4	3	6	4
	5
	7
	13
	67	68	69	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	
2P	3c	3b	3a	3h	3i	3d	3e	3b	3g	3f	3e	3c	3d	3l	3e	3b	3g	3e	3m	3c			
3P	2b	2c	2f	2a	2a	2b	2b	2d	2b	2f	2c	2d	2c	2a	2d	2e	2f	2f	2g	2a	2f		
5P	67	68	69	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	
7P	67	68	69	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	
13P	67	68	69	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	
X.88	-18	18	33	.	.	-18	9	-6	9	-18	9	-9	-6	18	.	-3	6	6	9	-9	.	6	
X.89	-5	-5	20	6	6	4	4	-5	4	4	-4	4	-5	4	15	4	-5	-1	-4	-4	-3	-1	
X.90	-39	-39	20	-21	-21	6	6	1	6	6	8	6	1	6	-3	4	1	-7	8	8	-3	-7	
X.91	-38	.	6	.	.	22	-14	20	7	-11	-6	.	-10	.	.	2	12	3	.	.	.	-6	
X.92	-39	39	20	21	21	6	6	1	6	6	8	-6	1	-6	3	4	-1	-7	8	-8	3	-7	
X.93	-4	-26	.	.	.	26	-10	.	5	-13	-14	-6	.	-8	7	.	.	4	
X.94	2	-2	-12	21	21	-16	-16	10	-16	-16	6	16	10	16	-6	-2	-10	-6	6	-6	-6	-6	
X.95	-5	5	20	-6	-6	4	4	-5	4	4	-4	-4	-5	-4	-15	4	5	-1	-4	4	3	-1	
X.96	2	2	-12	-21	-21	-16	-16	10	-16	-16	6	-16	10	-16	6	-2	10	-6	6	6	6	-6	
X.97	9	-3	.	.	.	36	-18	6	9	-18	18	.	-3	.	.	-6	.	6	-9	.	.	-3	
X.98	32	-32	.	8	8	-4	-4	-8	-4	-4	.	-4	-8	4	8	4	8	8	
X.99	32	32	.	-8	-8	-4	-4	-8	-4	-4	.	-4	-8	-4	-8	4	-8	-8	
X.100	-32	-32	.	-8	-8	4	4	-8	4	4	.	4	-8	4	-8	4	-8	-8	
X.101	-32	32	.	8	8	4	4	-8	4	4	.	-4	-8	-4	8	4	8	8	
X.102	-18	18	-8	-18	-18	9	-18	-6	-18	9	-8	18	-6	-9	.	6	-2	-8	8	-9	-2	-2	
X.103	-18	-18	-8	18	18	9	-18	-6	-18	9	-8	-18	-6	9	.	-6	-2	-8	-8	9	-2	-2	
X.104	-27	27	12	-54	27	.	.	-3	.	.	-6	.	-3	.	.	-6	3	-3	-6	6	-3	-3	
X.105	-27	27	12	27	-54	.	.	-3	.	.	-6	.	-3	.	.	-6	3	-3	-6	6	-3	-3	
X.106	-27	-27	12	-27	54	.	.	-3	.	.	-6	.	-3	.	.	-6	3	-3	-6	-6	-3	-3	
X.107	-27	-27	12	54	-27	.	.	-3	.	.	-6	.	-3	.	.	-6	3	-3	-6	-6	-3	-3	
X.108	34	-34	-16	10	-44	-2	-2	-10	-2	-2	8	-2	-10	-2	-8	-4	10	2	8	-8	1	2	
X.109	34	34	-16	-10	44	-2	-2	-10	-2	-2	8	-2	-10	-2	8	-4	-10	2	8	-8	-1	2	
X.110	34	-34	-16	-44	10	-2	-2	-10	-2	-2	8	-2	-10	-2	-8	-4	10	2	8	-8	1	2	
X.111	34	34	-16	44	-10	-2	-2	-10	-2	-2	8	-2	-10	-2	8	-4	-10	2	8	-8	-1	2	
X.112	34	-34	-16	10	10	-2	-2	-10	-2	-2	-16	-2	-10	-2	10	-4	10	-2	-16	16	10	2	
X.113	34	34	-16	-10	-10	-2	-2	-10	-2	-2	-16	-2	-10	-2	-10	-4	-10	-2	-16	-16	-10	2	
X.114	9	9	-21	.	.	9	9	-3	9	9	9	-3	9	-3	9	-3	9	-3	9	-9	-3	-3	
X.115	9	-9	-21	.	.	9	9	-3	9	9	9	-3	9	-3	9	-3	9	-3	9	-9	-3	-3	
X.116	-21	21	-18	45	-36	-3	-3	3	-3	-3	3	3	3	3	9	3	-3	3	3	-3	3	3	
X.117	-21	-21	-18	-45	36	-3	-3	3	-3	-3	3	-3	3	-3	9	3	-3	3	3	3	3	3	
X.118	9	.	25	.	.	-18	36	-18	-18	9	4	.	9	.	-12	.	-2	-2	.	.	.	1	
X.119	-21	-21	-18	36	-45	-3	-3	3	-3	-3	3	-3	3	-3	9	3	-3	3	3	3	3	3	
X.120	-21	21	-18	-36	45	-3	-3	3	-3	-3	3	3	3	3	9	3	-3	3	3	-3	3	3	
X.121	12	.	-7	.	.	-42	12	24	-6	21	-16	.	-12	8	8	.	.	-4	
X.122	-20	.	-16	.	.	4	40	24	-20	-2	-16	.	-12	8	8	.	.	-4	
X.123	18	.	-24	.	.	18	-36	-12	18	-9	.	.	6	12	.	.	.	-6	
X.124	18	.	-33	.	.	-36	18	-12	-9	18	18	.	6	.	.	-6	.	12	-9	.	.	-6	
X.125	39	.	-20	.	.	12	12	-2	-6	-6	16	.	-1	.	.	8	.	-14	-8	.	.	7	
X.126	-2	.	12	.	.	-32	-32	20	16	16	12	-10	.	.	-4	.	-12	-6	.	.	.	6	
X.127	5	.	-20	.	.	8	8	-10	-4	-4	-8	.	5	.	.	8	.	-2	4	.	.	1	
X.128	32	8	8	-16	-4	-4	.	8	.	.	8	
X.129	-32	-8	-8	16	4	4	.	-8	.	.	-8	
X.130	.	.	.	-16	-16	.	.	-16	.	.	.	-16	.	.	20	.	-16	2	
X.131	.	.	.	16	16	.	.	-16	.	.	.	-16	.	.	-20	.	8	-16	.	.	.	-2	
X.132	18	.	8	.	.	18	-36	-12	18	-9	-16	.	6	.	.	-12	.	-4	8	.	.	2	
X.133	27	.	-12	-6	.	-12	.	3	.	.	-12	.	-6	6	.	.	.	3	
X.134	27	.	-12	-6	.	-12	.	3	.	.	-12	.	-6	6	.	.	.	3	
X.135	-34	.	16	.	.	-4	-4	-20	2	2	16	.	10	.	.	-8	.	4	-8	.	.	-2	
X.136	-34	.	16	.	.	-4	-4	-20	2	2	16	.	10	.	.	-8	.	4	-8	.	.	-2	
X.137	-34	.	16	.	.	-4	-4	-20	2	2	-32	.	10	.	.	-8	.	4	16	.	.	-2	
X.138	-18	18	6	.	.	9	9	-6	9	9	-9	-9	-6	-9	.	3	6	6	-9	9	.	6	
X.139	-18	-18	6	.	.	9	9	-6	9	9	-9	9	-6	9	.	3	-6	6	-9	-9	.	6	
X.140	-9	.	21	.	.	18	18	-6	-9	-9	18	.	3	.	.	-6	.	-6	-9	.	.	3	
X.141	21	.	18	.	.	-6	-6	6	3	3	6	.	-3	.	.	6	.	6	-3	.	.	-3	
X.142	39	-39	28	-18	-18	-6	-6	15	-6	-6	4	6	15	6	9	.	-15	-5	4	-4	9	-5	
X.143	39	39	28	18	18	-6	-6	15	-6	-6	4	-6	15	-6	-9	.	15	-5	4	4	-9	-5	
X.144	21	.	18	.	.	-6	-6	6	3	3	6	.	-3	.	.	6	.	6	-3	.	.	-3	
X.145	16	16	.	8	8	16	16	.	16	16	.	16	.	16	-10	8	
X.146	-16	-16	.	8	8	-16	-16	.	-16	-16	.	-16	.	-16	-10	8	
X.147	16	-16	.	-8	-8	16	16	.	16	16	.	-16	.	16	10	-8	
X.148	-16	16	.	-8	-8	-16	-16	.	-16	-16	.	16	.	16	10	-8	
X.149	32	-32	.	24	24	-4	-4	8	-4	-4	.	4	8	4	-12	-4	-8	6	
X.150	-32	32	.	-24	-24	4	4	8	4	4	.	-4	8	4	12	-4	8	-6	
X.151	-32	32	.	24	24	4	4	8	4	4	.	-4	8	4	-12	-4	-8	6	
X.152	32	32	.	-24	-24	-4	-4	8	-4	-4	.	-4	8	-4	12	-4	8	-6	
X.153	.	.	.	-8	-8	-8	10	
X.154	.	.	.	-8	-8	-8	10	
X.155	.	.	.	8	8	8	-10	
X.156	.	.	.	8	8	8	-10	
X.157	.	.	27	
X.158	.	.	27	
X.159	-34	34	-48	-24	-24	2	2	10	2	2	.	-2	10	-2	-6	4	-10	6	.	.	12	6	
X.160	-34	-34	-48	24	24	2	2	10	2	2	.	2	10	2	6	4	10	6	.	.	-12	6	
X.161	27	-27	9	-9	-9	.	.	.	9	-9	.	.	.	-9	
X.162	27	27	9	-9	-9	.	.	.	-9	-9	.	.	.	-9	
X.163	-32	.	.	.	16	.	.	16	
X.164	18	.	-6	.	.	18	18	-12	-9	-9	-18	.	6	.	.	6	.	12					

2	3	3	3	1	2	2	5	5	5	5	3	2	2	1	1	1	.	.	4	4	3	4	3	6	7	2
3	4	3	3	4	3	1	1	1	.	.	5	5	5	5	4	4	4	.	.	1	1	1	1	3	3	2
5	1	1	1	1	1	1	.	.
13	1	1	1	1	1	1	1	.	.
2P	6 ₅₄	6 ₅₅	6 ₅₆	6 ₅₇	6 ₅₈	7 _a	8 _a	8 _b	8 _c	8 _d	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f	9 _g	9 _h	10 _a	10 _b	10 _c	10 _d	10 _e	12 ₁	12 ₂	12 ₃
3P	2 _f	2 _g	2 _g	2 _f	2 _g	7 _a	4 _e	4 _b	4 _b	4 _e	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f	9 _g	9 _h	5 _a	5 _a	5 _a	5 _a	5 _a	6 ₃	6 ₉	6 ₉
5P	6 ₅₄	6 ₅₅	6 ₅₆	6 ₅₇	6 ₅₈	7 _a	8 _a	8 _b	8 _c	8 _d	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f	9 _g	9 _h	2 _d	2 _a	2 _b	2 _e	2 _c	12 ₁	12 ₂	12 ₃
7P	6 ₅₄	6 ₅₅	6 ₅₆	6 ₅₇	6 ₅₈	7 _a	8 _a	8 _b	8 _c	8 _d	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f	9 _g	9 _h	10 _a	10 _b	10 _c	10 _d	10 _e	12 ₁	12 ₂	12 ₃
13P	6 ₅₄	6 ₅₅	6 ₅₆	6 ₅₇	6 ₅₈	7 _a	8 _a	8 _b	8 _c	8 _d	9 _a	9 _b	9 _c	9 _d	9 _e	9 _f	9 _g	9 _h	10 _a	10 _b	10 _c	10 _d	10 _e	12 ₁	12 ₂	12 ₃
X.88	1	-1	-1	1	1	5	-3	-3
X.89	-1	-1	-1	-1	-1	-3	-3	-3	-3	12	-4
X.90	-1	-1	-1	-1	-1	-6	4	4
X.91	3	-6	10	-6	-6	
X.92	-1	1	1	-1	1	-6	4	4	4
X.93	-2	.	.	1	-6	-10	2	2	2
X.94	-3	-3	-3	-3	-4	4
X.95	-1	1	1	-1	1	-3	-3	-3	-3	12	-4
X.96	-3	-3	-3	-3	-4	4
X.97	-2	-2	5	-3	5
X.98	1	1	1	1	1	1	1	1	-1	1	1	1	-1	16	.	.
X.99	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	1	16	.
X.100	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-16	.
X.101	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-16	.
X.102	-2	2	2	2	1																					

Character table of mE (continued)

	2	3	3	3	4	1	1	2	2	2	3	1	.	3	3	3	2	2	2	2	3	3	3	1	2	
	3	2	2	2	1	1	1	1	.	.	2	2	2	4	4	4	4	4	4	4	4	3	3	3	4	3
	5	1	1	1
	7	1	1	1
13	1	1
	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈	13 _a	13 _b	14 _a	14 _b	14 _c	15 _a	15 _b	15 _c	18 _a	18 _b	18 _c	18 _d	18 _e	18 _f	18 _g	18 _h	18 _i	18 _j	18 _k	18 _l	18 _m	
2P	6 ₃₆	6 ₃₇	6 ₃₈	6 ₃₉	13 _b	13 _a	7 _a	7 _a	7 _a	15 _b	15 _c	9 _a	9 _a	9 _a	9 _c	9 _b	9 _b	9 _c	9 _a	9 _a	9 _a	9 _d	9 _b	9 _b	9 _b	
3P	4 _d	4 _f	4 _d	4 _h	13 _a	13 _b	14 _a	14 _b	14 _c	5 _a	5 _a	5 _a	6 ₆	6 ₂	6 ₆	6 ₆	6 ₆	6 ₂	6 ₆	6 ₈	6 ₁₄	6 ₈	6 ₆	6 ₆	6 ₈	
5P	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈	13 _b	13 _a	14 _a	14 _b	14 _c	3 _a	3 _d	3 _f	18 _c	18 _b	18 _a	18 _g	18 _e	18 _f	18 _d	18 _j	18 _i	18 _k	18 _l	18 _k	18 _l	
7P	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈	13 _b	13 _a	2 _b	2 _a	2 _c	15 _a	15 _b	15 _c	18 _a	18 _b	18 _c	18 _d	18 _e	18 _f	18 _g	18 _h	18 _i	18 _j	18 _k	18 _l	18 _l	
13P	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈	1 _a	1 _a	14 _a	14 _b	14 _c	15 _a	15 _b	15 _c	18 _a	18 _b	18 _c	18 _d	18 _e	18 _f	18 _g	18 _h	18 _i	18 _j	18 _k	18 _l	18 _l	
X.88	1	-1	-1	1	-3	1	1	1	-3	1	1	1	1	1	1	
X.89	-6	-2	
X.90	1	.	1	-2	-6	-2	1	4	.	1	.	2	-2	.	.	
X.91	.	-1	
X.92	-1	.	-1	6	-2	
X.93	.	1	-4	-4	2	2	.	2	.	.	-1	.	.	
X.94	1	.	1	-1	3	-1	-1	-1	3	-1	1	1	-1	1	1	
X.95	1	3	1	1	1	3	1	-1	-1	-1	-1	
X.96	-1	.	-1	-1	-3	-1	-1	-1	-3	-1	-1	-1	-1	-1	-1	
X.97	
X.98	2	2	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	
X.99	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	
X.100	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	
X.101	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	
X.102	.	1	.	-1	-3	
X.103	.	1	.	1	-3	
X.104	-1	.	2	.	.	.	1	1	-1	
X.105	2	.	-1	.	.	.	1	1	-1	
X.106	-2	.	1	.	.	.	1	-1	1	
X.107	1	.	-2	.	.	.	1	-1	1	
X.108	1	1	1	1	1	1	1	-1	-1	-1	1	-1	
X.109	1	-1	1	1	1	-1	1	1	-1	1	1	1	
X.110	1	1	1	1	1	1	1	-1	-1	-1	1	-1	
X.111	1	-1	1	1	1	-1	1	1	-1	1	1	1	
X.112	1	1	1	1	1	1	1	-1	-1	-1	1	-1	
X.113	1	-1	1	1	1	-1	1	1	-1	1	1	1	
X.114	1	1	1	1	1	1	1	1	1	1	1	1	
X.115	.	-1	.	-1	1	1	1	1	1	1	1	1	1	1	1	1	
X.116	-2	-1	1	1	
X.117	2	-1	-1	-1	
X.118	.	-1	
X.119	-1	-1	2	-1	
X.120	1	-1	-2	1	
X.121	.	-1	
X.122	2	2	-1	4	.	.	4	-2	-2	.	-2	.	.	.	1	.	
X.123	.	1	.	.	-2	-2	.	.	.	-1	2	-1	
X.124	2	
X.125	
X.126	-2	.	-2	1	-2	.	1	.	2	.	1	.	
X.127	2	.	2	-1	2	.	-1	.	2	.	-1	.	
X.128	1	-2	1	2	.	-2	-1	2	.	2	.	-1	
X.129	1	-2	1	-2	.	-2	1	-2	.	2	.	1	
X.130	-2	1	1	2	.	.	.	
X.131	-2	1	1	2	.	.	.	
X.132	.	-1	3	
X.133	2	
X.134	2	
X.135	2	.	2	-1	2	.	-1	.	-2	.	-1	.	
X.136	2	.	2	-1	2	.	-1	.	-2	.	-1	.	
X.137	2	.	2	-1	2	.	-1	.	-2	.	-1	.	
X.138	.	1	.	-1	2	-1	-1	
X.139	.	1	.	1	2	-1	-1	
X.140	.	1	-1	2	-1	
X.141	.	1	
X.142	
X.143	
X.144	.	1	
X.145	1	1	1	-2	-4	-2	-2	-2	2	-2	-2	.	-2	-2	-2	
X.146	-1	1	-1	2	-4	2	2	2	2	2	2	.	2	2	2	
X.147	1	-1	-1	-2	4	-2	-2	-2	-2	-2	-2	.	2	-2	2	
X.148	-1	-1	1	2	4	2	2	2	-2	-2	-2	.	-2	-2	2	
X.149	-1	-3	-1	-1	-1	-3	-1	1	-1	1	-1	1	
X.150	1	3	1	1	1	-3	1	1	-1	1	1	1	
X.151	-3	.	1	1	1	-3	1	-1	-1	-1	-1	-1	
X.152	-3	.	-1	3	-1	-1	-1	3	-1	-1	-1	-1	
X.153	4	
X.154	4	.	.	-2	
X.155	-4	.	.	2	
X.156	-4	.	.	2	
X.157	1	-1	-1	-1	3	
X.158	1	-1	1	1	3	
X.159	-1	3	-1	-1	-1	3	-1	1	1	-1	
X.160	-1	-3	-1	-1	-1	-3	-1	-1	-1	-1	
X.161	
X.162	
X.163	.	.	.																							

Character table of mE (continued)

2	2	3	1	1	3	3	1	4	4	1	1	3	2	1			1	2
3	3	2	3	3	1	1	1	1	1			1	1	1	1	1	1	1
5					1	1						1	1	1			1	1
7							1				1					1	1	
13										1						1	1	
	18m	18n	18o	18p	20a	20b	21a	24a	24b	26a	26b	30a	30b	30c	39a	39b	42a	60a
2P	9c	9a	9e	9f	10a	10a	21a	12 ₃	12 ₂	13a	13b	15a	15b	39b	39a	21a	30a	
3P	6 ₁₄	6 ₂₃	6 ₁₀	6 ₁₁	20a	20b	7a	8a	8b	26a	26b	10a	10c	10b	13a	13b	14a	20a
5P	18m	18n	18o	18p	4a	4c	21a	24a	24b	26b	26a	6 ₃	6 ₁	6 ₄	39b	39a	42a	12 ₁
7P	18m	18n	18o	18p	20a	20b	3a	24a	24b	26b	26a	30a	30b	30c	39b	39a	6 ₁	60a
13P	18m	18n	18o	18p	20a	20b	21a	24a	24b	2a	2a	30a	30b	30c	3a	3a	42a	60a
X.88			1					1	-1	-1							1	
X.89																		
X.90	-2	-2																
X.91	-1																	
X.92	-2	2																
X.93																		
X.94	1	-1																
X.95	1	-1																
X.96	1	1																
X.97								1	1						-1	-1		
X.98	1	-1	-1	-1	1	-1					-1	1	1				1	
X.99	1	1	1	1	1	1					-1	1	-1				1	
X.100	1	1	1	1	1	-1	-1				-1	-1	-1				-1	
X.101	1	-1	-1	-1	-1	1					-1	-1	1				-1	
X.102					-1	1					1	1					-1	
X.103					-1	-1					1	1					-1	
X.104						-1											1	
X.105						-1											1	
X.106						-1											1	
X.107						-1											1	
X.108	-1	1	-2	1														
X.109	-1	-1	2	-1														
X.110	-1	1	1	-2														
X.111	-1	-1	-1	2														
X.112	-1	1	1	1														
X.113	-1	-1	-1	-1														
X.114					-1	-1		1	1			1	-1	1			-1	
X.115					-1	1		1	1			1	-1	-1			-1	
X.116																		
X.117								-1	1									
X.118																		
X.119																		
X.120																		
X.121								1	-1									
X.122												2						
X.123					2							-1	-1		1	1	-1	-1
X.124						-1	1	1								-1		
X.125	2																	
X.126	-1																	
X.127	-1																	
X.128	-1				-2							1	1					
X.129	-1				2							1	-1					
X.130	2	2	-1	-1								-2		1				
X.131	2	-2	1	1								-2		-1				
X.132					-2							-1	-1					
X.133						1											-1	
X.134						1											-1	
X.135	1																	
X.136	1																	
X.137	1																	
X.138												2		1				
X.139												2		-1				
X.140					-2		-1	-1			-1	1						
X.141																		
X.142																		
X.143																		
X.144																		
X.145			-1	-1		1											1	
X.146			-1	-1		1											-1	
X.147			1	1		1											1	
X.148			1	1		1											-1	
X.149	-1	1			-1	1						1	1				-1	
X.150	-1	-1			1	1						1	-1				1	
X.151	-1	1			1	-1						1	-1				1	
X.152	-1	-1			-1	-1						1	1				-1	
X.153			1	1						D	C				C	D		
X.154			1	1						C	D				D	C		
X.155			-1	-1						-C	-D				D	-C		
X.156			-1	-1						-D	-C				C	D		
X.157					-1	-1	-1	1	-1	1	-1	1	1		1	1	-1	-1
X.158					-1	1	-1	1	-1	-1	-1	-1	1		1	1	-1	-1
X.159	1	-1																
X.160	1	1																
X.161					1	-1		-1	1			-1	-1				1	
X.162					1	1		-1	1			-1	-1				1	
X.163	-2											2						
X.164												-2						
X.165																		
X.166						-1											1	
X.167						-1											-1	
X.168	1				-2							-1	-1				1	
X.169	1				2							-1	1				-1	
X.170																		
X.171															-D	-C		
X.172					-2		1	-1	1			1	-1		-C	-D	1	1
X.173	-1														-1	-1		
X.174					2			1	-1			1	1					

where $A = -6\zeta(3) - 3$, $B = -12\zeta(3) - 6$, $C = -\zeta(13)^{11} - \zeta(13)^8 - \zeta(13)^7 - \zeta(13)^6 - \zeta(13)^5 - \zeta(13)^2 - 1$, $D = -C - 1$, $E = 2D$, $F = 2C$.

B.4. Character table of $H(\text{Fi}'_{24}) = \langle b, p, r \rangle$

	21	21	19	20	14	14	17	17	16	13	8	12	10	5	8	5	6	15	15
3	7	7	4	2	4	4	3	3	1	2	7	6	7	7	5	6	4	1	2
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	3c	3d	3e	3f	3g	4a	4b
2P	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	3a	3b	3c	3d	3e	3f	3g	2a	2a
3P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	1a	1a	1a	1a	1a	1a	1a	4a	4b
5P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	3c	3d	3e	3f	3g	4a	4b
7P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	3c	3d	3e	3f	3g	4a	4b
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.3	21	21	21	21	9	9	5	5	5	1	21	3	-6	-6	3	3	3	21	5
X.4	21	21	21	21	-9	-9	5	5	5	-1	21	3	-6	-6	3	3	3	21	5
X.5	30	30	30	30			-2	-2	-2		-15	6	-6	12		-3		30	-2
X.6	35	35	35	35	15	15	3	3	3	-1	35	-1	8	8	8	-1	-1	35	3
X.7	35	35	35	35	-5	-5	3	3	3	-5	35	8	8	8	-1	8	-1	35	3
X.8	35	35	35	35	5	5	3	3	3	5	35	8	8	8	-1	8	-1	35	3
X.9	35	35	35	35	-15	-15	3	3	3	1	35	-1	8	8	8	-1	-1	35	3
X.10	42	42	42	42			10	10	10		-21	12	-3	6		-6		42	10
X.11	90	90	90	90	-30	-30	10	10	10	-6	90	9	9	9	9	9		90	10
X.12	90	90	90	90	30	30	10	10	10	6	90	9	9	9	9	9		90	10
X.13	140	140	140	140	-20	-20	12	12	12	-4	140	-4	5	5	-4	-4	5	140	12
X.14	140	140	140	140	20	20	12	12	12	4	140	-4	5	5	-4	-4	5	140	12
X.15	189	189	189	189	-9	-9	-3	-3	-3	-9	189		27	27				189	-3
X.16	189	189	189	189	9	9	-3	-3	-3	9	189		27	27				189	-3
X.17	210	210	210	210			18	18	18		-105	6	-15	30		-3		210	18
X.18	210	210	210	210	30	30	2	2	2	-10	210	3	21	21	3	3	3	210	2
X.19	210	210	210	210			-14	-14	-14		-105	6	-15	30		-3		210	-14
X.20	210	210	210	210	-30	-30	2	2	2	10	210	3	21	21	3	3	3	210	2
X.21	210	210	210	210			18	18	18		-105	24	12	-24		-12		210	18
X.22	280	280	280	280	-40	-40	-8	-8	-8	8	280	1	10	10	10	1	1	280	-8
X.23	280	280	280	280	-40	-40	-8	-8	-8	8	280	1	10	10	10	1	1	280	-8
X.24	280	280	280	280	40	40	-8	-8	-8	-8	280	1	10	10	10	1	1	280	-8
X.25	280	280	280	280	40	40	-8	-8	-8	-8	280	1	10	10	10	1	1	280	-8
X.26	315	315	315	315	-15	-15	11	11	11	9	315	18	-9	-9	-9	18		315	11
X.27	315	315	315	315	15	15	11	11	11	3	315	-9	-9	-9	18	-9		315	11
X.28	315	315	315	315	15	15	11	11	11	-9	315	18	-9	-9	-9	18		315	11
X.29	315	315	315	315	-75	-75	11	11	11	-3	315	-9	-9	-9	18	-9		315	11
X.30	378	378	58	-6	-36	-36	42	42	10	12		54	27		9			-6	-6
X.31	378	378	58	-6	-36	-36	42	42	10	-12		54	27		9			-6	-6
X.32	420	420	420	420	60	60	4	4	4	-4	420	6	-39	-39	6	6	-3	420	4
X.33	420	420	420	420	-60	-60	4	4	4	4	420	6	-39	-39	6	6	-3	420	4
X.34	420	420	420	420			4	4	4		-210	30	-3	6		-15		420	4
X.35	560	560	560	560			-16	-16	-16		560	20	20	20	2	20	2	560	-16
X.36	560	560	560	560			-16	-16	-16		560	2	-34	-34	2	2	2	560	-16
X.37	560	560	560	560			-16	-16	-16		560	2	-34	-34	2	2	2	560	-16
X.38	630	630	630	630			-10	-10	-10		-315	18	36	-72		-9		630	-10
X.39	672	672	672	672			32	32	32		-336	12	6	-12		-6		672	32
X.40	720	720	720	720			16	16	16		-360	-18	18	-36		9		720	16
X.41	720	720	720	720			16	16	16		-360	-18	18	-36		9		720	16
X.42	729	729	729	729	-81	-81	9	9	9	-9	729							729	9
X.43	729	729	729	729	81	81	9	9	9	9	729							729	9
X.44	768	-768					-64	64			-6	96	-24	-6		6			
X.45	768	768	768	768							-384	24	-24	48		-12		768	
X.46	840	840	840	840			8	8	8		-420	-12	-33	66		6		840	8
X.47	896	896	896	896	64	64					896	-4	32	32		-4	-4	896	
X.48	896	896	896	896	-64	-64					896	-4	32	32		-4	-4	896	
X.49	1260	1260	1260	1260			12	12	12		-630	-18	-9	18		9		1260	12
X.50	1280	1280	1280	1280							1280	-16	-16	-16	-16	-16	2	1280	
X.51	1280	-1280					64	-64			20	32	56	-7	8	-4	8		
X.52	1280	-1280					64	-64			20	32	56	-7	8	-4	8		
X.53	1458	1458	1458	1458			18	18	18		-729							1458	18
X.54	1512	1512	1512	1512			-24	-24	-24		-756		-27	54				1512	-24
X.55	1701	1701	-27	37	27	27	117	117	21	19		81					9	-27	-27
X.56	1701	1701	-27	37	-27	-27	117	117	21	-19		81					9	-27	-27
X.57	1890	1890	1890	1890			-30	-30	-30		-945		27	-54				1890	-30
X.58	2016	2016	-32	-32			96	96	-32	32		120	36		6	3	6	32	32
X.59	2016	2016	-32	-32			96	96	-32	-32		120	36		6	3	6	32	32
X.60	2268	2268	348	-36	-144	-144	60	60	-4				-81		27			-36	-36
X.61	2268	2268	348	-36	144	144	60	60	-4				-81		27			-36	-36
X.62	3584	-3584			-192	192	-128	128			56	32	-16	2	44	-4	8		
X.63	3584	-3584			128	-128	-128	128			56	176	-16	2	8	-22	8		
X.64	3584	-3584			-128	128	-128	128			56	176	-16	2	8	-22	8		
X.65	3584	-3584			192	-192	-128	128			56	32	-16	2	44	-4	8		
X.66	3780	3780	580	-60			36	36	-28			216	27		-18			-60	-60
X.67	4480	-4480			160	-160	-32	32			70	112	-128	16	28	-14	-8		
X.68	4480	-4480			-160	160	-32	32			70	112	-128	16	28	-14	-8		
X.69	4480	-4480			160	-160	-32	32			70	-32	88	-11	28	4	-8		
X.70	4480	-4480			-160	160	-32	32			70	-32	88	-11	28	4	-8		
X.71	4480	-4480			160	160	-32	32			70	-32	88	-11	28	4	-8		
X.72	4480	-4480			-160	160	-32	32			70	-32	88	-11	28	4	-8		
X.73	5670	5670	870	-90	180	180	150	150	54	12		162	-81					-90	6
X.74	5670	5670	870	-90	180	180	-42	-42	-10	-36			162		27			-90	6
X.75	5670	5670	870	-90	-180	-180	-42	-42	-10	36			162		27			-90	6
X.76	5670	5670	870	-90	-180	-180	150	150	54	-12		162	-81					-90	6
X.77	7560	7560	1160	-120			-120	-120	8			108	-189		18			-120	72
X.78	7560	7560	1160	-120	-360	-360	168	168	40	-24		-54	54		45			-120	-24
X.79	7560	7560	1160	-120	360	360	168	168	40	24		-54	54		45			-120	-24
X.80	7680	-7680					-128	128			120	-96	-96	12	-24	12	12		
X.81	7680	-7680					-128	128											

Character table of $H(\text{Fi}'_{24})$ (continued)

3	8	9	10	5	12	8	3	8	3	8	3	8	6	7	5	5	5	5	7	3
3	5	4	3	6	2	3	3	3	3	3	3	3	4	4	3	4	4	4	4	2
5
7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
2P	3e	3c	3b	3f	3g	3a	3b	3c	3d	3e	3f	3g	3h	3i	3j	3k	3l	3m	3n	3o
3P	2a	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n	2o	2p	2q	2r	2s	2t
5P	6a	6b	6c	6d	6e	6f	6g	6h	6i	6j	6k	6l	6m	6n	6o	6p	6q	6r	6s	6t
7P	65	66	67	68	69	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.3	3	-6	3	3	3	-1	5	-3	3	5	2	1	-1	1	3	3	3	3	3	3
X.4	3	-6	3	3	3	-1	5	3	3	5	2	2	-1	-1	3	3	3	3	3	3
X.5	5	-6	6	6	6	-2	1	3	3	5	2	-2	4	-1	3	3	3	3	3	3
X.6	8	-8	-1	-1	-1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
X.7	-1	8	8	8	8	3	-2	2	3	3	3	3	3	3	3	3	3	3	3	3
X.8	-1	8	8	8	8	3	-2	2	3	3	3	3	3	3	3	3	3	3	3	3
X.9	8	-8	-1	-1	-1	3	3	-3	-3	3	3	3	3	3	3	3	3	3	3	3
X.10	-3	12	-6	12	4	-5	3	-3	-3	3	3	3	3	3	3	3	3	3	3	3
X.11	9	9	9	9	9	1	10	-3	-3	10	1	1	1	1	1	1	1	1	1	1
X.12	9	9	9	9	9	1	10	-3	-3	10	1	1	1	1	1	1	1	1	1	1
X.13	-4	5	-4	-4	-4	12	-2	2	12	-3	-3	3	3	3	3	3	3	3	3	3
X.14	-4	5	-4	-4	-4	12	-2	2	12	-3	-3	3	3	3	3	3	3	3	3	3
X.15	27	3	.	.	3	3	3	3	3	3	3	3	3	3	3	3
X.16	27	3	.	.	3	3	3	3	3	3	3	3	3	3	3	3
X.17	-15	6	-3	6	6	-9	3	3	3	3	3	3	3	3	3	3	3	3	3	3
X.18	3	21	3	3	3	-1	2	3	3	2	5	-1	-1	3	3	3	3	3	3	3
X.19	-15	6	-3	6	-2	7	7	1	4	4	-2	2	2	3	3	3	3	3	3	3
X.20	3	21	3	3	3	-1	2	-3	-3	2	5	-1	-1	3	3	3	3	3	3	3
X.21	12	24	-12	24	.	-9	.	.	-9
X.22	10	10	1	1	1	1	8	-1	-1	-8	-2	-2	-2	1	1	10	A	A	A	A
X.23	10	10	1	1	1	1	8	-1	-1	-8	-2	-2	-2	1	1	10	A	A	A	A

Character table of $H(\text{Fi}'_{24})$ (continued)

Character table of $H(\text{Fi}'_{24})$ (continued)

	2	6	5	5	5	4	4	9	7	9	6	4	8	8	7	5	5	6	6	6	6	6	7
	3	1	1	1	1	1	1	5	5	3	4	5	2	2	2	3	3	2	2	2	2	2	1
	5	1	1	1	1	1	1	5	5	3	4	5	2	2	2	3	3	2	2	2	2	2	1
	7	1	1	1	1	1	1	5	5	3	4	5	2	2	2	3	3	2	2	2	2	2	1
	10a	10b	10c	10d	10e	10f	12 ₁	12 ₂	12 ₃	12 ₄	12 ₅	12 ₆	12 ₇	12 ₈	12 ₉	12 ₁₀	12 ₁₁	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	
2P	5a	5a	5a	5a	5a	5a	6 ₂	6 ₃	6 ₂	6 ₅	6 ₈	6 ₉	6 ₆	6 ₃	6 ₂₀	6 ₂₀	6 ₇	6 ₁₄	6 ₇	6 ₅	6 ₁₄	6 ₉	
3P	10a	10c	10b	10d	10e	10f	4a	4a	4a	4a	4a	4e	4e	4b	4a	4a	4d	4g	4h	4b	4i	4c	
5P	2a	2c	2c	2b	2d	2e	12 ₁	12 ₂	12 ₃	12 ₄	12 ₅	12 ₆	12 ₇	12 ₈	12 ₉	12 ₁₀	12 ₁₁	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	
7P	10a	10c	10b	10d	10e	10f	12 ₁	12 ₂	12 ₃	12 ₄	12 ₅	12 ₆	12 ₇	12 ₈	12 ₉	12 ₁₀	12 ₁₁	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.2	1	1	1	1	1	-1	1	1	1	1	1	1	-1	-1	1	1	1	1	1	1	1	-1	
X.3	1	1	1	1	1	-1	3	-6	3	3	3	3	1	1	2	3	3	-3	1	-1	-1	1	-3
X.4	1	1	1	1	1	1	3	-6	3	3	3	3	-1	-1	2	3	3	3	1	-1	-1	1	3
X.5	6	-6	6	.	-3	.	.	.	-2	.	.	-3	-2	4	-3	.	
X.6	-1	8	-1	8	-1	-1	-1	-1	-1	-1	3	3	3	3	3	3	
X.7	8	8	8	-1	8	-2	-2	-2	-1	-1	-2	3	.	3	3	-2	
X.8	8	8	8	-1	8	2	2	2	-1	-1	2	3	.	3	3	2	
X.9	-1	8	-1	8	-1	1	1	1	-1	-1	-3	3	3	3	3	-3	
X.10	2	2	2	2	.	.	12	-3	12	.	-6	.	.	.	1	.	.	-1	4	4	-1	.	
X.11	9	9	9	9	9	-3	-3	1	.	.	-3	-2	1	1	-2	-3	
X.12	9	9	9	9	9	3	3	1	.	.	5	5	-2	4	.	-2	
X.13	-4	5	-4	-4	-4	2	2	-3	5	5	-2	4	.	.	4	2	
X.14	-4	5	-4	-4	-4	-2	-2	-3	5	5	2	4	.	.	5	.	
X.15	-1	-1	-1	-1	1	1	.	27	3	.	.	5	.	.	5	.	
X.16	-1	-1	-1	-1	-1	-1	.	27	3	.	.	5	.	.	5	.	
X.17	6	-15	6	.	-3	.	.	-3	.	.	.	-1	6	.	-1	.	
X.18	3	21	3	3	3	-1	-1	5	3	3	3	-2	-1	-1	-2	3	
X.19	6	-15	6	.	-3	.	.	1	.	.	-5	-2	4	-5	.	.	
X.20	3	21	3	3	3	1	1	5	3	3	-3	-2	-1	-1	-2	-3	
X.21	24	12	24	.	-12	-1	.	.	-1	.	.	
X.22	1	10	1	10	1	-1	-1	-2	1	1	-1	.	1	-2	.	-1	
X.23	1	10	1	10	1	-1	-1	-2	1	1	-1	.	1	-2	.	-1	
X.24	1	10	1	10	1	1	1	-2	1	1	1	.	1	-2	.	1	
X.25	1	10	1	10	1	1	1	-2	1	1	1	.	1	-2	.	1	
X.26	18	-9	18	-9	18	.	.	-1	.	.	-1	2	-1	-1	.	-1	
X.27	-9	-9	18	-9	18	-3	-3	-1	.	.	-3	-1	-1	-2	-1	-3	
X.28	18	-9	18	-9	18	.	.	-1	.	.	-3	-1	-1	-2	-1	3	
X.29	-9	-9	18	-9	18	3	3	1	.	.	3	-1	-1	2	-3	-2	
X.30	3	-1	-1	3	1	1	18	3	-6	-3	.	6	6	3	.	.	2	.	2	-3	.	-2	
X.31	3	-1	-1	3	-1	-1	18	3	-6	-3	.	-6	-6	3	.	.	-2	.	2	-3	.	2	
X.32	6	-39	6	6	6	2	2	1	-3	-3	-6	4	-2	-2	4	-6	
X.33	6	-39	6	6	6	-2	-2	1	-3	-3	6	4	-2	-2	4	6	
X.34	30	-3	30	.	-15	.	.	1	.	.	2	-2	4	2	.	.	
X.35	20	20	20	2	20	.	.	-4	2	2	.	-4	2	.	.	.	
X.36	2	-34	2	2	2	.	.	2	2	2	.	2	2	.	.	.	
X.37	2	-34	2	2	2	.	.	2	2	2	.	2	2	.	.	.	
X.38	18	36	18	.	-9	.	.	-4	.	.	-3	2	-4	-3	.	.	
X.39	2	2	2	2	.	.	12	6	12	.	-6	.	.	2	.	.	.	-4	-4	.	.	.	
X.40	-18	18	-18	.	9	.	.	-2	.	.	.	-2	4	.	.	.	
X.41	-18	18	-18	.	9	.	.	-2	.	.	.	-2	4	.	.	.	
X.42	-1	-1	-1	-1	-1	-1	-3	.	.	-3	.	.	
X.43	-1	-1	-1	-1	1	1	-3	.	.	-3	.	.	
X.44	-8	2	.	.	-2	.	.	
X.45	-2	-2	-2	-2	.	.	24	-24	24	.	-12	-4	-4	-4	-4	.	.	
X.46	-12	-33	-12	.	6	.	.	-1	.	.	-4	-4	-4	-4	.	.	
X.47	1	1	1	1	-1	-1	-4	32	-4	-4	-4	.	.	.	-4	-4	4	.	.	.	4	.	
X.48	1	1	1	1	1	1	-4	32	-4	-4	-4	.	.	.	-4	-4	-4	.	.	.	-4	.	
X.49	-18	-9	-18	.	9	.	.	3	.	.	2	6	.	-2	.	.	
X.50	-16	-16	-16	-16	-16	2	2	
X.51	-4	.	.	4	.	
X.52	-4	.	.	4	.	
X.53	-2	-2	-2	-2	3	.	.	3	.	.	
X.54	2	2	2	2	4	.	.	4	.	.	
X.55	6	2	2	-2	2	2	-27	.	-3	.	.	1	1	-3	-3	-3	-3	.	-3	.	.	1	
X.56	6	2	2	-2	2	2	-27	.	-3	.	.	-1	-1	-3	-3	3	3	.	-3	.	.	-1	
X.57	27	3	.	.	-1	.	.	-1	.	.	
X.58	6	-2	-2	-2	.	.	8	-4	8	2	-1	-8	-8	-4	2	2	.	.	.	2	.	.	
X.59	6	-2	-2	-2	.	.	8	-4	8	2	-1	8	-8	-4	2	2	.	.	.	2	.	.	
X.60	3	-1	-1	3	1	1	.	-9	.	-9	.	.	.	3	.	.	4	.	-4	3	.	-4	
X.61	3	-1	-1	3	-1	-1	.	-9	.	-9	.	.	.	3	.	.	-4	.	-4	3	.	4	
X.62	-4	.	.	.	-2	2	
X.63	-4	.	.	.	-2	2	
X.64	-4	.	.	.	-2	2	
X.65	-4	.	.	.	-2	2	
X.66	72	3	-24	6	.	.	.	-9	-6	.	.	
X.67	-2	.	.	2	.	
X.68	-2	.	.	2	.	
X.69	-2	.	.	2	.	
X.70	-2	.	.	2	.	
X.71	-2	.	.	2	.	
X.72	-2	.	.	2	.	
X.73	54	-9	-18	.	.	6	6	3	.	.	-2	.	2	.	2	.	
X.74	18	.	-9	.	.	6	.	6	.	.	4	.	4	3	.	-4	
X.75	18	.	-9	.	.	6	.	6	.	.	-4	.	4	3	.	4	
X.76	54	-9	-18	.	.	-6	-6	3	.	.	2	.	2	.	-2	.	
X.77	36	-21	-12	-6	.	.	.	3	.	.	.	-4	-6	.	.	.	
X.78	-18	6	6	-15	.	.	6	6	-6	.	-2	.	2	-3	.	2	
X.79	-18	6	6	-15	.	.	-6	-6	-6	.	2	.	2	-3	.	-2	
X.80	
X.81	
X.82	-6	2	2	2	.	.	32	-16	8	8	-4	-4	-4	.	2	2	
X.83	-6	2	2	2	.	.	32	-16	8	8	-4	-4	-4	.	2	2	
X.84	-54	.	-6	.	.	2	2	.	3	3	-6	.	-6	.	.		

Character table of $H(\text{Fi}'_{24})$ (continued)

	2	1	5	5	3	1	4	2	2	2	2	2	2	4	3	3	3	1	1	5	5	3	3	4	4
	3	1	.	4	4	4	2	3	3	2	2	2	2	.	1	1	1	1	1	2	2	2	2	1	1
	5	1
	7
	15a	16a	18a	18b	18c	18d	18e	18f	18g	18h	18i	18j	20a	20b	20c	20d	21a	21b	24a	24b	24c	24d	24e	24f	
2P	15a	8b	9a	9b	9c	9a	9c	9d	9e	9d	9e	9e	10a	10b	10d	10c	21b	21a	12c	12i	12j	12a	12i	12j	
3P	5a	16a	63	63	63	66	64	62	62	62	62	62	20a	20d	20c	20b	7a	7a	8a	8a	8a	8a	8i	8c	
5P	3a	16a	18a	18b	18c	18d	18f	18e	18i	18j	18g	18h	4a	4c	4d	4c	21a	21b	24a	24b	24c	24d	24i	24g	
7P	15a	16a	18a	18b	18c	18d	18e	18f	18g	18h	18i	18j	20a	20d	20c	20b	3a	3a	24a	24b	24c	24d	24i	24g	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
X.2	1	-1	1	1	1	1	1	1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	-1	-1	1	-1	
X.3	1	-1	1	-1	-1	-1	.	.	1	1	1	1	-1	1	
X.4	1	1	1	1	1	1	.	.	-1	-1	-1	-1	-1	-1	
X.5	.	.	-3	3	-1	-1	-1	.	
X.6	-1	-1	-1	-1	-1	-1	2	2	-1	-1	-1	-1	-1	-1	
X.7	-1	2	2	2	2	2	-1	-1	1	1	1	1	-2	-2	1	1	-1	-1	
X.8	1	2	2	2	2	2	-1	-1	-1	-1	-1	-1	2	2	-1	-1	-1	1	
X.9	1	-1	-1	-1	-1	2	2	1	1	1	1	-1	.	
X.10	-1	.	3	-3	.	3	2	-1	-1	.	.	1	.	
X.11	-1	-1	-3	-3	.	.	
X.12	-1	-1	3	3	.	-1	
X.13	.	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	2	2	-1	-1	.	.	
X.14	.	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	.	-2	-2	1	1	.	.	
X.15	-1	-1	-1	1	1	1	1	1	.	
X.16	-1	1	-1	-1	-1	-1	1	.	
X.17	.	-3	3	.	-3	-1	.	
X.18	-1	-1	-1	-1	.	1	
X.19	.	-3	3	.	-3	1	.	
X.20	1	1	1	1	.	-1	
X.21	.	3	-3	3	-1	.	
X.22	.	1	1	1	1	B	B	C	C	C	C	C	-1	-1	-1	-1	.	.	
X.23	.	1	1	1	1	B	B	C	C	C	C	C	-1	-1	-1	-1	.	.	
X.24	.	1	1	1	1	B	B	-C	-C	-C	-C	-C	1	1	1	1	.	.	
X.25	.	1	1	1	1	B	B	-C	-C	-C	-C	-C	1	1	1	1	.	.	
X.26	-1	1	1	
X.27	1	-3	-3	
X.28	1	1	-1	
X.29	-1	3	3	.	.	1	.	
X.30	.	9	.	.	1	-1	-1	1	-1	-1	
X.31	.	9	.	.	1	-1	1	-1	1	1	
X.32	2	2	-1	-1	.	.	
X.33	-2	-2	1	1	.	.	
X.34	.	.	3	-3	
X.35	.	-1	-1	-1	-1	2	2	
X.36	.	-1	-1	-1	-1	-1	-1	E	E	E	E	E	
X.37	.	-1	-1	-1	-1	-1	-1	E	E	E	E	E	
X.38	1	
X.39	-1	-3	.	3	-3	2	
X.40	F	G	
X.41	F	G	
X.42	-1	1	-1	-1	-1	-1	1	1	-1	.	
X.43	-1	-1	-1	1	1	1	1	1	-1	.	
X.44	-1	.	-12	1	1	
X.45	1	-2	1	1	
X.46	.	.	3	-3	
X.47	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	
X.48	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	
X.49	
X.50	.	2	2	2	2	2	2	-1	-1	
X.51	.	4	-2	1	.	1	1	E	E	E	E	E	-1	-1	
X.52	.	4	-2	1	.	1	1	E	E	E	E	E	-1	-1	
X.53	1	-2	-1	-1	.	.	.	1	.	
X.54	-1	2	
X.55	-1	-2	-2	-1	-1	-1	-1	.	.	
X.56	.	1	-2	.	2	.	.	.	1	1	1	1	.	.	
X.57	-1	
X.58	.	6	3	.	-2	2	
X.59	.	6	3	.	-2	2	
X.60	-1	-1	1	-1	1	
X.61	-1	1	-1	1	-1	
X.62	1	.	4	-2	1	.	-2	-2	
X.63	1	.	-8	4	-2	.	1	1	-1	1	-1	1	
X.64	1	.	-8	4	-2	.	1	1	1	-1	1	-1	
X.65	1	.	4	-2	1	.	-2	-2	
X.66	.	9	.	.	.	1	
X.67	.	-4	2	-1	.	-1	-1	1	1	-1	1	-1	
X.68	.	-4	2	-1	.	-1	-1	-1	1	-1	1	-1	
X.69	.	-4	2	-1	.	-B	-B	-C	C	-C	-C	C	
X.70	.	-4	2	-1	.	-B	-B	-C	C	-C	-C	C	
X.71	.	-4	2	-1	.	-B	-B	C	-C	C	-C	-C	
X.72	.	-4	2	-1	.	-B	-B	C	-C	C	-C	-C	
X.73	
X.74	-1	
X.75	1	
X.76	
X.77	.	-9	.	.	-1	
X.78	.	-9	.	.	-1	-1	
X.79	.	-9	.	.	-1	1	
X.80	1	1	
X.81	1	1	
X.82	.	.	6	3	.	-2	-2	-6	6	
X.83	.	.	6	3	.	-2	-2	-6	-6	
X.84	.	1	-2	-2	1	1	.	.	
X.85	-1	2	2	-1	-1	.	.	
X.86	1	1	1	-2	-2	.	.	
X.87	-1																			

Character table of $H(\text{Fi}'_{24})$ (continued)

	21	21	19	20	14	14	17	17	16	13	8	12	10	5	8	5	6
	7	7	4	2	4	4	3	3	1	2	7	6	7	7	5	6	4
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	3c	3d	3e	3f	3g
2P	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	3a	3b	3c	3d	3e	3f	3g
3P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	1a	1a	1a	1a	1a	1a	1a
5P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	3c	3d	3e	3f	3g
7P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	3c	3d	3e	3f	3g
X.89	9072	9072	1392	-144	144	144	48	48	48	48	.	162	162
X.90	9072	9072	1392	-144	-144	-144	48	48	48	-48	.	162	162
X.91	10080	10080	-160	-160	.	.	96	96	-32	-32	.	-48	180	.	30	15	-6
X.92	10080	10080	-160	-160	.	.	96	96	-32	32	.	-48	180	.	30	15	-6
X.93	10752	-10752	128	-128	.	.	-84	192	-120	-30	.	12	.
X.94	11340	11340	1740	-180	-360	-360	12	12	-52	24	.	162	81	.	27	.	.
X.95	11340	11340	1740	-180	360	360	12	12	-52	-24	.	162	81	.	27	.	.
X.96	12096	12096	-192	-192	.	.	-192	-192	64	.	.	72	216	.	36	18	.
X.97	13608	13608	-216	296	216	216	72	72	72	8	.	-81	-9
X.98	13608	13608	-216	296	-216	-216	72	72	72	-8	.	-81	-9
X.99	13608	13608	-216	296	216	216	72	72	72	8	.	-81	-9
X.100	13608	13608	-216	296	-216	-216	72	72	72	-8	.	-81	-9
X.101	15309	15309	-243	333	-243	-243	189	189	93	-27
X.102	15309	15309	-243	333	243	243	189	189	93	27
X.103	15360	-15360	-256	256	.	.	-120	480	-48	-12	.	30	.
X.104	16128	-16128	-320	320	.	.	-126	288	144	36	.	18	.
X.105	17010	17010	-270	370	-270	-270	-126	-126	66	26	.	81	9
X.106	17010	17010	-270	370	270	270	-126	-126	66	-26	.	81	9
X.107	20160	20160	-320	-320	.	.	192	192	-64	.	.	480	36	.	-12	-15	-12
X.108	20160	20160	-320	-320	.	.	192	192	-64	.	.	-168	36	.	-12	-15	6
X.109	20160	20160	-320	-320	.	.	192	192	-64	.	.	-168	36	.	-12	-15	6
X.110	22680	22680	3480	-360	-360	-360	120	120	-8	-24	.	-162	162	.	-27	.	.
X.111	22680	22680	3480	-360	360	360	120	120	-8	24	.	-162	162	.	-27	.	.
X.112	22680	22680	3480	-360	.	.	216	216	88	.	.	-81	.	.	-54	.	.
X.113	24192	24192	3712	-384	-576	-576	-108	-216	.	36	.	.
X.114	24192	24192	3712	-384	576	576	-108	-216	.	36	.	.
X.115	25515	25515	-405	555	-135	-135	-117	-117	-21	57	.	243
X.116	25515	25515	-405	555	135	135	-117	-117	-21	-57	.	243
X.117	25515	25515	-405	555	135	135	315	315	27	15	.	243
X.118	25515	25515	-405	555	-135	-135	315	315	27	-15	.	243
X.119	26880	-26880	320	-320	.	.	420	96	-336	42	24	-12	24
X.120	26880	-26880	-192	192	.	.	-210	-96	-192	-48	.	-6	.
X.121	30240	30240	4640	-480	.	.	-96	-96	-96	.	.	108	-270	.	-36	.	.
X.122	30618	30618	4698	-486	324	324	-54	-54	42	-36
X.123	30618	30618	4698	-486	-324	-324	-54	-54	42	36
X.124	32256	-32256	.	.	-768	-768	-128	128	.	.	504	-144	-144	18	72	18	.
X.125	32256	-32256	.	.	-768	768	-128	128	.	.	504	-144	-144	18	72	18	.
X.126	32256	-32256	.	.	192	-192	-128	128	.	.	504	288	-144	18	-36	-36	.
X.127	32256	-32256	.	.	-192	192	-128	128	.	.	504	288	-144	18	-36	-36	.
X.128	34020	34020	5220	-540	.	.	-252	-252	-60	.	.	243
X.129	34560	-34560	192	-192	.	.	540	.	216	-27	.	.	.
X.130	34560	-34560	192	-192	.	.	540	.	216	-27	.	.	.
X.131	34560	-34560	192	-192	.	.	-270	.	216	54	.	.	.
X.132	34560	-34560	192	-192	.	.	-270	.	216	54	.	.	.
X.133	35840	-35840	.	-640	640	-256	256	.	.	.	560	32	272	-34	8	-4	8
X.134	35840	-35840	.	-640	640	-256	256	.	.	.	560	32	272	-34	8	-4	8
X.135	40320	40320	-640	-640	-64	.	-120	72	.	-24	24	3	.
X.136	40320	40320	-640	-640	-64	.	-120	72	.	-24	24	3	.
X.137	40320	40320	-640	-640	64	.	-120	72	.	-24	24	3	.
X.138	40320	-40320	.	.	-480	480	224	-224	.	.	630	144	144	-18	36	-18	.
X.139	40320	40320	-640	-640	.	.	-384	-384	128	.	64	-120	72	.	-24	24	3
X.140	40320	40320	-640	-640	.	.	-384	-384	128	.	.	240	-252	.	12	6	12
X.141	40320	-40320	.	.	480	-480	224	-224	.	.	630	144	144	-18	36	-18	.
X.142	40320	40320	-640	-640	64	.	204	72	.	-24	24	-6
X.143	40320	40320	-640	-640	-64	.	204	72	.	-24	24	-6
X.144	40320	40320	-640	-640	.	.	384	384	-128	.	.	240	-252	.	12	6	12
X.145	43008	-43008	512	-512	.	.	-336	192	-48	-12	.	12	.
X.146	49152	-49152	-384	384	192	48	.	24	.	.
X.147	51030	51030	-810	1110	-270	-270	198	198	6	42	.	-243
X.148	51030	51030	-810	1110	270	270	198	198	6	-42	.	-243
X.149	53760	-53760	-384	384	.	.	-420	384	-168	-42	.	24	.
X.150	57344	-57344	.	.	512	-512	896	-64	-256	32	-16	8	-16
X.151	57344	-57344	.	.	-512	512	896	-64	-256	32	-16	8	-16
X.152	60480	60480	-960	-960	.	.	-192	-192	64	.	.	144	108	.	-36	-45	.
X.153	60480	60480	-960	-960	.	.	192	192	-64	.	.	-72	108	.	18	-18	.
X.154	60480	60480	-960	-960	.	.	192	192	-64	.	.	-72	108	.	18	-18	.
X.155	60480	60480	-960	-960	.	.	-192	-192	64	.	.	-72	108	.	18	-18	.
X.156	60480	60480	-960	-960	.	.	-192	-192	64	.	.	-72	108	.	18	-18	.
X.157	76545	76545	-1215	1665	-405	-405	-351	-351	-63	27
X.158	76545	76545	-1215	1665	-405	-405	81	81	-15	-45
X.159	76545	76545	-1215	1665	405	405	-351	-351	-63	-27
X.160	76545	76545	-1215	1665	405	405	81	81	-15	45
X.161	80640	-80640	-576	576	.	.	-630	-288	72	18	.	-18	.
X.162	80640	80640	-1280	-1280	-168	-504	.	.	24	12	-12
X.163	80640	-80640	-64	64	.	.	-630	-288	72	18	.	-18	.
X.164	80640	-80640	-64	64	.	.	-630	-288	72	18	.	-18	.
X.165	81920	-81920	1280	-256	128	-16	-64	32	8
X.166	107520	-107520	256	-256	.	.	-840	-96	-336	-84	.	-6	.
X.167	107520	-107520	256	-256	.	.	-840	192	96	24	.	12	.

Character table of $H(\text{Fi}'_{24})$ (continued)

3	8	12	10	5	8	9	10	5	11	8	8	8	8	8	8	8	8	8	8	6	7
7	7	6	7	7	5	4	3	6	2	3	3	3	3	3	3	3	3	3	3	4	3
5	1
7	1
	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	6 ₇	6 ₈	6 ₉	6 ₁₀	6 ₁₁	6 ₁₂	6 ₁₃	6 ₁₄	6 ₁₅	6 ₁₆	6 ₁₇	6 ₁₈	6 ₁₉	6 ₂₀	6 ₂₁
2P	3a	3b	3c	3d	3e	3c	3b	3f	3b	3b	3a	3b	3b	3a	3c	3c	3e	3e	3b	3g	3e
3P	2a	2a	2a	2a	2a	2b	2b	2a	2c	2f	2g	2e	2d	2f	2f	2g	2g	2f	2g	2a	2b
5P	6a	6c	63	64	65	66	67	68	69	610	611	612	613	614	615	616	617	618	619	620	621
7P	61	62	63	64	65	66	67	68	69	610	611	612	613	614	615	616	617	618	619	620	621
X.89		162	162			-30	-6		18	-6					6	6				-6	
X.90		162	162			-30	-6		18	-6					6	6				-6	
X.91		-48	180		30	20	32	15	-16						12	12	6	6		-6	-10
X.92		-48	180		30	20	32	15	-16						12	12	6	6		-6	-10
X.93	84	-192	120	30				-12		-16	4				-4	-4	4	16	-16	16	
X.94		162	81		27	-15	-6		18	-6		-6	-6		-3	-3	-9	-9	-6		3
X.95		162	81		27	-15	-6		18	-6		6	6		-3	-3	-9	-9	-6		3
X.96		72	216		36	24	24	18	-24						-24	-24	-12	-12			-12
X.97		-81					-9		-1	-9		-9	-9						-9	-9	
X.98		-81					-9		-1	-9		-9	-9						-9	-9	
X.99		-81					-9		-1	-9		-9	-9						-9	-9	
X.100		-81					-9		-1	-9		9	9						-9	-9	
X.101																					
X.102																					
X.103	120	-480	48	12				-30		8	-8				8	8	-8	16	-16	-8	
X.104	126	-288	-144	-36				-18		-8	-10				10	-8	8	-16	16	8	
X.105		81					9		1	9		-9	-9						9	9	
X.106		81					9		1	9		9	9						9	9	
X.107		480	36		-12	4	-32	-15	-32						-12	-12	12	12		-12	4
X.108		-168	36		-12	4	40	-15	-8						-12	-12	12	12		6	4
X.109		-168	36		-12	4	40	-15	-8						-12	-12	12	12		6	4
X.110		-162	162		-27	-30	6		-18	-6		-6	-6		6	6	9	9	-6		-3
X.111		-162	162		-27	-30	6		-18	-6		6	6		6	6	9	9	-6		-3
X.112		-81			-54	15									-9	-9	-18	-18			-6
X.113		-108	-216		36	40	4	-12				12	12								4
X.114		-108	-216		36	40	4	-12				-12	-12								4
X.115		243					27	3	-9			9	9						-9		
X.116		243					27	3	-9			-9	-9						-9		
X.117		243					27	3	-9			-9	-9						-9		
X.118		243					27	3	-9			9	9						-9		
X.119	-420	-96	336	-42	-24			12		8	-20				20	8	-8	-8	8	-8	-24
X.120	210	96	192	48			6	24	-6						6				-24		
X.121		108	-270		-36	50	-4	12	12						6	6			12		-4
X.122																					
X.123																					
X.124	-504	144	144	-18	-72			-18		4	8	12	-12	-8	-8	8	8	-8	-4		
X.125	-504	144	144	-18	-72			-18		4	8	-12	12	-8	-8	8	8	-8	-4		
X.126	-504	-288	144	-18	36			36		-8	8			-8	-8	8	-4	4	8		
X.127	-504	-288	144	-18	36			36		-8	8			-8	-8	8	-4	4	8		
X.128		243				-45									-9	-9					
X.129	-540		-216	27						-12				12	-12	12					
X.130	-540		-216	27						-12				12	-12	12					
X.131	270		-216	-54						6				-6	-12	12					
X.132	270		-216	-54						6				-6	-12	12					
X.133	-560	-32	-272	34	-8			4		8	16	16	-16	-16	8	-8	-8	8	-8	-8	
X.134	-560	-32	-272	34	-8			4		8	16	16	-16	-16	8	-8	-8	8	-8	-8	
X.135		-120	72		-24	8	8	24	8											3	8
X.136		-120	72		-24	8	8	24	8											3	8
X.137		-120	72		-24	8	8	24	8											3	8
X.138	-630	-144	-144	18	-36			18		-4	-14	12	-12	14	8	-8	4	-4	4		
X.139		-120	72		-24	8	8	24	8											3	8
X.140		240	-252		12	-28	-16	6	-16						-12	-12	12	12		12	-4
X.141	-630	-144	-144	18	-36			18		-4	-14	-12	12	14	8	-8	4	-4	4		
X.142		204	72		-24	8	-28	24	-4											-6	8
X.143		204	72		-24	8	-28	24	-4											-6	8
X.144		240	-252		12	-28	-16	6	-16						12	12	-12	-12		12	-4
X.145	336	-192	48	12			-12		-16	16				-16	8	-8	16	-16	16		
X.146	384	-384	-192	-48			-24														
X.147		-243					-27		-3	9		-9	-9						9		
X.148		-243					-27		-3	9		9	9						9		
X.149	420	-384	168	42			-24			-12				12	-12	12					
X.150	-896	64	256	-32	16		-8					16	-16							16	
X.151	-896	64	256	-32	16		-8					-16	16							16	
X.152		144	108		-36	12	48	-45	-48						12	12	-12	-12			12
X.153		-72	108		18	12	-24	-18	24						-12	-12	-6	-6			-6
X.154		-72	108		18	12	-24	-18	24						-12	-12	-6	-6			-6
X.155		-72	108		18	12	-24	-18	24						12	12	6	6			-6
X.156		-72	108		18	12	-24	-18	24						12	12	6	6			-6
X.157																					
X.158																					
X.159																					
X.160																					
X.161	630	288	-72	-18				18		-24	-18				18	12	-12			24	
X.162		-168	-504		24	-56	40	12	-8											-12	-8
X.163	630	288	-72	-18				18		8	-2				2	-4	4	16	-16	-8	
X.164	630	288	-72	-18				18		8	-2				2	-4	4	16	-16	-8	
X.165	-1280	256	-128	16	64			-32													-8
X.166	840	96	336	84				6		-8	8				-8	-8	8	-16	16	8	
X.167	840	-192	-96	-24				-12		16	8				-8	16	-16	-16	16	-16	

Character table of $H(\text{Fi}'_{24})$ (continued)

	2	5	5	5	5	7	5	5	5	5	5	5	5	7	5	3	1	8	9	8	8	8	8	8	8	6	
	3	4	4	4	4	2	3	3	3	3	3	3	3	3	1	2	2	1	2	1	1	1	
	5	
	7	
		622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	7a	8a	8b	8c	8d	8e	8f	8g	8h	8i
2P		3d	3d	3d	3d	3e	3e	3d	3e	3e	3d	3e	3f	3f	3e	3g	3g	7a	4a	4b	4b	4j	4j	4f	4f	4g	
3P		2d	2d	2e	2e	2i	2d	2g	2g	2f	2f	2e	2g	2f	2h	2c	2i	7a	8a	8b	8c	8d	8e	8f	8g	8h	8i
5P		623	622	625	624	626	627	628	629	630	631	632	633	634	635	636	637	7a	8a	8b	8c	8d	8e	8f	8g	8h	8i
7P		622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	1a	8a	8b	8c	8d	8e	8f	8g	8h	8i
X.89	6	-6	6	6	6	6	6	-6
X.90	-6	6	6	6	6	6	6	-6
X.91	-8	3	3	-2	2	-2
X.92	8	3	3	-2	2	2
X.93	-2	4	-4	2	.	.	4	-4
X.94	-6	-3	-3	-3	-3	-3	-3	-3	-3	-1	-4	-4	.	-4	.	.	.	4	.
X.95	6	3	-3	-3	-3	-3	-3	-3	-3	-1	-4	4	.	4	.	.	.	-4	.
X.96	-6	-6	4
X.97	-1	-1	-1	.	.	-8
X.98	1	-1	1	.	.	-8
X.99	-1	-1	-1	.	.	-8
X.100	1	-1	1	.	.	8
X.101	9	-3	-3	1	1	-3	1	1	3	.
X.102	-9	-3	-3	-1	-1	-3	1	-1	3	.
X.103	4	4	-4	-4	-4	-2	2	.	.	.	2
X.104	-4	-4	4	4	.	.	2	-2
X.105	-1	1	-1	.	10	6	6	-2	-2	2	2	2	-2	.
X.106	1	1	1	.	-10	6	-6	2	2	2	2	2	2	.
X.107	-3	-3	-4	4
X.108	-3	-3	-4	-2
X.109	-3	-3	-4	-2
X.110	6	-3	-3	-3	-3	-3	-3	-3	-3	-1	-8
X.111	-6	3	-3	-3	-3	-3	-3	-3	-3	1	8
X.112	-2	8
X.113	6	6
X.114	-6	-6
X.115	-3	-3	-9	-3	1	1	3	-1	1	3	.
X.116	3	3	-9	3	-1	1	3	-1	-1	-3	.
X.117	3	3	3	3	3	-1	-1	3	-1	-3	.
X.118	-3	-3	3	-3	-3	1	-1	3	1	-3	.
X.119	-2	4	-4	2	.	.	4	-4
X.120	-6	6
X.121	6	6
X.122	-6	.	.	-4	-2	2	4	.	.
X.123	-6	.	.	4	-2	2	-4	.	.
X.124	-6	-6	6	6	.	.	2	-4	4	-2	.	2	-2
X.125	6	6	-6	-6	.	.	2	-4	4	-2	.	2	-2
X.126	-6	-6	6	6	.	.	2	-2	-2	-6	-4	4
X.127	6	6	-6	-6	.	-6	2	2	-2	-2	6	-4	4
X.128	4	.	.	.	4	-4	.	.	.
X.129	M	M	M	M	.	.	3	.	-3	1
X.130	M	M	M	M	.	.	3	.	-3	1
X.131	-6	.	.	6	1
X.132	-6	.	.	6	1
X.133	-10	-10	10	10	.	-4	-2	4	-4	2	4	4	-4
X.134	10	10	-10	-10	.	4	-2	4	-4	2	-4	4	-4
X.135	8	-1	-1
X.136	8	-1	-1
X.137	-8	-1	1
X.138	6	6	-6	-6	.	6	-2	-2	2	2	-6	-2	2
X.139	-8	6	6	-4	-4	.	.	-1	1
X.140
X.141	-6	-6	6	6	.	-6	-2	-2	2	2	6	-2	2
X.142	4	2	-2
X.143	-4	2	2
X.144	-6	-6	4	-4
X.145	4	4	-4	-4
X.146	-2
X.147	3	-6	-6	-6	-2	2	2	2	2	2	.
X.148	-3	6	-6	6	2	-2	2	2	2	-2	.
X.149	-6	.	.	6
X.150	8	8	-8	-8	.	-4	4
X.151	-8	-8	8	8	.	4	-4
X.152	3	3	4
X.153	6	6	2
X.154	6	6	2
X.155	-6	-6	-2
X.156	-6	-6	-2
X.157	-9	-3	3	3	-1	-3	1	-1	-3	.
X.158	-9	9	3	-1	-1	3	-1	3	-1	3
X.159	9	-3	-3	-3	1	-3	1	1	-3	.
X.160	9	9	-3	1	1	1	-3	1	3	.
X.161	6	.	.	-6	.	.	6	-6
X.162
X.163	-2	4	-4	2	.	-2	2
X.164	-2	4	-4	2	.	-2	2
X.165	-1
X.166	-4	-4	4	4	.	2	-2
X.167	8	-4																				

3	6	6	6	5	3	2	2	6	5	5	5	4	4	9	5	9	6	4	8	8	7	5	5
5	.	.	.	4	4	4	3	3	1	1	1	1	1	1	.	.	.	4	5	2	2	3	3
7
8j	8k	8l	9a	9b	9c	9d	9e	10a	10b	10c	10d	10e	10f	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r
2P	4r	4s	4t	9a	9b	9c	9d	9e	10a	10b	10c	10d	10e	10f	12i	12j	12k	12l	12m	12n	12o	12p	12q
3P	8j	8k	8l	3c	3c	3c	3d	3d	10a	10c	10b	10d	10e	10f	4a	4a	4a	4a	4e	4e	4b	4a	4a
5P	8j	8k	8l	9a	9b	9c	9e	9d	2a	2c	2c	2b	2d	2e	12i	12j	12k	12l	12m	12n	12o	12p	12q
7P	8j	8k	8l	9a	9b	9c	9d	9e	10a	10c	10b	10d	10e	10f	12i	12j	12k	12l	12m	12n	12o	12p	12q
X.89	-3	1	1	-3	-1	-1	54	18	-18	.	.	6	6	6	.	.
X.90	-3	1	1	-3	-1	-1	54	18	-18	.	.	-6	-6	6	.	.
X.91	.	.	.	-6	-3	40	-20	-8	.	10	-5	8	-4	-2	-2
X.92	.	.	.	-6	-3	40	-20	-8	.	10	-5	-8	-8	-4	-2
X.93	.	.	.	-6	-3	40	-20	-8	.	10	-5	-8	-8	-4	-2
X.94	.	.	.	-6	-3	.	.	8	54	9	-18	-9	.	-6	-6	-3	.	.
X.95	54	9	-18	-9	.	6	-6	-3	.	.
X.96	6	-2	-2	-2	.	.	48	-24	.	12	-6	.	8	.	.	.
X.97	3	J	K	-1	1	1	27	.	3	.	.	-1	-1	.	3	3
X.98	3	K	J	-1	-1	-1	27	.	3	.	.	1	1	.	3	3
X.99	3	K	J	-1	1	1	27	.	3	.	.	-1	-1	.	3	3
X.100	3	J	K	-1	-1	-1	27	.	3	.	.	1	1	.	3	3
X.101	-1	-1	-1	-6	-2	-2	2	2	2	2
X.102	-1	-1	1	-6	-2	-2	2	2	2	-2
X.103	.	.	.	12	6
X.104	-8
X.105	-27	.	-3	.	.	-1	-1	.	-3	-3
X.106	-27	.	-3	.	.	1	1	.	-3	-3
X.107	.	.	.	6	-6	32	-4	32	-4	5	.	4	4	-4	-4
X.108	.	.	.	-3	3	32	-4	-16	-4	5	.	.	4	-10	14
X.109	.	.	.	-3	3	32	-4	-16	-4	5	.	.	4	14	-10
X.110	-54	18	18	9	.	6	6			

Character table of $H(\text{Fi}'_{24})$ (continued)

2	4	4	4	1	1	5	5	3	1	4	2	2	2	2	2	2	4	3	3	3	1	1	5		
3	1	1	1	1	1	1	4	4	4	4	2	3	3	3	2	2	1	1	1	1	1	1	2		
5	1	1	1	1	1	1	1	.		
7	1	1	1	1	1	1	1	.		
	12 ₃₁	12 ₃₂	12 ₃₃	12 ₃₄	14 _a	15 _a	16 _a	18 _a	18 _b	18 _c	18 _d	18 _e	18 _f	18 _g	18 _h	18 _i	18 _j	20 _a	20 _b	20 _c	20 _d	21 _a	21 _b	24 _a	
2P	6 ₁₇	6 ₃₁	6 ₁₁	6 ₃₁	7 _a	15 _a	8 _b	9 _a	9 _b	9 _c	9 _a	9 _e	9 _d	9 _e	9 _d	9 _d	9 _e	10 _a	10 _b	10 _d	10 _c	21 _b	21 _a	12 ₃	
3P	4 _q	4 _n	4 _p	4 _n	14 _a	5 _a	16 _a	6 ₃	6 ₃	6 ₃	6 ₆	6 ₄	6 ₄	6 ₂₂	6 ₂₄	6 ₂₃	6 ₂₅	20 _a	20 _d	20 _c	20 _b	7 _a	7 _a	8 _a	
5P	12 ₃₁	12 ₃₄	12 ₃₃	12 ₃₂	14 _a	3 _a	16 _a	18 _a	18 _b	18 _c	18 _d	18 _f	18 _e	18 _i	18 _j	18 _h	4 _a	4 _c	4 _d	4 _c	21 _a	21 _b	24 _a		
7P	12 ₃₁	12 ₃₂	12 ₃₃	12 ₃₄	2 _a	15 _a	16 _a	18 _a	18 _b	18 _c	18 _d	18 _e	18 _f	18 _g	18 _h	18 _i	18 _j	20 _a	20 _d	20 _c	20 _b	3 _a	3 _a	24 _a	
X.89	1	-1	-1	-1	.	.	.	
X.90	1	-1	-1	-1	.	.	.	
X.91	2	-6	-3	.	2	
X.92	-2	-6	-3	.	2	
X.93	1	.	.	6	3	
X.94	1	
X.95	-1	
X.96	2	
X.97	-1	L	-1	-L	.	.	1	
X.98	-1	L	-1	-L	.	.	-1	
X.99	-1	-L	-1	-L	.	.	-1	
X.100	-1	-L	1	L	.	.	-1	
X.101	2	.	-2	
X.102	-1	.	-12	-6	2	.	2	.	.	-1	-1	
X.103	-2	
X.104	-1	
X.105	1	
X.106	-1	
X.107	6	-6	.	-2	
X.108	-3	3	.	1	
X.109	-3	3	.	1	
X.110	-1	
X.111	1	
X.112	9	.	.	1	1	1	-1	1	.	.	
X.113	9	.	.	1	1	-1	-1	-1	.	.	
X.114	-1	3	
X.115	-1	-3	
X.116	-1	-3	
X.117	-1	
X.118	1	
X.119	
X.120	12	.	-3	
X.121	-9	.	-1	
X.122	-1	1	-1	1	.	.	.	
X.123	-1	-1	1	-1	.	.	.	
X.124	-1	
X.125	-1	
X.126	-1	
X.127	-1	
X.128	
X.129	E	E	E	-1	1	1
X.130	E	E	E	-1	1	1
X.131	.	.	.	-1	-F	-G	
X.132	.	.	.	-1	-G	-F	
X.133	4	-2	1	.	1	1	-1	1	-1	1	
X.134	4	-2	1	.	1	1	1	-1	-1	-1	
X.135	3	-3	.	-1	
X.136	3	-3	.	-1	
X.137	3	-3	.	-1	
X.138	3	-3	.	-1	
X.139	-6	-3	.	2	
X.140	
X.141	
X.142	-6	6	.	2	6	
X.143	-6	6	.	2	-6	
X.144	-6	-3	.	2	
X.145	-1	.	.	12	.	-3	
X.146	2	1	.	.	-6	-3	
X.147	
X.148	
X.149	12	6	-3	
X.150	4	-2	1	.	1	1	-1	1	-1	1	3	
X.151	4	-2	1	.	1	1	1	-1	-1	-1	
X.152	
X.153	
X.154	2	
X.155	-2	
X.156	
X.157	1	
X.158	-1	
X.159	-1	
X.160	1	
X.161	
X.162	6	3	.	-2	
X.163	
X.164	
X.165	1	.	.	-8	4	-2	.	-2	-2	-1	-1	
X.166	-12	-6	
X.167	6	3	

Character table of $H(\text{Fi}'_{24})$ (continued)

2	5	3	3	4	4	4	4	4	1	4	4	2	1	1
3	2	2	2	1	1	1	1	1	1	3	3	3	1	1
5	1
7	1	1
	24b	24c	24d	24e	24f	24g	24h	24i	30a	36a	36b	36c	42a	42b
2P	12 ₁	12 ₁₀	12 ₉	12 ₁₂	12 ₁₄	12 ₁₄	12 ₈	12 ₁₂	15a	18a	18a	18b	21a	21b
3P	8a	8a	8a	8i	8c	8c	8b	8i	10a	12 ₂	12 ₂	12 ₂	14a	14a
5P	24b	24c	24d	24i	24g	24f	24h	24e	6 ₁	36a	36b	36c	42a	42b
7P	24b	24c	24d	24i	24g	24f	24h	24e	30a	36a	36b	36c	6 ₁	6 ₁
X.89
X.90
X.91	-2	-2	1	.	.	.
X.92	-2	-2	1	.	.	.
X.93	-1
X.94	.	.	.	-1	-1	-1
X.95	.	.	.	1	1	-1
X.96
X.97	1	1	1
X.98	-1	-1	-1
X.99	1	1	1
X.100	-1	-1	-1
X.101
X.102	1	1
X.103
X.104	1
X.105	1	1	1
X.106	-1	-1	-1
X.107	2	2	2	.	.	.
X.108	-7	5	-1	.	.	.
X.109	5	-7	-1	.	.	.
X.110	.	.	.	1	1
X.111	.	.	.	-1	-1
X.112	-1
X.113	-3	-3
X.114	-3	-3
X.115	3
X.116	-3
X.117	-3
X.118	3
X.119
X.120
X.121	3	3
X.122
X.123
X.124	1
X.125	1
X.126	1
X.127	1
X.128	1
X.129	-1	-1	.
X.130	-1	-1	.
X.131	G	F	.
X.132	F	G	.
X.133
X.134
X.135	.	3	-3	-5	7	1	.	.	.
X.136	.	-3	3	7	-5	1	.	.	.
X.137	.	-3	3	-5	7	1	.	.	.
X.138
X.139	.	3	-3	-2	-2	1	.	.	.
X.140
X.141	-2	-2	-2	.	.	.
X.142	-6	-2	-2	-2	.	.	.
X.143	6	-2	-2	-2	.	.	.
X.144	-2	-2	1	.	.	.
X.145	1
X.146	-1	.	.	.	-1	-1	.
X.147	-3
X.148	3
X.149
X.150	-1
X.151	-1
X.152
X.153
X.154	N	-N
X.155	-N	N
X.156
X.157
X.158
X.159
X.160
X.161
X.162
X.163	.	.	-N	N
X.164	.	.	N	-N
X.165	1	1	.
X.166
X.167

, where $A = 12\zeta(3) + 2$, $B = -3\zeta(3) - 2$, $C = -\zeta(3)$, $D = -12\zeta(3) - 6$, $E = -2\zeta(3) - 1$, $F = 2\zeta(21)_3\zeta(21)_7^4 + 2\zeta(21)_3\zeta(21)_7^2 + 2\zeta(21)_3\zeta(21)_7 + \zeta(21)_3 + \zeta(21)_7^4 + \zeta(21)_7^2 + \zeta(21)_7 + 1$, $G = -F + 1$, $H = -3E$, $I = 6\zeta(3) + 1$, $J = -4\zeta(5)^3 - 4\zeta(5)^2 - 1$, $K = 4\zeta(5)^3 + 4\zeta(5)^2 + 3$, $L = 2\zeta(5)^3 + 2\zeta(5)^2 + 1$, $M = 18\zeta(3) + 9$, $N = -4\zeta(12)_4\zeta(12)_3 - 2\zeta(12)_4$.

B.5. Character table of $A_1(\text{Fi}'_{24}) = \langle y, q, s \rangle$

2	19	19	17	14	19	19	14	17	10	9	8	5	11	14	14	13
3	9	9	6	6	4	4	4	3	7	9	7	7	4	4	1	3
5	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	4a	4b	4c	4d
2P	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a
3P	1a	2a	2b	2c	2d	2e	2f	2g	1a	1a	1a	1a	4a	4b	4c	4d
5P	1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	4a	4b	4c	4d
7P	1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	4a	4b	4c	4d
11P	1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	4a	4b	4c	4d
13P	1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	4a	4b	4c	4d
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.3	78	78	-34	22	14	14	6	-2	15	-3	6	-3	-6	-10	-6	6
X.4	78	78	-34	-22	14	14	-6	-2	15	-3	6	-3	6	10	-6	6
X.5	352	-352			32	-32			-8	28	10	1				
X.6	352	-352			32	-32			-8	28	10	1				
X.7	429	429	77	79	45	45	15	13	6	24	15	-3	7	15	15	13
X.8	429	429	77	79	45	45	15	13	6	24	15	-3	7	15	15	13
X.9	1001	1001	-231	49	41	41	-31	7	56	29	2	2	-21	-15	-1	1
X.10	1001	1001	-231	-49	41	41	31	-7	56	29	2	2	-21	-15	-1	1
X.11	1430	1430	-154	-170	86	86	6	6	-1	-28	26	-1	-14	22	-26	14
X.12	1430	1430	-154	170	86	86	-6	-6	1	-28	26	-1	14	-22	26	14
X.13	3003	3003	539	-203	59	59	21	-37	105	6	15	6	21	-43	-11	11
X.14	3003	3003	539	203	59	59	-21	-37	105	6	15	6	-21	43	11	11
X.15	3080	3080	616	-280	136	136	-56	40	119	2	20	2	-56	-56	-24	24
X.16	3080	3080	616	280	136	136	56	40	119	2	20	2	56	56	24	24
X.17	4160	-4160			192	-192			128	-52	38	2				
X.18	10725	10725	-715	-575	165	165	-15	-43	15	114	24	6	85	33	-15	29
X.19	10725	10725	-715	575	165	165	15	-43	15	114	24	6	-85	-33	15	29
X.20	11648	-11648			128	-128			308	-16	-16	-16				
X.21	11648	-11648			128	-128			308	-16	-16	-16				
X.22	13650	13650	1330	-350	210	210	-30	114	105	123	33	15	-70	-30	-30	-14
X.23	13650	13650	1330	350	210	210	30	114	105	123	33	15	70	30	30	-14
X.24	27456	-27456			448	-448			240	240	78	24				
X.25	27456	-27456			448	-448			-120	-84	60	-3				
X.26	27456	-27456			448	-448			-120	-84	60	-3				
X.27	30030	30030	1694	1330	526	526	66	62	-21	-102	60	6	14	82	66	38
X.28	30030	30030	1694	-1330	526	526	-66	62	-21	-102	60	6	-14	-82	-66	38
X.29	32032	32032	-2464	1568	544	544	96	-32	91	-44	64	10	-56	-128	64	64
X.30	32032	32032	-2464	-1568	544	544	-96	-32	91	-44	64	10	56	128	-64	64
X.31	43680	43680	-4256	1120	416	416	-96	-32	399	-60	48	-6	-56	-128	64	
X.32	43680	43680	-4256	-1120	416	416	96	-32	399	-60	48	-6	56	128	-64	
X.33	45045	45045	4389	945	309	309	129	133	441	90	-18	9	189	81	1	29
X.34	45045	45045	4389	-945	309	309	-129	133	441	90	-18	9	-189	-81	-1	29
X.35	48048	48048	-1232	448	432	432	192	48	-84	258	6	-12	-112			-16
X.36	48048	48048	-1232	-448	432	432	-192	48	-84	258	6	-12	112			-16
X.37	50050	50050	770	-350	130	130	130	-126	-35	235	19	-8	-70	2	-30	14
X.38	50050	50050	-5390	1050	450	450	170	-46	595	73	10	19	-210	-102	10	10
X.39	50050	50050	770	350	130	130	-130	-126	-35	235	19	-8	70	-2	30	-14
X.40	50050	50050	-5390	-1050	450	450	170	-46	595	73	10	19	210	102	-10	-10
X.41	75075	75075	7315	-875	515	515	165	51	735	-93	-3	-12	105	-107	5	-5
X.42	75075	75075	7315	875	515	515	-165	-51	-735	93	3	12	-105	107	-5	-5
X.43	75075	75075	1153	2625	835	835	225	131	-210	150	5	-12	105	-107	33	65
X.44	75075	75075	1153	-2625	835	835	-225	-131	-210	150	5	-12	-105	107	-33	65
X.45	75075	75075	-875	-125	-125	-125	165	83	420	-12	42	-15	-175	85	5	-5
X.46	75075	75075	875	125	-125	-125	-165	-83	-420	-12	-42	15	175	-85	-5	-5
X.47	81081	81081	3465	2331	633	633	-117	-87		162	81		-105	123	75	33
X.48	81081	81081	3465	-2331	633	633	117	-87		162	81		105	-123	-75	33
X.49	96096	-96096			544	-544			1092	192	-6	30				
X.50	105600	-105600			-640	640			120	-24	48	-24				
X.51	105600	-105600			-640	640			120	-24	48	-24				
X.52	105600	-105600			-640	640			120	-24	48	-24				
X.53	105600	-105600			-640	640			120	-24	48	-24				
X.54	114400	114400	-8800	2400	480	480	160	32	685	28	-8	-26	-120	-192		64
X.55	114400	114400	-8800	-2400	480	480	-160	32	685	28	-8	-26	120	192		64
X.56	123200	-123200			960	-960			-280	404	62	-28				
X.57	123200	-123200			960	-960			-280	404	62	-28				
X.58	133056	133056			-192	192			-108	-54	54					
X.59	133056	133056			192	-192			108	54	-54					
X.60	150150	150150	8470	350	70	70	270	54	525	57	48	-24	210	-2	50	-34
X.61	150150	150150	8470	-350	70	70	-270	54	525	57	48	-24	-210	2	-50	34
X.62	205920	205920	1056	4320	864	864	-96	160	-279	-144	72	18	216		64	64
X.63	205920	205920	1056	-4320	864	864	96	160	-279	-144	72	18	-216		-64	64
X.64	228800	-228800			320	-320			-160	380	-52	29				
X.65	228800	-228800			320	-320			-160	380	-52	29				
X.66	277200	277200	-18480		720	720		-48	1260	-306	-90	18				16
X.67	289575	289575	12375	225	615	615	-495	183	405	162	-81		-195	33	-15	15
X.68	289575	289575	12375	-225	615	615	495	183	405	162	-81		195	-33	15	15
X.69	292864	-292864			2048	-2048			1552	-32	112	-32				
X.70	300300	300300	-7700	4900	1420	1420	60	-84	-210	-291	114	-21	140	124	-100	-4
X.71	300300	300300	-7700	-4900	1420	1420	-60	84	-210	-291	114	-21	-140	-124	100	-4
X.72	320320	320320	14784	4480	1344	1344	-256	192	406	-116	64	-8	-336	-192	-64	
X.73	320320	320320	14784	-4480	1344	1344	256	-192	406	-116	64	-8	336	192	64	
X.74	360855	360855	18711	-3645	1431	1431	-189	279	729				-189	-189	-61	-9
X.75	360855	360855	18711	3645	1431	1431	189	-279	729				189	189	61	-9
X.76	370656	370656	15840	3744	1248	1248	-96	405	324				-120	192		
X.77	370656	370656	15840	-3744	1248	1248	96	-405	-324				120	-192		
X.78	450450	450450	25410	-1050	210	210	150	-350	1575	171	-18	9	210	-90	-10	-6
X.79	450450	450450	25410	1050	-210	-210	-150	350	-1575	-171	18	-9	-210	90	10	6
X.80	450450	450450	6930	7350	1170	1170	-150	-110	-315	-72	9	9	210	-150	10	130
X.81	450450	450450	6930	-7350	1170	1170	150	110	-315	-72	9	-9	-210	150	-10	-130
X.82	471744	-471744			2112	-2112			-648	162						
X.83	576576	576576	14784	-5376	576	576	384	-320	126	180	72	18	336	-192	-64	
X.84	576576	576576	14784	5376	576	576	-384	-320	126	180	72	18	-336	192	64	

Character table of $A_1(\text{Fi}'_{24})$ (continued)

	2	11	13	11	11	11	12	11	10	5	10	9	8	8	8	8	8	8	5	9	9	10
	3	2	1	2	2	2	1	1	1	1	7	9	7	5	5	6	4	4	7	4	4	3
	5	1	2	1	.	.	1	1	.	.	1
	7	1	1	1
	11
	13
		4e	4f	4g	4h	4i	4j	4k	4l	5a	61	62	63	64	65	66	67	68	69	610	611	612
2P	2d	2e	2d	2g	2g	2e	2d	2g	5a	3a	3b	3c	3a	3a	3b	3a	3a	3d	3b	3b	3a	
3P	4e	4f	4g	4h	4i	4j	4k	4l	5a	2a	2a	2a	2b	2c	2b	2c	2c	2a	2e	2d	2d	
5P	4e	4f	4g	4h	4i	4j	4k	4l	1a	61	62	63	64	65	66	68	67	69	610	611	612	
7P	4e	4f	4g	4h	4i	4j	4k	4l	5a	61	62	63	64	65	66	67	68	69	610	611	612	
11P	4e	4f	4g	4h	4i	4j	4k	4l	5a	61	62	63	64	65	66	68	67	69	610	611	612	
13P	4e	4f	4g	4h	4i	4j	4k	4l	5a	61	62	63	64	65	66	67	68	69	610	611	612	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	1	-1	1	1	-1	1	-1	1	1	1	1	1	1	-1	1	-1	1	1	1	1	1
X.3	-6	-2	2	2	2	2	2	2	2	3	15	-3	6	-7	-5	-7	1	1	-3	5	5	-1
X.4	-6	-2	-2	2	2	-2	2	-2	3	15	-3	6	-7	5	-7	-1	-1	-3	5	5	-1	
X.5	.	.	.	-8	8	.	.	.	2	8	-28	-10	.	.	.	A	A	-1	4	-4	8	
X.6	.	.	.	-8	8	.	.	.	2	8	-28	-10	.	.	.	A	A	-1	4	-4	8	
X.7	5	-3	7	5	5	-1	5	-1	4	6	24	15	14	16	-4	-8	-8	-3	.	.	6	
X.8	5	-3	7	5	5	1	5	1	4	6	24	15	14	-16	-4	8	8	-3	.	.	6	
X.9	-11	9	-3	5	5	9	-3	-3	1	56	29	2	-6	4	-15	-14	-14	2	5	5	8	
X.10	-11	9	3	5	5	-9	-3	3	1	56	29	2	-6	-4	-15	14	14	2	5	5	8	
X.11	-6	6	-6	10	10	-2	2	2	5	-1	-28	26	-19	19	8	1	1	-1	-4	-4	-1	
X.12	-6	6	6	10	10	-2	2	-2	5	-1	-28	26	-19	-19	8	-1	-1	-1	-4	-4	-1	
X.13	11	-5	5	3	3	5	-5	-3	3	105	6	15	17	-5	26	7	7	6	14	14	7	
X.14	11	-5	-5	3	3	-5	-5	3	3	105	6	15	17	5	26	-7	-7	6	14	14	-7	
X.15	24	8	-8	.	.	-8	8	.	5	119	2	20	31	-19	22	-7	-7	2	10	10	7	
X.16	24	8	8	.	.	8	8	.	5	119	2	20	31	19	22	7	7	2	10	10	7	
X.17	.	.	.	-16	16	.	.	.	10	-128	52	-38	-2	-12	12	.	
X.18	-15	-11	-3	9	9	9	-7	-3	.	15	114	24	5	-35	14	-29	-29	6	-6	-6	15	
X.19	-15	-11	3	9	9	-9	-7	3	.	15	114	24	5	35	14	29	29	6	-6	-6	15	
X.20	-2	-308	16	16	16	16	-16	-4	
X.21	-2	-308	16	16	16	16	-16	-4	
X.22	10	2	-6	10	10	-14	10	2	.	105	123	33	25	-35	7	-35	-35	15	3	3	9	
X.23	10	2	6	10	10	14	10	-2	.	105	123	33	25	35	7	35	35	15	3	3	9	
X.24	.	.	-16	16	.	.	.	6	-240	-240	-78	-24	-16	16	16	
X.25	.	.	-16	16	.	.	.	6	120	84	-60	A	A	3	20	-20	-8	
X.26	.	.	-16	16	.	.	.	6	120	84	-60	A	A	3	20	-20	-8	
X.27	26	-2	6	18	18	10	2	6	5	-21	-102	60	29	-11	-34	7	7	6	-14	-14	-5	
X.28	26	-2	-6	18	18	-10	2	-6	5	-21	-102	60	29	11	-34	-7	-7	6	-14	-14	-5	
X.29	-24	.	8	.	.	.	8	.	7	91	-44	64	-79	-61	20	-7	-7	10	4	4	-5	
X.30	-24	.	-8	.	.	.	8	.	7	91	-44	64	-79	61	20	7	7	10	4	4	-5	
X.31	-24	.	-8	.	.	.	8	.	5	399	-60	48	-71	-59	-44	7	7	-6	20	20	-1	
X.32	-24	.	8	.	.	.	8	.	5	399	-60	48	-71	59	-44	-7	-7	-6	20	20	-1	
X.33	41	5	21	1	1	9	1	5	-5	441	90	-18	-21	-9	42	21	21	9	-6	-6	9	
X.34	41	5	-21	1	1	-9	1	-5	-5	441	90	-18	-21	9	-42	-21	21	9	-6	-6	9	
X.35	.	-16	16	16	16	.	.	.	-2	-84	258	6	-44	16	10	-56	-56	-12	18	18	12	
X.36	.	-16	16	16	16	.	.	.	-2	-84	258	6	-44	-16	10	56	56	-12	18	18	12	
X.37	10	18	10	18	18	2	-6	-6	.	-35	235	19	5	-35	-13	49	49	-8	-5	-5	13	
X.38	-50	-14	6	-2	-2	-14	6	-2	.	595	73	10	-35	15	-71	21	21	19	9	9	3	
X.39	10	18	-10	18	18	-2	-6	6	.	-35	235	19	5	35	-13	-49	-49	-8	-5	-5	13	
X.40	-50	-14	-6	-2	-2	14	6	2	.	595	73	10	-35	-15	-71	-21	-21	19	9	9	3	
X.41	55	-13	1	7	7	13	-1	9	.	735	-93	-3	25	25	79	7	7	-12	11	11	-1	
X.42	-5	19	25	3	3	1	11	9	.	-210	150	51	30	60	-6	.	.	-12	-2	-2	-2	
X.43	-5	19	-25	3	3	-1	11	-9	.	-210	150	51	30	-60	-6	.	.	-12	-2	-2	-2	
X.44	15	3	9	-1	-1	-3	-9	-7	.	420	-12	42	-10	-20	-64	-14	-14	15	28	28	4	
X.45	55	-13	-1	7	7	-13	-1	-9	.	735	-93	-3	25	-25	79	-7	-7	-12	11	11	-1	
X.46	15	3	-9	-1	-1	3	-9	7	.	420	-12	42	-10	20	-64	14	14	15	28	28	4	
X.47	5	-7	1	-3	-3	-13	-3	-9	6	.	162	81	90	90	-18	.	.	-6	-6	.	.	
X.48	5	-7	-1	-3	-3	13	-3	9	6	.	162	81	90	-90	-18	.	.	-6	-6	.	.	
X.49	.	.	-8	-4	-1092	-192	6	-30	32	-32	-20	
X.50	-120	24	-48	.	.	.	A	A	24	-8	8	8	
X.51	-120	24	-48	.	.	.	A	A	24	-8	8	8	
X.52	-120	24	-48	.	.	.	A	A	24	-8	8	8	
X.53	-120	24	-48	.	.	.	A	A	24	-8	8	8	
X.54	-40	.	8	.	.	-8	.	.	.	685	28	-8	-25	15	-52	-15	-15	-26	12	12	-3	
X.55	-40	.	-8	.	.	-8	.	.	.	685	28	-8	-25	-15	-52	15	15	-26	12	12	-3	
X.56	.	.	-16	16	280	-404	-62	.	.	.	G	G	28	12	-12	24	
X.57	.	.	-16	16	280	-404	-62	.	.	.	G	G	28	12	-12	24	
X.58	.	.	-16	16	.	.	.	6	.	.	108	54	-54	12	-12	.	
X.59	.	.	-16	16	.	.	.	6	.	.	108	54	-54	12	-12	.	
X.60	-30	22	-6	-6	6	10	-6	.	.	525	57	48	55	-55	73	35	35	-24	25	25	13	
X.61	-30	22	6	-6	-6	10	6	.	.	525	57	48	55	55	73	-35	-35	-24	25	25	13	
X.62	24	.	24	.	.	.	-5	-8	.	-279	-144	72	-69	-99	-24	15	15	18	.	.	9	
X.63	24	.	-24	.	.	.	-8	-5	-279	-144	72	-69	99	-24	-15	-15	18	.	.	.	9	
X.64	.	.	-16	16	160	-380	52	.	.	.	J	J	-29	4	-4	32	
X.65	.	.	-16	16	160	-380	52	.	.	.	J	J	-29	4	-4	32	
X.66	-80	16	.	16	.	-16	.	.	.	1260	-306	-90	60	.	-66	.	.	18	-18	-18	12	
X.67	35	23	5	3	3	-23	-5	-3	.	405	162	-81	-45	-45	-18	-27	-27	.	-6	-6	21	
X.68	35	23	-5	3	3	23	-5	3	.	405	162	-81	-45	45	-18	27	27	.	-6	-6	21	
X.69	14	-1552	32	-112	32	-32	32	-16	
X.70	-20	-20	-4	20	20	12	-4	4	.	-210	-291	114	-50	50	49	14	14	-21	-11	-11	-2	
X.71	-20	-20	4	20	20	-12	-4	-4	.	-210	-291	114	-50	-50	49	-14	-14	-21	-11	-11	-2	
X.72	16	.	-16	.	.	.	16	-5	406	-116	64	114	-34	-12	14</							

Character table of $A_1(\text{Fi}'_{24})$ (continued)

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	3	3	6	5	5	3	3	3	3	3	3	3	4	4	4	4	3	3	3	1	1	1	1	1	1
	5																								
	7																			1					
	11																								
	13																								
	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	7a	8a	8b	8c	8d	8e
2P	3a	3b	3c	3c	3c	3c	3b	3a	3a	3c	3c	3d	3d	3d	3d	3d	3d	3c	3c	7a	4f	4d	4d	4d	4d
3P	2e	2c	2c	2b	2g	2g	2g	2f	2g	2e	2d	2f	2f	2c	2c	2g	2g	2g	2f	7a	8a	8b	8c	8d	8e
5P	613	614	615	616	617	618	619	620	621	622	623	625	624	627	626	628	629	630	631	7a	8a	8b	8c	8d	8e
7P	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	1a	8a	8b	8c	8d	8e
11P	613	614	615	616	617	618	619	620	621	622	623	625	624	627	626	628	629	630	631	7a	8a	8b	8c	8d	8e
13P	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	7a	8a	8b	8c	8d	8e
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	-1	-1	1	1	1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	1	1	-1	-1	-1	-1	-1	-1
X.3	-1	-5	4	2	-2	-2	1	3	1	2	2	-3	-3	1	1	1	1	-2		1	2	2	2	-2	-2
X.4	-1	5	-4	2	-2	-2	1	-3	1	2	2	3	3	-1	-1	1	1	-2		1	-2	2	-2	-2	2
X.5	-8				-6	6				-2	2	B	B	C	C	-3	3			2					
X.6	-8				-6	6				-2	2	B	B	C	C	-3	3			2					
X.7	6	-2	7	5	7	7	4		-2	3	3	-3	-3	1	1	1	1	1	3	2	3	1	3	1	3
X.8	6	2	-7	5	7	7	4		-2	3	3	3	3	-1	-1	1	1	-3		2	-3	1	-3	1	-3
X.9	8	-5	4	-6	2	2	-7	-4	2	2	2	-4	-4	4	4	2	2	2	4		1	-3	-3	1	1
X.10	8	5	-4	-6	2	2	-7	-4	2	2	2	4	4	-4	-4	2	2	2	-4		-1	3	3	1	-1
X.11	-1	-8	8	8	-6	-6		3	-3	2	2	-3	-3	1	1	3	3			2	-6	-2	-2	2	2
X.12	-1	8	8	8	-6	-6		-3	-3	2	2	3	3	-1	-1	3	3			2	-6	-2	-2	2	2
X.13	-7	-14	-5	-1	-1	-1	2	-3	-7	-1	-1	-6	-6	-2	-2	2	-1	3		3	-3	1	1	-1	1
X.14	-7	-14	-5	-1	-1	-1	2	-3	-7	-1	-1	-6	-6	-2	-2	2	-1	3		3	-3	1	1	-1	1
X.15	-7	-10	-10	4	4	4	-2	-11	7	4	4	-2	-2	2	2	-2	-2	4	-2			4	-4	4	-4
X.16	7	10	10	4	4	4	-2	-11	7	4	4	2	2	-2	-2	-2	-2	4	2			4	4	4	4
X.17						-6	-6			-6	6								2						
X.18	15	-8	-8	-4	8	8	-10	-3	5			-6	-6	-2	-2	2	2	-4		1	-3	1	5	-3	1
X.19	15	8	8	-4	8	8	-10	3	5			6	6	2	2	2	2	-4		1	3	1	-5	-3	-1
X.20	4								-8	8															
X.21	4								-8	8															
X.22	9	1	1	7	9	9	15	-3	9	-3	-3	-3	-3	1	1	3	3	3	-3		-2	-2	-2	-2	-2
X.23	9	-1	-1	7	9	9	15	3	9	-3	3	3	3	-1	-1	3	3	3	3		2	-2	-2	-2	-2
X.24	-16				-18	18				2	-2									2					
X.25	8				12	-12				-4	4	F	F	C	C	-3	3			2					
X.26	8				12	-12				-4	4	F	F	C	C	-3	3			2					
X.27	-5	-20	16	2	-4	-4	-10	-3	5	4	4	-6	-6	-2	-2	2	2	2			6	2	-2	-2	2
X.28	-5	20	-16	2	-4	-4	-10	3	5	4	4	6	6	2	2	2	2	2			-6	2	2	-2	-2
X.29	-5	2	20	2	-8	-8	4	3	1	4	4	6	6	2	2	-2	-2	-2				4	4	-4	-4
X.30	-5	-2	-20	2	-8	-8	4	-3	1	4	4	-6	-6	-2	-2	-2	-2	-2				-4	-4	4	4
X.31	-1	-14	4	10	-8	-8	4	-3	1	-4	-4	-6	-6	-2	-2	-2	-2	-2				-4	4	4	-4
X.32	-1	14	-4	10	-8	-8	4	3	1	-4	-4	6	6	2	2	-2	-2	-2				-4	-4	4	4
X.33	9			6	-2	-2	10	15	19	6	6	3	3	3	3	1	1	-2			-3	1	-3	5	1
X.34	9			6	-2	-2	10	-15	19	6	6	-3	-3	-3	-3	1	1	-2			3	1	3	5	-1
X.35	12	16	-2	-8	6	-6		-12	6	6	-6	-6	-6	-2	-2				-6						
X.36	12	-16	2	-8	6	-6		12	6	6	6	6	6	2	2										
X.37	13	1	1	5	3	3	3	-11	-3	-5	-5	4	4	4	4				-3	1		2	2	2	2
X.38	3	-3	6	-8	2	2	-7	23	5	6	6	-1	-1	3	3	-1	-1	-4	2		-2	2	2	-2	-2
X.39	13	-1	-1	5	3	3	3	11	-3	-5	-5	-4	-4	-4	-4				-3	-1		2	2	-2	-2
X.40	3	3	-6	-8	2	2	-7	-23	5	6	6	1	1	-3	-3	-1	-1	-4	-2		2	2	-2	-2	2
X.41	-1	-11	-11	-11	-3	-3	-9	9	9	5	5	-6	-6	-2	-2				-3	-3		1	-5	5	1
X.42	-2	6	15	3	11	11	2	-12	-10	7	7					-4	-4	-1	3		-3	3	1	3	1
X.43	-2	-6	-15	3	11	11	2	12	-10	7	7					-4	-4	-1	-3		3	-3	1	3	-1
X.44	4	16	-2	-10	2	2	8	12	-10	-2	-2	3	3	-5	-5	-1	-1	2	-6		-3	-1	1	3	-3
X.45	-1	11	11	-11	-3	-3	-9	-9	9	5	5	6	6	2	2				-3	3		-1	-5	-5	-1
X.46	4	-16	2	-10	2	2	8	-12	-10	-2	-2	-3	-3	5	5	-1	-1	2	6		3	-1	-1	3	3
X.47	-18	9	9	9	9	9	6	-6	-6	-3	-3								-3	-3		3	-7	3	-1
X.48	-18	-9	9	9	9	9	6	-6	-6	-3	-3								3	3		-3	-7	-3	3
X.49	20			-6	6				-10	10						6	-6								
X.50	-8								-8	8				A	A				-2						
X.51	-8								-8	8				A	A				-2						
X.52	-8								-8	8				A	A				-2						
X.53	-8								-8	8				A	A				-2						
X.54	-3	-30	6	2	8	8	-4	7	-1			-2	-2	-6	-6	2	2	2	-2	-1		4	-4	-4	4
X.55	-3	30	-6	2	8	8	-4	-7	-1			2	2	6	6	2	2	2	2	-1		4	4	-4	-4
X.56	-24			-18	18				-6	6	H	H	D	D											
X.57	-24			-18	18				-6	6	H	H	D	D											
X.58				-6	6				-6	6						6	-6								
X.59				-6	6				-6	6						6	-6								
X.60	13	-1	8	-8			9	9	-9	-8	-8			8	8						2	2	2	-2	-2
X.61	13	1	-8	-8			9	9	-9	-8	-8			-8	-8						-2	2	2	-2	-2
X.62	9			-8	-8	-8	-3	-5		-6	-6	6	6	-2	-2	4			1		-4	-4	4	4	
X.63	9			-8	-8	-8	3	-5		6	6	-6	-6	-2	-2	4			1		-4	4	4	-4	
X.64	-32			12	-12				4	-4	B	B	B	B	-3	3			-2						
X.65	-32			12	-12				4	-4	B	B	B	B	-3	3			-2						
X.66	12			6	6	30			12	6	6					-6	-6	6							
X.67	21	-18	9	9	-9	-9	6	3	3	3	3					-3	-3	-1	-3	-1		1	-5	5	
X.68	21																								

Character table of $A_1(\text{Fi}'_{24})$ (continued)

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7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2P	4f	4h	4i	9a	9b	9c	5a	5b	5c	5d	5e	5f	11a	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	12s
3P	8f	8g	8h	3b	3c	3d	10a	10b	10c	10d	10e	10f	11a	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	12s
5P	8f	8g	8h	9a	9b	9c	2a	2c	2d	2e	2b	2f	11a	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	12s
7P	8f	8g	8h	9a	9b	9c	10a	10b	10c	10d	10e	10f	11a	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	12s
11P	8f	8g	8h	9a	9b	9c	10a	10b	10c	10d	10e	10f	11a	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	12s
13P	8f	8g	8h	9a	9b	9c	10a	10b	10c	10d	10e	10f	11a	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	12s
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	-1	1	1	1	1	1	-1	1	1	1	-1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1
X.3	2	1	3	1	3	1	3	-3	-1	-1	1	1	1	-3	5	-1	3	3	3	-3	-3	-1	2	-3
X.4	2	1	3	1	3	1	3	-3	-1	-1	1	-1	1	3	-5	1	-3	3	3	-3	-3	1	-2	3
X.5	2	1	3	1	3	1	3	-3	-1	-1	1	-1	1	3	-5	1	-3	3	3	-3	-3	1	-2	3
X.6	2	1	3	1	3	1	3	-3	-1	-1	1	-1	1	3	-5	1	-3	3	3	-3	-3	1	-2	3
X.7	1	-1	1	1	1	1	1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.8	1	1	1	1	1	1	1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.9	1	1	1	1	1	1	1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.10	1	1	1	1	1	1	1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.11	2	1	3	1	3	1	3	-3	-1	-1	1	1	1	3	-5	1	-3	3	3	-3	-3	1	-2	3
X.12	2	1	3	1	3	1	3	-3	-1	-1	1	1	1	3	-5	1	-3	3	3	-3	-3	1	-2	3
X.13	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.14	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.15	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.16	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.17	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.18	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.19	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.20	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.21	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.22	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.23	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.24	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.25	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.26	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.27	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.28	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.29	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.30	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.31	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.32	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.33	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.34	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.35	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.36	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.37	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.38	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.39	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.40	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.41	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.42	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.43	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.44	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.45	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.46	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.47	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.48	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.49	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.50	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.51	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.52	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.53	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.54	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.55	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.56	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.57	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.58	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.59	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.60	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.61	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.62	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.63	3	1	-1	3	1	-1	3	-3	-1	-1	1	1	1	3	-13	2	-6	5	5	2	5	-1	-1	-5
X.64	3	1	-1	3	1	-1	3	-3	-1	-1	1													

Character table of $A_1(\text{Fi}'_{24})$ (continued)

	2	6	6	6	6	4	5	5	5	6	6	4	4	4	4	5	4	4	1	3	2
	3	2	2	2	2	3	2	2	2	1	1	2	2	2	2	1	1	1	1	1	1
	5	1	1
	7
	11
	13
	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	12 ₁₇	12 ₁₈	12 ₁₉	12 ₂₀	12 ₂₁	12 ₂₂	12 ₂₃	12 ₂₄	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈	13 _a	14 _a	14 _b	
2P	6 ₁₂	6 ₁₇	6 ₁₂	6 ₁₇	6 ₃	6 ₁₉	6 ₁₉	6 ₁₁	6 ₁₀	6 ₁₂	6 ₂₃	6 ₂₃	6 ₂₈	6 ₂₈	6 ₂₂	6 ₂₈	6 ₂₈	13 _a	14 _a	14 _b	
3P	4 _g	4 _h	4 _e	4 _i	4 _a	4 _i	4 _h	4 _g	4 _f	4 _k	4 _g	4 _e	4 _i	4 _h	4 _j	4 _i	4 _i	13 _a	14 _a	14 _b	
5P	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	12 ₁₇	12 ₁₈	12 ₁₉	12 ₂₀	12 ₂₁	12 ₂₂	12 ₂₃	12 ₂₄	12 ₂₅	12 ₂₆	12 ₂₈	12 ₂₇	13 _a	14 _a	14 _b	
7P	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	12 ₁₇	12 ₁₈	12 ₁₉	12 ₂₀	12 ₂₁	12 ₂₂	12 ₂₃	12 ₂₄	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈	13 _a	2 _a	2 _c	
11P	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	12 ₁₇	12 ₁₈	12 ₁₉	12 ₂₀	12 ₂₁	12 ₂₂	12 ₂₃	12 ₂₄	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈	13 _a	14 _a	14 _b	
13P	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	12 ₁₇	12 ₁₈	12 ₁₉	12 ₂₀	12 ₂₁	12 ₂₂	12 ₂₃	12 ₂₄	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈	1 _a	14 _a	14 _b	
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	-1	1	1	1	-1	1	1	-1	1	1	-1	1	1	1	-1	-1	-1	1	1	-1	-1
X.3	-1	2	-3	2	.	-1	-1	-1	1	-1	2	.	-1	-1	-2	-1	-1	.	1	1	-1
X.4	1	2	-3	2	.	-1	-1	1	1	-1	-2	.	-1	-1	2	1	1	.	1	-1	-1
X.5	.	-2	.	2	.	2	-2	-1	1	.	C	C	1	-2	.	.
X.6	.	-2	.	2	.	2	-2	-1	1	.	C	C	1	-2	.	.
X.7	4	-1	2	-1	1	2	2	-2	.	2	1	-1	-1	-1	-1	-1	-1	.	2	2	-2
X.8	-4	-1	2	-1	-1	2	2	2	.	2	-1	-1	-1	-1	1	1	1	.	2	-2	-2
X.9	.	2	-2	2	.	-1	-1	3	-3	.	.	-2	2	2
X.10	.	-2	-2	2	.	-1	-1	-3	-3	.	.	-2	2	2
X.11	-3	-2	-3	-2	-2	-2	-2	.	-1	.	.	1	1	-2	-1	-1	-1	.	2	-2	-2
X.12	-3	-2	-3	-2	-2	-2	-2	.	-1	.	.	1	1	-2	-1	-1	-1	.	2	-2	-2
X.13	-3	-3	5	3	3	.	.	2	-2	1	-1	-1	.	.	-1
X.14	1	3	5	3	-3	.	.	-2	-2	1	1	-1	.	.	1
X.15	1	.	3	.	-2	1	-2	-2	-2	-1	-2	.	.	.	-2	.	-1
X.16	-1	.	3	.	2	.	2	2	-1	2	2	.	-1
X.17	.	2	.	-2	.	-2	2	-2	2	-2	.	.
X.18	-3	.	-3	.	4	.	.	.	-2	-1	1	-1	-1
X.19	3	.	-3	.	-4	.	.	.	-2	-1	1	1	1
X.20
X.21
X.22	-3	1	1	1	-1	1	1	-3	-1	1	3	1	1	1	1	-1	-1
X.23	3	1	1	1	1	1	1	3	-1	1	-3	1	1	1	-1	1	1
X.24	.	2	.	-2	.	4	-4	4	-4	-2	.	.
X.25	.	-4	.	4	.	-2	2	1	-1	.	C	C	.	-2	.	.
X.26	.	-4	.	4	.	-2	2	1	-1	.	C	C	.	-2	.	.
X.27	-3	.	5	.	2	.	.	-2	-1	.	2	.	.	.	-2
X.28	3	.	5	.	-2	.	.	-2	-1	.	2	.	.	.	2
X.29	-1	.	-3	.	-2	.	.	2	.	2
X.30	1	.	-3	.	2	.	.	-2	-1	-2
X.31	1	.	-3	.	2	.	.	-2	-1	-2
X.32	-1	.	-3	.	-2	.	.	2	-1	2
X.33	-3	-2	-1	-2	.	-2	-2	.	2	1	.	2	1	1	.	-1	-1
X.34	-3	-2	-1	-2	.	-2	-2	.	2	1	.	2	1	1	.	1	1
X.35	4	-2	.	-2	-4	-2	-2	4	2	.	-2	-2	-2	-2
X.36	-4	-2	.	-2	-4	-2	-2	4	2	.	-2	-2	-2	-2
X.37	1	3	1	3	-1	3	3	1	3	-3	1	1	1	1	-1
X.38	3	-2	1	-2	1	1	-3	1	3	.	-2	1	1	-2	1	1	1
X.39	-1	3	1	3	1	3	3	-1	3	-3	-1	1	1	1	1
X.40	-3	-2	1	-2	1	1	3	1	3	.	-2	1	1	2	-1	-1
X.41	1	1	1	1	-3	1	1	1	-1	-1	1	1	-2	-2	1
X.42	4	3	-2	3	3	.	-2	2	2	1	1	.	.	.	1
X.43	-4	3	-2	3	-3	.	2	-2	2	-1	1	.	.	.	-1
X.44	.	2	-6	2	2	2	2	-1	-1	.	-1	-1
X.45	-1	1	1	1	3	1	1	-1	-1	-1	1	-2	-2	-1
X.46	.	2	-6	2	-2	2	2	-1	-1	.	1	1
X.47	2	-3	2	-3	-3	.	.	2	2	.	-1	-1	.	.	-1
X.48	-2	-3	2	-3	3	.	.	-2	2	.	1	-1	.	.	1
X.49	.	-2	.	2	.	-4	4	2	-2
X.50	1	2	.
X.51	1	2	.
X.52	1	2	.
X.53	1	2	.
X.54	-1	.	-1	2	.	1	2	2	-1	-1	-1
X.55	1	.	-1	-2	.	1	-2	2	-1	-1	-1
X.56	.	2	.	-2	.	-2	2	-2	2	-1	.	.
X.57	.	2	.	-2	.	-2	2	-2	2	-1	.	.
X.58	.	2	.	-2	.	-2	2	-2	2	-1	.	.
X.59	.	2	.	-2	.	-2	2	-2	2	1	.	.
X.60	-3	.	3	.	.	-3	-3	3	1	1
X.61	3	.	3	.	.	-3	-3	-3	1	1
X.62	-3	.	3	1	1	1
X.63	3	.	3	1	1	-1
X.64	.	-4	.	4	.	-2	2	1	-1	.	C	C	.	2	.	.
X.65	.	-4	.	4	.	-2	2	1	-1	.	C	C	.	2	.	.
X.66	.	-2	4	-2	.	-2	-2	.	-2	-4	.	-2	-2	-2	.	.	.	1	.	.	.
X.67	-1	3	-1	3	3	.	.	2	2	1	-1	-1	.	.	1	.	.	.	-1	1	1
X.68	1	3	-1	3	-3	.	.	-2	2	1	1	-1	.	.	-1	.	.	.	-1	-1	-1
X.69
X.70	2	2	-2	2	2	-1	-1	-1	1	2	2	-2	-1	-1	.	1	1
X.71	-2	2	-2	2	-2	-1	-1	1	1	2	-2	-2	-1	-1	.	-1	-1
X.72	-2	.	-2	-2	.	-2	-2
X.73	-2	.	-2	-2	.	-2	-2
X.74	-3	.	-3	1	1	-2	2
X.75	-3	.	-3	1	1	-2	-2
X.76	-1	.	-1	2	.	1	2	2	-1	1	1
X.77	1	.	-1	-2	.	1	-2	2	-1	-1	-1
X.78	5	-2	-3	-2	.	1	1	-1	-1	2	.	1	1	1	1	1	1
X.79	5	1	1	1	-3	-2	-2	2	.	-3	-1	1	1	1	1	-1	-1
X.80	-5	1	1	1	-3	-2	-2	-2	.	-3	1	1	1	1	-1	1	1
X.81	-5	-2	-3	-2	.	1	1	1	-1	-1	-2	.	1	1	.	-1	-1
X.82	.	6	.</																		

[illegible]

Character table of $A_1(\text{Fi}'_{24})$ (continued)

	2	19	19	17	14	19	19	14	17	10	9	8	5	11	14	14
	3	9	9	6	6	4	4	4	3	7	9	7	7	4	4	1
	5	2	2	1	2	1	1	1	1	1	1	1	1	1	1	1
	7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	4a	4b	4c
2P		1a	1a	1a	1a	1a	1a	1a	1a	3a	3b	3c	3d	2a	2e	2e
3P		1a	2a	2b	2c	2d	2e	2f	2g	1a	1a	1a	1a	4a	4b	4c
5P		1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	4a	4b	4c
7P		1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	4a	4b	4c
11P		1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	4a	4b	4c
13P		1a	2a	2b	2c	2d	2e	2f	2g	3a	3b	3c	3d	4a	4b	4c
X.85	577368	577368	-21384	-5832	216	216	216	-72	729	216	216	56
X.86	577368	577368	-21384	5832	216	216	216	-72	729	-216	-216	-56
X.87	579150	579150	-6930	-5850	1230	1230	-90	174	-405	324	81	.	.	-210	102	-90
X.88	579150	579150	-6930	5850	1230	1230	90	174	-405	324	81	.	.	210	-102	90
X.89	600600	600600	21560	-7000	920	920	-120	-136	525	-96	12	-42	-280	-248	40	40
X.90	600600	600600	21560	7000	920	920	120	-136	525	-96	12	-42	280	248	-40	-40
X.91	640640	-640640	.	.	-1152	1152	.	.	812	-232	128	-16
X.92	675675	675675	10395	4725	-165	-165	405	411	.	135	-54	54	525	21	-75	-75
X.93	675675	675675	10395	-4725	-165	-165	-405	411	.	135	-54	54	-525	-21	75	75
X.94	720720	720720	-33264	-1680	1104	1104	-336	-112	1386	-18	36	-18	336	48	-16	-16
X.95	720720	720720	-33264	1680	1104	1104	336	-112	1386	-18	36	-18	-336	-48	16	16
X.96	800800	-800800	.	.	1120	-1120	.	.	3220	-128	-74	34
X.97	800800	800800	12320	.	2080	2080	.	32	-560	-614	-20	34
X.98	800800	800800	-12320	-5600	800	800	-160	-416	-245	520	16	34	280	64	.	.
X.99	800800	800800	-12320	5600	800	800	160	-416	-245	520	16	34	-280	-64	.	.
X.100	800800	800800	-36960	.	-480	-480	.	160	1960	196	-56	20
X.101	852930	852930	-29646	2430	1026	1026	-594	18	729	594	-162	46
X.102	852930	852930	-29646	-2430	1026	1026	594	18	729	-594	162	-46
X.103	873600	-873600	.	.	-640	640	.	.	1680	420	-84	-66
X.104	938223	938223	-2673	9477	1647	1647	-27	207	-729	-27	-27	5
X.105	938223	938223	-2673	-9477	1647	1647	27	207	-729	27	27	-5
X.106	960960	-960960	.	.	3392	-3392	.	.	-672	-348	138	30
X.107	972972	972972	-24948	9828	1836	1836	324	-180	.	-243	.	.	.	-252	-252	36
X.108	972972	972972	-24948	-9828	1836	1836	-324	-180	.	-243	.	.	.	252	252	-36
X.109	1029600	-1029600	.	.	1440	-1440	.	.	-720	252	90	9
X.110	1029600	1029600	.	.	-1440	1440	.	.	720	252	90	9
X.111	1164800	1164800	-17920	.	-2560	-2560	.	512	560	344	128	20
X.112	1201200	1201200	-30800	2800	560	560	240	176	420	-30	-48	51	.	-80	-80	.
X.113	1201200	1201200	18480	.	-2000	-2000	.	560	420	-30	114	-30
X.114	1201200	1201200	-30800	-2800	560	560	-240	176	420	-30	-48	51	.	80	80	.
X.115	1360800	1360800	30240	.	1440	1440	.	288	.	486
X.116	1360800	1360800	30240	.	-1440	-1440	.	288	.	486
X.117	1372800	1372800	49280	-6400	640	640	.	128	1560	-312	-24	12	.	-256	.	.
X.118	1372800	-1372800	.	.	1920	-1920	.	.	1920	12	156	-42
X.119	1372800	1372800	49280	6400	640	640	.	128	1560	-312	-24	12	.	256	.	.
X.120	1441792	1441792	.	-8192	-512	640	64	-8
X.121	1441792	1441792	.	8192	-512	640	64	-8
X.122	1441792	-1441792	-512	640	64	-8
X.123	1441792	-1441792	512	640	64	-8
X.124	1791153	1791153	-5103	5103	-2511	-2511	-81	81	567	-81	-81
X.125	1791153	1791153	-5103	-5103	-2511	-2511	81	81	-567	81	81
X.126	1830400	-1830400	.	.	2560	-2560	.	.	880	1096	16	16
X.127	1876446	1876446	37422	-5346	-1890	-1890	270	-306	729	378	-162	110
X.128	1876446	1876446	37422	5346	-1890	-1890	-270	-306	729	-378	162	-110
X.129	1965600	1965600	30240	.	-480	-480	.	-480	.	-270	108	54
X.130	2027025	2027025	-24255	-7875	-1455	-1455	45	33	.	-324	81	.	105	93	45	45
X.131	2027025	2027025	-24255	7875	-1455	-1455	-45	33	.	-324	81	.	-105	-93	-45	-45
X.132	2050048	2050048	.	10752	2048	2048	512	.	-1232	-224	-80	-8
X.133	2050048	2050048	.	-10752	2048	2048	-512	.	-1232	-224	-80	-8
X.134	2316600	2316600	-43560	-1800	-840	-840	-360	24	405	-162	.	.	120	24	120	120
X.135	2316600	2316600	-43560	1800	-840	-840	360	24	405	-162	.	.	-120	-24	-120	-120
X.136	2402400	-2402400	.	.	3360	-3360	.	.	-420	264	-78	48
X.137	2402400	2402400	12320	5600	-160	-160	-480	160	-735	-384	48	-6	-280	64	.	.
X.138	2402400	2402400	12320	-5600	-160	-160	480	160	-735	-384	48	-6	280	-64	.	.
X.139	2555904	2555904	32768	4096	-384	192	-96	-24	512	.	.	.
X.140	2555904	2555904	-32768	-4096	-384	192	-96	-24	512	.	.	.
X.141	2555904	2555904	-32768	4096	-384	192	-96	-24	-512	.	.	.
X.142	2555904	2555904	32768	-4096	-384	192	-96	-24	-512	.	.	.
X.143	2594592	-2594592	.	.	2400	-2400	.	.	2268	-648	-162
X.144	2729376	2729376	7776	7776	-864	-864	-864	-288	-729	216	.	-64
X.145	2729376	2729376	7776	-7776	-864	-864	864	-288	-729	-216	.	64
X.146	3326400	-3326400	.	.	-4800	4800	.	.	.	216	108	54
X.147	4392960	-4392960	.	.	-2048	2048	.	.	-912	-480	-48	-48
X.148	4717440	-4717440	.	.	2688	-2688	.	.	-2268	-648
X.149	4804800	-4804800	.	.	-3520	3520	.	.	1680	-120	132	42
X.150	5111808	-5111808	-768	384	-192	-48

Character table of $A_1(\text{Fi}'_{24})$ (continued)

	2	13	11	13	11	11	11	12	11	10	5	10	9	8	8	8	8	8	8	5	9
	3	3	2	1	2	2	2	1	1	1	1	7	9	7	5	5	6	4	4	7	4
	5	1	2	1	.	.	1	.	.	1	1	.	.
	7	1
	11
	13
		4d	4e	4f	4g	4h	4i	4j	4k	4l	5a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	6 ₇	6 ₈	6 ₉	6 ₁₀
2P		2e	2d	2e	2d	2g	2g	2e	2d	2g	5a	3a	3b	3c	3a	3a	3b	3a	3a	3d	3b
3P		4d	4e	4f	4g	4h	4i	4j	4k	4l	5a	2a	2a	2a	2b	2c	2b	2c	2c	2a	2e
5P		4d	4e	4f	4g	4h	4i	4j	4k	4l	1a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	6 ₈	6 ₇	6 ₉	6 ₁₀
7P		4d	4e	4f	4g	4h	4i	4j	4k	4l	5a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	6 ₇	6 ₈	6 ₉	6 ₁₀
11P		4d	4e	4f	4g	4h	4i	4j	4k	4l	5a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	6 ₈	6 ₇	6 ₉	6 ₁₀
13P		4d	4e	4f	4g	4h	4i	4j	4k	4l	5a	6 ₁	6 ₂	6 ₃	6 ₄	6 ₅	6 ₆	6 ₇	6 ₈	6 ₉	6 ₁₀
X.85		72	-24	24	-24	.	.	-24	-8	.	-7	729	.	.	81	-81	.	27	27	.	.
X.86		72	-24	24	24	.	.	24	-8	.	-7	729	.	.	81	81	.	-27	-27	.	.
X.87		-18	-10	30	14	6	6	-10	-10	6	.	-405	324	81	-45	-45	36	27	27	.	-12
X.88		-18	-10	30	-14	6	6	10	-10	-6	.	-405	324	81	-45	45	36	-27	-27	.	-12
X.89		72	40	24	24	.	.	-8	-8	.	.	525	-96	12	5	65	-40	-7	-7	-42	-16
X.90		72	40	24	-24	.	.	8	-8	.	.	525	-96	12	5	-65	-40	7	7	-42	-16
X.91		-10	-812	232	-128	16	72
X.92		75	-45	11	-3	-5	-5	-11	3	-3	.	.	135	-54	.	.	27	.	.	54	15
X.93		75	-45	11	3	-5	-5	11	3	3	.	.	135	-54	.	.	27	.	.	54	15
X.94		-80	-16	16	-16	.	.	16	16	.	-5	1386	-18	36	-54	66	-54	-42	-42	-18	6
X.95		-80	-16	16	16	.	.	-16	16	.	-5	1386	-18	36	-54	-66	-54	42	42	-18	6
X.96		8	-8	-3220	128	74	-34	32
X.97		-96	.	32	.	32	32	-560	-614	-20	80	.	-46	.	.	34	10
X.98		.	40	.	24	.	.	.	8	.	.	-245	520	16	-35	25	-8	7	7	34	8
X.99		.	40	.	-24	.	.	.	8	.	.	-245	520	16	-35	-25	-8	-7	-7	34	8
X.100		32	.	-32	1960	196	-56	120	.	-132	.	.	-20	-12
X.101		-54	6	18	-6	-18	-18	6	-2	-6	5	729	.	.	-81	-81	.	-27	-27	.	.
X.102		-54	6	18	6	-18	-18	-6	-2	6	5	729	.	.	-81	81	.	27	27	.	.
X.103		-32	32	-1680	-420	84	66	28
X.104		63	15	15	21	-9	-9	21	-1	-3	-2	-729	.	.	-81	-81	.	27	27	.	.
X.105		63	15	15	-21	-9	-9	-21	-1	3	-2	-729	.	.	-81	81	.	-27	-27	.	.
X.106		16	-16	.	.	.	10	672	348	-138	-30	28
X.107		60	-36	-20	-12	-12	-12	-12	12	12	-3	.	-243	.	.	.	81	.	.	.	-27
X.108		60	-36	-20	12	-12	-12	12	12	-12	-3	.	-243	.	.	.	81	.	.	.	-27
X.109		-8	8	720	-252	-90	.	.	.	J	J	-9	36
X.110		-8	8	720	-252	-90	.	.	.	J	J	-9	36
X.111		560	344	128	80	.	8	.	.	20	-40
X.112		-16	.	-16	.	16	16	-16	.	-16	.	420	-30	-48	-20	100	34	28	28	51	2
X.113		-16	-80	.	-16	.	-16	-16	.	-16	.	420	-30	114	-60	.	66	.	.	-30	34
X.114		-16	.	-16	.	16	16	16	.	16	.	420	-30	-48	-20	-100	34	-28	-28	51	2
X.115		-96	.	-32	486	.	.	.	-54	.	.	.	-18
X.116		-96	.	-32	486	.	.	.	-54	.	.	.	-18
X.117		1560	-312	-24	-40	80	32	8	8	12	-8
X.118		32	-32	-1920	-12	-156	42	-12
X.119		1560	-312	-24	-40	-80	32	-8	-8	12	-8
X.120		-8	-512	640	64	.	-128	.	64	64	-8	.
X.121		-8	-512	640	64	.	128	.	-64	-64	-8	.
X.122		-8	512	-640	-64	.	.	.	K	K	8	.
X.123		-8	512	-640	-64	.	.	.	K	K	8	.
X.124		81	9	33	-9	9	9	-33	9	15	3
X.125		81	9	33	9	9	9	33	9	-15	3
X.126		-880	-1096	-16	-16	-40
X.127		54	-30	-18	-30	18	18	6	10	-6	-4	729	.	.	-81	81	.	27	27	.	.
X.128		54	-30	-18	30	18	18	-6	10	6	-4	729	.	.	-81	-81	.	-27	-27	.	.
X.129		-96	.	32	.	-32	-32	-270	108	.	.	.	-54	.	.	54	-30
X.130		9	85	-15	17	-3	-3	-11	-3	9	.	.	-324	81	-90	90	-36	.	.	.	12
X.131		9	85	-15	-17	-3	-3	11	-3	-9	.	.	-324	81	-90	-90	-36	.	.	.	12
X.132		-2	-1232	-224	-80	.	-48	.	.	.	-8	32
X.133		-2	-1232	-224	-80	.	48	.	.	.	-8	32
X.134		-24	40	-8	8	.	.	-24	-8	.	.	405	-162	.	45	-45	18	27	27	.	6
X.135		-24	40	-8	-8	.	.	24	-8	.	.	405	-162	.	45	45	18	-27	-27	.	6
X.136		420	-264	78	-48	24
X.137		-64	-40	.	-24	.	.	.	-8	.	.	-735	-384	48	35	-25	8	-7	-7	-6	-16
X.138		-64	-40	.	24	.	.	.	-8	.	.	-735	-384	48	35	25	8	7	7	-6	-16
X.139		4	-384	192	-96	-64	64	-64	64	64	-24	.
X.140		4	-384	192	-96	64	-64	64	-64	-64	-24	.
X.141		4	-384	192	-96	64	64	64	64	64	-24	.
X.142		4	-384	192	-96	-64	-64	-64	-64	-64	-24	.
X.143		-24	24	.	.	.	-8	-2268	648	162	-24
X.144		-24	.	24	8	.	1	-729	.	.	81	81	.	27	27	.	.
X.145		-24	.	-24	8	.	1	-729	.	.	81	-81	.	-27	-27	.	.
X.146		-16	16	-216	-108	-54	-24
X.147		10	912	480	48	48	32
X.148		-10	2268	648	-24
X.149		16	-16	-1680	120	-132	-42	-8
X.150		8	768	-384	192	48	.

Character table of $A_1(\text{Fi}'_{24})$ (continued)

	2	9	10	10	5	5	5	8	8	8	8	8	7	7	5	5	5	5	5	5	3	8	
3	3	4	3	3	6	5	5	3	3	3	3	3	3	3	4	4	4	4	3	3	3	1	
5		
7		
11		1	.	
13		
	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	7a	8a
2P	3b	3a	3a	3b	3c	3c	3c	3c	3b	3a	3a	3c	3c	3d	3d	3d	3d	3d	3c	3c	3c	7a	4f
3P	2d	2d	2e	2c	2c	2b	2g	2g	2g	2f	2g	2e	2d	2f	2f	2c	2c	2g	2g	2g	2f	7a	8a
5P	611	612	613	614	615	616	617	618	619	620	621	622	623	625	624	627	626	628	629	630	631	7a	8a
7P	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	7a	8a
11P	611	612	613	614	615	616	617	618	619	620	621	622	623	625	624	627	626	628	629	630	631	7a	8a
13P	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	7a	8a
X.85		9	9							-9	9											1	a
X.86		9	9							9	9											1	.
X.87	-12	-21	-21	-18	9	9	9	9	-12	3	3	-3	-3							-3	-3	-2	-6
X.88	-12	-21	-21	18	-9	9	9	9	-12	-3	3	-3	-3							-3	3	-2	6
X.89	-16	-19	-19	20	-16	-22	-4	-4	-8	-15	5	-4	-4	6	6	2	2	2	2	2	2	.	.
X.90	-16	-19	-19	-20	16	-22	-4	-4	8	15	5	-4	-4	-6	-6	-2	-2	2	2	2	2	.	.
X.91	-72	36	-36																			.	.
X.92	15			27			-6	-6	3		-6	-6					6	6				.	.
X.93	15			-27			-6	-6	3		-6	-6					6	6				.	-1
X.94	6	-6	-6	-6	12		-4	-4	2	-6	2			6	6	-6	6	2	-4			.	.
X.95	6	-6	-6	-6	-12		-4	-4	2	6	2			-6	-6	6	6	2	-4			.	.
X.96	-32	28	-28				-6	6				2	-2			6	6		-6	6		.	.
X.97	10	16	16			-28	-4	-4	2		-16	4	4					2	-2	-4		.	.
X.98	8	11	11	16	-2	-8	-8	-8	-8	-7	13	-4	-4	2	2	-2	-2	-2	-2	4	2	.	.
X.99	8	11	11	-16	2	-8	-8	-8	-8	7	13	-4	-4	-2	-2	2	2	-2	-2	-4	-2	.	.
X.100	-12	-24	-24			12	-8	-8	-20		-8							4	4	4		.	.
X.101		9	9							-9	-9											1	-6
X.102		9	9							9	9											1	6
X.103	-28	-16	16				12	-12				4	-4				6	-6				-1	-3
X.104		-9	-9							-9	-9											.	.
X.105		-9	-9							9	-9											-1	3
X.106	-28	32	-32				-6	6				-2	2				6	-6				.	.
X.107	-27			27						9												-4	-4
X.108	-27			-27						9												.	.
X.109	-36	-48	48				-6	6				6	-6	F	F	B	B	-3	3			-2	.
X.110	-36	-48	48				-6	6				6	-6	F	F	B	B	-3	3			-2	.
X.111	-40	-16	16			-28	-16	-16	8		-16	8	8				-4	-4	-4			.	.
X.112	2	-28	-28	-8	10	-2	8	8	2	-12	-4	-4	-3	-3	1	1	-1	-1	2	6		.	.
X.113	34	4	4			-6	2	2	2	2	20	10	10				2	2	2			.	.
X.114	2	-28	-28	8	-10	-2	8	8	2	12	-4	-4	-4	3	3	-1	-1	-1	-1	2	-6	.	.
X.115	-18								18													.	.
X.116	-18								18													.	.
X.117	-8	-8	-8	-28	8	-4	8	8	-16		8	-8	-8			8	8	-4	-4	-4		2	.
X.118	12						12	-12				12	-12				6	-6				2	.
X.119	-8	-8	-8	-28	-4	8	8	8	-16		8	-8	-8			-8	-8	-4	-4	-4		2	.
X.120				16	16											-8	-8					2	.
X.121				-16	-16											8	8					2	.
X.122																A	A					2	.
X.123																A	A					2	.
X.124																						.	3
X.125																						.	-3
X.126	40	16	-16							9	-9		8	-8								-2	-6
X.127		9	9							9	-9											.	.
X.128		9	9							-9	-9											-2	6
X.129	-30					12	12	-6			12	12					6	6				.	.
X.130	12			-18	9	-9	9	9	12	-6	6	-3	-3							3	-3	-3	.
X.131	12			18	-9	-9	9	9	12	6	6	-3	-3							3	3	3	.
X.132	32	-16	-16	-48	-12				-16		8	8	8	8							-4	.	.
X.133	32	-16	-16	48	12				16		8	8	-8	-8							4	.	.
X.134	6	21	21	-18	-18	18			-6	3	-3									-6	6	-1	.
X.135	6	21	21	18	18	18			-6	-3	-3									-6	-6	-1	.
X.136	-24	-12	12				18	-18			-18	18										.	.
X.137	-16	17	17	-16	2	8	-8	-8	-8	15	-5	-4	4	6	6	2	2	-2	-2	4	6	.	.
X.138	-16	17	17	16	-2	8	-8	-8	-8	-15	-5	-4	-4	-6	-6	-2	-2	-2	-2	4	-6	.	.
X.139				-8	-8	8										-8	-8				1	.	
X.140				8	8	-8										8	8				1	.	
X.141				-8	-8	-8										-8	-8				1	.	
X.142				8	8	8										8	8				1	.	
X.143																					.	.	
X.144	24	-12	12				-18	18			-6	6									-1	.	
X.145		-9	-9							9	9											.	.
X.146		-9	-9							-9	9											-1	.
X.147	24						12	-12				-12	12					6	-6			.	.
X.148	-32	16	-16								8	-8										-2	.
X.149	24	12	-12																			.	.
X.150	8	-16	16				-12	12			4	-4						-6	6			.	.

Character table of $A_1(\text{Fi}'_{24})$ (continued)

2	8	8	8	8	8	6	6	4	3	2	5	4	5	5	3	4	2	6	8	5	5	8	8	6	7
3	1	1	1	1	1	1	1	4	4	3	1	1	1	1	1	1	1	3	3	4	4	2	2	3	2
5	2	2	1	1	1	1	.	1
7	1
11
13
	8b	8c	8d	8e	8f	8g	8h	9a	9b	9c	10a	10b	10c	10d	10e	10f	11a	12 ₁	12 ₂	12 ₃	12 ₄	12 ₅	12 ₆	12 ₇	12 ₈
2P	4d	4d	4d	4d	4f	4h	4i	9a	9b	9c	5a	5a	5a	5a	5a	5a	11a	6 ₁	6 ₁₃	6 ₁₀	6 ₂	6 ₁₃	6 ₁₃	6 ₁₀	6 ₁₃
3P	8b	8c	8d	8e	8f	8g	8h	3b	3b	3d	10a	10b	10c	10d	10e	10f	11a	4a	4b	4b	4a	4d	4d	4d	4d
5P	8b	8c	8d	8e	8f	8g	8h	9a	9b	9c	2a	2c	2d	2e	2b	2f	11a	12 ₁	12 ₂	12 ₃	12 ₄	12 ₅	12 ₆	12 ₇	12 ₈
7P	8b	8c	8d	8e	8f	8g	8h	9a	9b	9c	10a	10b	10c	10d	10e	10f	11a	12 ₁	12 ₂	12 ₃	12 ₄	12 ₅	12 ₆	12 ₇	12 ₈
11P	8b	8c	8d	8e	8f	8g	8h	9a	9b	9c	10a	10b	10c	10d	10e	10f	1a	12 ₁	12 ₂	12 ₃	12 ₄	12 ₅	12 ₆	12 ₇	12 ₈
13P	8b	8c	8d	8e	8f	8g	8h	9a	9b	9c	10a	10b	10c	10d	10e	10f	11a	12 ₁	12 ₂	12 ₃	12 ₄	12 ₅	12 ₆	12 ₇	12 ₈
X.85	-4	-4	-4	-4	-7	-7	1	1	1	1	.	-9	-9	.	.	-3	-3	.	-3
X.86	-4	-4	-4	-4	-7	7	1	1	1	-1	.	9	9	.	.	-3	-3	.	-3
X.87	2	2	2	2	2	-2	2	15	3	-6	6	3	3	.	3
X.88	2	-2	2	-2	2	2	2	-15	-3	6	-6	3	3	.	3
X.89	4	4	4	4	.	.	.	3	3	5	1	4	8	-3	-3	.	-3
X.90	4	-4	4	-4	.	.	.	3	3	-5	-1	-4	-8	-3	-3	.	-3
X.91	8	-4	2	10	.	-2	2	-12	12	.	.
X.92	3	-3	3	-3	-1	3	-1	3	3	.	.	3	.
X.93	3	3	3	3	-1	3	-1	-3	-3	.	.	3	.
X.94	-5	-5	-1	-1	1	-1	.	6	-6	-6	-6	-2	-2	10	-2
X.95	-5	5	-1	-1	1	1	.	-6	6	6	6	-2	-2	10	-2
X.96	-2	-2	-2	12	-12	.	.
X.97	-2	-2	-2	-6	.
X.98	4	-4	-4	4	.	.	.	-5	1	1	-5	7	-8	-8	3	3	.	-3
X.99	4	4	-4	-4	.	.	.	-5	1	1	5	-7	8	8	3	3	.	-3
X.100	-8	4	-2	8	8	-4	8
X.101	-2	2	2	-2	-2	5	5	1	1	-1	1	1	9	9	.	.	-3	-3	.	3
X.102	-2	-2	2	2	-2	-2	5	-5	1	1	-1	-1	1	-9	-9	.	.	-3	-3	.	3
X.103	-6	2
X.104	-5	1	-5	1	-1	-3	-1	.	.	.	-2	2	2	2	2	-2	.	-9	-9	.	.	3	3	.	3
X.105	-5	-1	-5	-1	-1	3	-1	.	.	.	-2	-2	2	2	2	2	.	9	9	.	.	3	3	.	3
X.106	-6	-6	.	-10	2	-2
X.107	4	-3	3	1	1	-3	-1	.	.	.	-9	-9	.	.	-3	.
X.108	4	-3	-3	1	1	-3	1	.	.	.	9	9	.	.	-3	.
X.109
X.110
X.111	2	2	2	-1
X.112	3	4	-8	.	-4	-4	2	-4
X.113	-6	-4	-4	2	-4
X.114	3	-4	8	.	-4	-4	2	-4
X.115	1	-6	.
X.116	1	-6	.
X.117	-3	-16	-4
X.118	6
X.119	-3	16	4
X.120	4	-2	-2	-8	8
X.121	4	-2	-2	-8	-8
X.122	4	-2	-2	8
X.123	4	-2	-2	8
X.124	-3	3	-3	3	-3	3	1	.	.	.	3	3	-1	-1	-3	-1	1
X.125	-3	-3	-3	-3	-3	-3	1	.	.	.	3	-3	-1	-1	-3	1	1
X.126
X.127	-2	-2	2	2	2	2	-4	4	-9	-9	.	.	-3	-3	.	3
X.128	-2	2	2	-2	2	2	-4	-4	9	9	.	.	-3	-3	.	3
X.129	-1	-6	.
X.130	1	-3	-3	-7	1	-1	-1	-6	6	.	.	6	.
X.131	1	3	-3	7	1	1	-1	6	-6	.	.	6	.
X.132	-2	4	1	-2	2	-2	-2	.	2
X.133	-2	4	1	-2	2	-2	-2	.	-2
X.134	4	-4	4	-4	15	3	6	-6	-3	-3	-6	-3
X.135	4	4	4	4	-15	-3	-6	6	-3	-3	-6	-3
X.136	6	-12	12	.	.
X.137	4	-4	-4	4	.	.	.	3	3	5	-11	-8	8	5	5	8	-1
X.138	4	4	-4	-4	.	.	.	3	3	-5	11	8	-8	5	5	8	-1
X.139	-3	.	.	4	-4	.	-2	.	-1	-16	.	.	.	8
X.140	-3	.	.	4	4	.	2	.	-1	-16	.	.	.	8
X.141	-3	.	.	4	-4	.	2	.	-1	-16	.	.	.	8
X.142	-3	.	.	4	4	.	-2	.	-1	-16	.	.	.	8
X.143	8	-12	12	.	.
X.144	-4	4	4	-4	1	1	1	1	1	1	1	-9	-9	.	.	3	3	.	-3
X.145	-4	-4	4	4	1	-1	1	1	1	-1	1	9	9	.	.	3	3	.	-3
X.146
X.147	6	6	.	-10	.	2	-2
X.148	10	.	-2	2	.	.	2	12	-12	.	.
X.149	-6
X.150	-6	.	.	-8	-2

Character table of $A_1(\text{Fi}'_{24})$ (continued)

2	7	5	8	6	6	6	6	4	5	5	5	6	6	4	4	4	5	4	4	
3	2	3	1	2	2	2	2	3	2	2	2	1	1	2	2	2	1	1	1	
5	
7	
11	
13	
	12 ₁	12 ₂	12 ₁₁	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	12 ₁₇	12 ₁₈	12 ₁₉	12 ₂₀	12 ₂₁	12 ₂₂	12 ₂₃	12 ₂₄	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈
2P	6 ₁₃	6 ₂₂	6 ₁₃	6 ₁₂	6 ₁₃	6 ₁₂	6 ₁₂	6 ₃	6 ₁₉	6 ₁₉	6 ₁₁	6 ₂₀	6 ₁₂	6 ₂₃	6 ₂₃	6 ₂₄	6 ₂₅	6 ₂₂	6 ₂₇	6 ₂₈
3P	4 _b	4 _b	4 _c	4 _g	4 _h	4 _e	4 _i	4 _a	4 _i	4 _h	4 _g	4 _f	4 _k	4 _g	4 _e	4 _i	4 _h	4 _j	4 _i	4 _i
5P	12 ₉	12 ₁₀	12 ₁₁	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	12 ₁₇	12 ₁₈	12 ₁₉	12 ₂₀	12 ₂₁	12 ₂₂	12 ₂₃	12 ₂₄	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈
7P	12 ₉	12 ₁₀	12 ₁₁	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	12 ₁₇	12 ₁₈	12 ₁₉	12 ₂₀	12 ₂₁	12 ₂₂	12 ₂₃	12 ₂₄	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈
11P	12 ₉	12 ₁₀	12 ₁₁	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	12 ₁₇	12 ₁₈	12 ₁₉	12 ₂₀	12 ₂₁	12 ₂₂	12 ₂₃	12 ₂₄	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈
13P	12 ₉	12 ₁₀	12 ₁₁	12 ₁₂	12 ₁₃	12 ₁₄	12 ₁₅	12 ₁₆	12 ₁₇	12 ₁₈	12 ₁₉	12 ₂₀	12 ₂₁	12 ₂₂	12 ₂₃	12 ₂₄	12 ₂₅	12 ₂₆	12 ₂₇	12 ₂₈
X.85	3	-1	3	.	-3	1
X.86	-3	.	1	-3	.	-3	1
X.87	3	3	3	-1	-3	-1	-3	-3	.	.	.	2	-1	-1	-1	.	.	-1	.	.
X.88	-3	-3	-3	1	-3	-1	-3	3	.	.	-2	.	-1	1	-1	.	.	1	.	.
X.89	1	-2	1	-3	.	1	.	2	1	.	-2	.	.	-2	.	.
X.90	-1	2	-1	3	.	1	.	-2	1	.	-2	.	.	2	.	.
X.91
X.92	.	6	.	.	-2	.	-2	-6	1	1	3	-1	.	.	.	-2	-2	-2	.	.
X.93	.	6	.	.	-2	.	-2	-6	1	1	-3	-1	.	.	.	-2	-2	2	.	.
X.94	6	-6	2	2	.	2	2	-2	-2	2	2	.	.	-2	.	.
X.95	-6	6	-2	-2	-2	-2	-2	-2	2
X.96	-2	.	.	4	-4	-2
X.97	-4	.	-4	.	2	2	.	.	2	.	.	-2	2	.	.	.
X.98	1	4	-9	-3	.	1	.	-2	-1	.	-2
X.99	-1	-4	9	3	.	1	.	2	-1	.	-2
X.100	4
X.101	3	.	1	3	.	3	1
X.102	-3	.	-1	-3	.	3	1
X.103	4	.	-4	.	2	-2	2	-2	.	.	.
X.104	3	.	-1	3	.	3	-1
X.105	-3	.	1	-3	.	3	-1
X.106	-2	.	2	.	2	-2	.	3	.	.	.	2	-2	.	.	.
X.107	3	3	.	3	1
X.108	3	3	-3	1
X.109	-2	.	2	.	2	-2	-1	1	.	C	C
X.110	-2	.	2	.	2	-2	-1	1	.	C	C
X.111
X.112	4	-2	4	.	4	.	4	.	-2	-2	.	2	.	.	.	1	1	2	-1	-1
X.113	2	4	2	.	2	2	.	2	-4	.	.	2	2	.	.	.
X.114	-4	2	-4	.	4	.	4	.	-2	-2	.	2	.	.	-2	1	1	-2	1	1
X.115	-2
X.116	-2
X.117	8	8
X.118	-4	.	4	.	-2	2	-2	2	.	.	.
X.119	-8	-8
X.120
X.121
X.122
X.123
X.124
X.125
X.126
X.127	-3	.	-1	-3	.	3	1
X.128	3	.	1	3	.	3	1
X.129	4	.	4	.	-2	-2	.	2	.	.	.	-2	-2	.	.	.
X.130	-6	-3	.	2	-3	-2	-3	3	.	.	.	2	.	-1	1	.	.	1	.	.
X.131	6	3	.	-2	-3	-2	-3	-3	.	.	-2	.	.	1	1	.	.	-1	.	.
X.132
X.133
X.134	3	.	3	-1	.	1	2	-2	.	2	-2
X.135	-3	.	-3	1	.	1	-2	-2	.	1	-2	-2
X.136	6	.	-6
X.137	-5	4	-3	3	.	-1	.	2	1	.	2
X.138	5	-4	3	-3	.	-1	.	-2	1	.	2
X.139	-4
X.140	-4
X.141	4
X.142
X.143	-6	.	6
X.144	-3	.	-1	-3	.	-3	-1
X.145	3	.	1	3	.	-3	-1
X.146	-4	.	4	.	4	-4	-2	2	.	.	.
X.147
X.148
X.149	4	.	-4	.	-4	4	2	-2	.	.	.
X.150

[illegible]

Character table of $A_1(\text{Fi}'_{24})$ (continued)

	2	2	2	5	5	5	5	4	4	4	4	1	2	3	2	2	3	3	2	2	2	2	2	2
	3	5	7	11	13																			
	22a	22b	22c	24a	24b	24c	24d	24e	24f	24g	24h	26a	28a	30a	30b	30c	36a	36b	36c	42a	42b	42c	60a	60a
2P	11a	11a	11a	12 ₅	12 ₆	12 ₆	12 ₅	12 ₁₃	12 ₅₀	12 ₁₃	12 ₂₀	13a	14a	15a	15a	18g	18g	18b	21a	21a	21a	30a	30a	30a
3P	22a	22b	22c	8b	8c	8d	8e	8g	8a	8g	8a	26a	28a	10a	10e	10b	12 ₃	12 ₃	12 ₄	14b	14a	14b	20a	20a
5P	22a	22b	22c	24a	24b	24c	24d	24g	24h	24e	24f	26a	28a	6 ₁	6 ₄	6 ₅	36a	36b	36c	42c	42b	42a	12 ₁	12 ₁
7P	22c	22b	22a	24a	24b	24c	24d	24g	24h	24e	24f	26a	4a	30a	30b	30c	36a	36b	36c	6 ₇	6 ₁	6 ₈	60a	60a
11P	2b	2a	2b	24a	24b	24c	24d	24e	24f	24g	24h	26a	28a	30a	30b	30c	36a	36b	36c	42c	42b	42a	60a	60a
13P	22c	22b	22a	24a	24b	24c	24d	24e	24f	24g	24h	2a	28a	30a	30b	30c	36a	36b	36c	42a	42b	42c	60a	60a
X.85	.	.	.	-1	1	-1	1	-1	-1	-1	1	-1	.	.	.	-1	1	-1	1	1
X.86	.	.	.	-1	-1	-1	-1	-1	1	-1	1	1	1	1	1	1	1	1	1	1
X.87	.	.	.	-1	-1	-1	-1	1	.	1	-1	1	-1	1	1
X.88	.	.	.	-1	1	-1	1	-1	.	-1	1	1	1	1	1
X.89	.	.	.	1	1	1	1	1	1	1	1	1
X.90	.	.	.	1	-1	1	-1	-1	-1	1	1	1
X.91
X.92	1	.	1
X.93	-1	.	-1
X.94
X.95	1
X.96	-1
X.97
X.98	.	.	.	1	-1	-1	1	1	1	1	1	1
X.99	.	.	.	1	1	-1	-1	-1	-1	-1	1	1
X.100
X.101	-1	1	-1	1	-1	-1	1	-1	-1	-1	-1	1	1	1	1	-1
X.102	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	1	1
X.103	.	-2
X.104	.	.	.	1	1	1	1	-1	-1	-1	1	1
X.105	.	.	.	1	-1	1	-1	-1	1	-1	1	1	-1	1	1	-1
X.106
X.107	-1	.	-1
X.108	1	.	1
X.109	C	-1	C	.	.
X.110	C	-1	C	.	.
X.111	-1	-1	-1
X.112	1	1	.	.	.
X.113	-1	-1	.	.	.
X.114
X.115	1	1	1	I	.	-I	-1
X.116	1	1	1	-I	.	I	-1
X.117	-1	-1	1	-1	1
X.118	-2
X.119	1	1	-1	-1	1
X.120	1	.	-2	.	2	.	.	.	1	-1	1	1	1
X.121	1	.	-2	.	-2	.	.	.	-1	-1	-1	1	1
X.122	-1	.	2	C	1	C	.	.
X.123	-1	.	2	C	1	C	.	.
X.124	1	1	1
X.125	1	1	1
X.126
X.127	.	.	.	1	1	-1	-1	-1	-1	1	.	.	.	-1	1	-1	1	1
X.128	.	.	.	1	-1	-1	1	-1	-1	-1	.	.	.	1	1	1	1	-1
X.129	1	-1	1
X.130	.	.	.	-2	.	.	2	-1	.	-1
X.131	.	.	.	-2	.	.	-2	1	.	1
X.132
X.133	-2	-2	.	.	.
X.134	.	.	.	1	-1	1	-1	1	-1	-1	-1	1	1
X.135	.	.	.	1	1	1	1	-1	1	-1	1	1	1
X.136
X.137	.	.	.	1	-1	-1	1	1	1	-1	1	1
X.138	.	.	.	1	1	-1	-1	-1	-1	1	1	1
X.139	-1	-1	-1	1	1	1	-1	-1	.	.	.	-1	1	1	1	-1
X.140	1	-1	1	1	1	-1	-1	-1	-1	1	1	-1
X.141	1	-1	1	-1	1	-1	-1	1	1	1	1	1
X.142	-1	-1	-1	-1	1	1	1	1	-1	1	-1	1
X.143
X.144	-1	1	-1	-1	1	1	-1	-1	1	1	1	-1	-1	-1	1	1
X.145	-1	1	-1	-1	-1	1	1	1	1	1	-1	1	-1	1	-1	1
X.146	1
X.147
X.148	.	-2
X.149
X.150	.	2

where $A = 16\zeta(3) + 8$, $B = 6\zeta(3) + 3$, $C = -2\zeta(3) - 1$, $D = 4\zeta(3) + 2$, $E = -2\zeta(11)^9 - 2\zeta(11)^5 - 2\zeta(11)^4 - 2\zeta(11)^3 - 2\zeta(11) - 1$, $F = 18\zeta(3) + 9$, $G = 112\zeta(3) + 56$, $H = 12\zeta(3) + 6$, $I = 4\zeta(12)_4\zeta(12)_3 + 2\zeta(12)_4$, $J = 96\zeta(3) + 48$, $K = -128\zeta(3) - 64$.

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DEPARTMENT OF MATHEMATICS, YALE UNIVERSITY, NEW HAVEN, CT. 06511, USA

DEPARTMENT OF MATHEMATICS, CORNELL UNIVERSITY, ITHACA, N.Y. 14853, USA